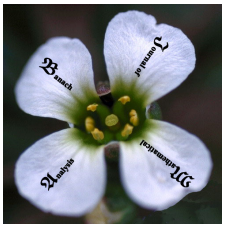

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NONCOMMUTATIVE SPECTRAL SYNTHESIS FOR THE INVOLUTIVE BANACH ALGEBRA ASSOCIATED WITH A TOPOLOGICAL DYNAMICAL SYSTEM

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Abstract. If $\Sigma = (X, \sigma)$ is a topological dynamical system, where X is a compact Hausdorff space and σ is a homeomorphism of X, then a crossed product involutive Banach algebra $\ell^1(\Sigma)$ is naturally associated with these data. If X consists of one point, then $\ell^1(\Sigma)$ is the group algebra of the integers, hence the general $\ell^1(\Sigma)$ could be regarded as a noncommutative ℓ^1-algebra. In this paper, we study spectral synthesis for the closed ideals of $\ell^1(\Sigma)$ in two versions, one modeled after $C(X)$, and one modeled after $\ell^1(\mathbb{Z})$. We identify the closed ideals which are equal to (what is the analogue of) the kernel of their hull, and determine when this holds for all closed ideals, i.e., when spectral synthesis holds. In both models, this is the case precisely when Σ is free.

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