



Banach J. Math. Anal. 7 (2013), no. 1, 132–141

BANACH **J**OURNAL OF **M**ATHEMATICAL **A**NALYSIS

ISSN: 1735-8787 (electronic)

www.emis.de/journals/BJMA/

A COMBINATORIAL APPROACH TO MUSIELAK–ORLICZ SPACES

JOSCHA PROCHNO

Communicated by D. E. Alspach

ABSTRACT. In this paper we show that, using combinatorial inequalities and Matrix-Averages, we can generate Musielak–Orlicz spaces, *i.e.*, we prove that $\text{Ave}_\pi \max_{1 \leq i \leq n} |x_i y_{i\pi(i)}| \sim \|x\|_{\Sigma M_i}$, where the Orlicz functions M_1, \dots, M_n depend on the matrix $(y_{ij})_{i,j=1}^n$. We also provide an approximation result for Musielak–Orlicz norms which already in the case of Orlicz spaces turned out to be very useful.

DEPARTMENT OF MATHEMATICAL AND STATISTICAL SCIENCES, UNIVERSITY OF ALBERTA,
505 CENTRAL ACADEMIC BUILDING, EDMONTON T6G 2G1, ALBERTA, CANADA.

E-mail address: prochno@ualberta.ca

Date: Received: 9 April 2012; Revised: 8 July 2012; Accepted: 22 August 2012.

2010 Mathematics Subject Classification. Primary 39B82; Secondary 44B20, 46C05.

Key words and phrases. Orlicz space, Musielak–Orlicz space, combinatorial inequality.