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## REFINEMENTS AND REVERSES OF MEANS INEQUALITIES FOR HILBERT SPACE OPERATORS

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ABSTRACT. In this paper we derive some improvements of means inequalities for Hilbert space operators. More precisely, we obtain refinements and reverses of the arithmetic-geometric operator mean inequality. As an application, we also deduce an improved variant for the refined arithmetic–Heinz mean inequality. We also present some eigenvalue inequalities for differences of certain operator means.

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