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GEOMETRY OF THE LEFT ACTION OF THE p -SCHATTEN GROUPS

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ABSTRACT. Let $\mathcal{H}$ be an infinite dimensional Hilbert space, $\mathcal{B}_p(\mathcal{H})$ the p -Schatten class of $\mathcal{H}$ and $U_p(\mathcal{H})$ be the Banach-Lie group of unitary operators which are p -Schatten perturbations of the identity. Let A be a bounded self-adjoint operator in $\mathcal{H}$. We show that

$$\mathcal{O}_A := \{UA : U \in U_p(\mathcal{H})\}$$

is a smooth submanifold of the affine space $A + \mathcal{B}_p(\mathcal{H})$ if only if A has closed range. Furthermore, it is a homogeneous reductive space of $U_p(\mathcal{H})$. We introduce two metrics: one via the ambient Finsler metric induced as a submanifold of $A + \mathcal{B}_p(\mathcal{H})$, the other, by means of the quotient Finsler metric provided by the homogeneous space structure. We show that $\mathcal{O}_A$ is a complete metric space with the rectifiable distance of these metrics.

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