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PSEUDO ASYMPTOTIC SOLUTIONS OF FRACTIONAL ORDER SEMILINEAR EQUATIONS

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ABSTRACT. Using a generalization of the semigroup theory of linear operators, we prove existence and uniqueness of mild solutions for the semilinear fractional order differential equation

$$D_t^{\alpha+1}u(t) + \mu D_t^\beta u(t) - Au(t) = f(t, u(t)), \quad t > 0, \quad 0 < \alpha \leq \beta \leq 1, \mu \geq 0,$$

with the property that the solution can be written as $u = f + h$ where f belongs to the space of periodic (resp. almost periodic, compact almost automorphic, almost automorphic) functions and h belongs to the space $P_0(\mathbb{R}_+, X) := \{\phi \in BC(\mathbb{R}_+, X) : \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \|\phi(s)\| ds = 0\}$. Moreover, this decomposition is unique.

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