



MATRIX INEQUALITIES RELATED TO HÖLDER INEQUALITY

HUSSIEN ALBADAWI

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ABSTRACT. Matrix inequalities of Hölder type are obtained. Among other inequalities, it is shown that if $2 \leq p, q < \infty$ and $r > 1$ with $1/p + 1/q = 1 - 1/r$, then for any $A_i, B_i \in M_n(\mathbb{C})$ and $\alpha_i \in [0, 1]$ ($i = 1, 2, \dots, m$) with $\sum_{i=1}^m \alpha_i = 1$, we have

$$\left| \sum_{i=1}^m \alpha_i^{1/r} B_i A_i \right| \leq \left(\sum_{i=1}^m |A_i|^p \right)^{1/p}$$

whenever $\sum_{i=1}^m |B_i^*|^q \leq I$. Related unitarily invariant norm inequalities are also presented.

PREPARATORY YEAR DEANSHIP, KING FAISAL UNIVERSITY, AHSAA, SAUDI ARABIA.
E-mail address: albadawi1@gmail.com

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