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COMPLETE MONOTONICITY OF A FUNCTION INVOLVING THE RATIO OF GAMMA FUNCTIONS AND APPLICATIONS

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ABSTRACT. In the paper, necessary and sufficient conditions are presented for a function involving a ratio of gamma functions to be logarithmically completely monotonic. This extends and generalizes the main result of Guo and Qi [Taiwanese J. Math. 7 (2003), no. 2, 239–247] and others. As applications, several inequalities involving the volume of the unit ball in $\mathbb{R}^n$ are derived, which refine, generalize and extend some known inequalities.

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