



ON LINEAR FUNCTIONAL EQUATIONS AND COMPLETENESS OF NORMED SPACES

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Communicated by Z. Páles

ABSTRACT. The aim of this note is to give a type of characterization of Banach spaces in terms of the stability of functional equations. More precisely, we prove that a normed space X is complete if there exists a functional equation of the type

$$\sum_{i=1}^n a_i f(\varphi_i(x_1, \dots, x_k)) = 0 \quad (x_1, \dots, x_k \in D)$$

with given real numbers $a_1, \dots, a_n$, given mappings $\varphi_1, \dots, \varphi_n: D^k \rightarrow D$ and unknown function $f: D \rightarrow X$, which has a Hyers–Ulam stability property on an infinite subset D of the integers.

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Date: Received: 3 July 2012; Accepted: 31 October 2012.

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2010 *Mathematics Subject Classification.* Primary 46B99; Secondary 39B82.

Key words and phrases. Hyers–Ulam stability, normed space, completeness, Banach space.