



Banach J. Math. Anal. 6 (2012), no. 2, 132–158

BANACH **J**OURNAL OF **M**ATHEMATICAL **A**NALYSIS

ISSN: 1735-8787 (electronic)

www.emis.de/journals/BJMA/

BOUNDS FOR THE RATIO OF TWO GAMMA FUNCTIONS—FROM WENDEL’S AND RELATED INEQUALITIES TO LOGARITHMICALLY COMPLETELY MONOTONIC FUNCTIONS

FENG QI¹ AND QIU-MING LUO^{2*}

Communicated by S. S. Dragomir

ABSTRACT. In the survey paper, along one of several main lines of bounding the ratio of two gamma functions, the authors retrospect and analyse Wendel’s double inequality, Kazarinoff’s refinement of Wallis’ formula, Watson’s monotonicity, Gautschi’s double inequality, Kershaw’s first double inequality, and the (logarithmically) complete monotonicity results of functions involving ratios of two gamma or q -gamma functions obtained by Bustoz, Ismail, Lorch, Muldoon, and other mathematicians.

¹COLLEGE OF MATHEMATICS, INNER MONGOLIA UNIVERSITY FOR NATIONALITIES, TONGLIAO CITY, INNER MONGOLIA AUTONOMOUS REGION, 028043, CHINA.

E-mail address: qifeng618@gmail.com, qifeng618@hotmail.com, qifeng618@qq.com

URL: <http://qifeng618.wordpress.com>

²DEPARTMENT OF MATHEMATICS, CHONGQING NORMAL UNIVERSITY, CHONGQING CITY, 401331, CHINA.

E-mail address: luomath@126.com, luomath2007@163.com

Date: Received: 2 April 2012; Accepted: 1 May 2012

*Corresponding author.

2010 *Mathematics Subject Classification.* Primary 33B15; Secondary 26A48, 26A51, 26D20, 33D05, 44A10, 46-02, 46F12, 46T20.

Key words and phrases. Bound; ratio of two gamma functions; logarithmically completely monotonic function; gamma function; q -gamma function.