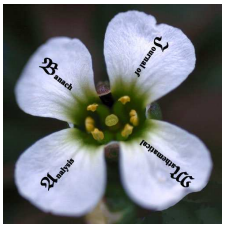

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COMPREHENSIVE SURVEY ON AN ORDER PRESERVING OPERATOR INEQUALITY

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Dedicated to Professor Masatoshi Fujii and Professor Eizaburo Kamei on their retirements with respect and affection.

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Abstract. In 1987, we established an operator inequality as follows; $A \geq B \geq 0 \Longrightarrow (A^{\frac{r}{2}}A^pA^{\frac{r}{2}})^{\frac{1}{q}} \geq (A^{\frac{r}{2}}B^pA^{\frac{r}{2}})^{\frac{1}{q}}$ holds for (*) $p \geq 0$, $q \geq 1$, $r \geq 0$ with $(1+r)q \geq p+r$. It is an extension of Löwner-Heinz inequality. The purpose of this paper is to explain geometrical background of the domain by (*), and to give brief survey of recent results of its applications.

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