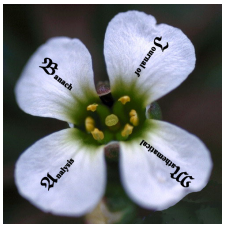

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UPPER BEURLING DENSITY OF SYSTEMS FORMED BY TRANSLATES OF FINITE SETS OF ELEMENTS IN $L^p(\mathbb{R}^d)$

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Abstract. In this paper, we prove that if a finite disjoint union of translates $\bigcup_{k=1}^{n}\{f_k(x-\gamma)\}_{\gamma\in\Gamma_k}$ in $L^p(\mathbb{R}^d)$ $(1<p<\infty)$ is a p'-Bessel sequence for some $1<p'<\infty$, then the disjoint union $\Gamma=\bigcup_{k=1}^{n}\Gamma_k$ has finite upper Beurling density, and that if $\bigcup_{k=1}^{n}\{f_k(x-\gamma)\}_{\gamma\in\Gamma_k}$ is a (C_q)-system with $1/p+1/q=1$, then Γ has infinite upper Beurling density. Thus, no finite disjoint union of translates in $L^p(\mathbb{R}^d)$ can form a p'-Bessel (C_q)-system for any $1<p'<\infty$. Furthermore, by using techniques from the geometry of Banach spaces, we obtain that, for $1<p\leq 2$, no finite disjoint union of translates in $L^p(\mathbb{R}^d)$ can form an unconditional basis.

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