



THE REFINED SOBOLEV SCALE, INTERPOLATION, AND ELLIPTIC PROBLEMS

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ABSTRACT. The paper gives a detailed survey of recent results on elliptic problems in Hilbert spaces of generalized smoothness. The latter are the isotropic Hörmander spaces $H^{s,\varphi} := B_{2,\mu}$, with $\mu(\xi) = \langle \xi \rangle^s \varphi(\langle \xi \rangle)$ for $\xi \in \mathbb{R}^n$. They are parametrized by both the real number s and the positive function φ varying slowly at $+\infty$ in the Karamata sense. These spaces form the refined Sobolev scale, which is much finer than the Sobolev scale $\{H^s\} \equiv \{H^{s,1}\}$ and is closed with respect to the interpolation with a function parameter. The Fredholm property of elliptic operators and elliptic boundary-value problems is preserved for this new scale. Theorems of various type about a solvability of elliptic problems are given. A local refined smoothness is investigated for solutions to elliptic equations. New sufficient conditions for the solutions to have continuous derivatives are found. Some applications to the spectral theory of elliptic operators are given.

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