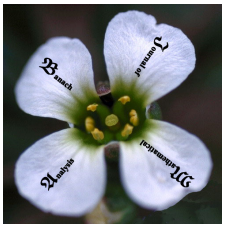

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SOME OPTIMIZATION PROBLEMS ON RANKS AND INERTIAS OF MATRIX-VALUED FUNCTIONS SUBJECT TO LINEAR MATRIX EQUATION RESTRICTIONS

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Abstract. Matrix rank and inertia optimization problems are a class of discontinuous optimization problems in which the decision variables are matrices running over certain matrix sets, while the ranks and inertias of the variable matrices are taken as integer-valued objective functions. In this paper, we establish a group of explicit formulas for calculating the maximal and minimal values of the rank and inertia objective functions of the Hermitian matrix-valued function $A_1 - B_1XB_1^*$ subject to the common Hermitian solution of a pair of consistent matrix equations $B_2XB_2^* = A_2$ and $B_3XB_3^* = A_3$, and Hermitian solution of the consistent matrix equation $B_4X = A_4$, respectively. Many consequences are obtained, in particular, necessary and sufficient conditions are established for the triple matrix equations $B_1XB_1^* = A_1$, $B_2XB_2^* = A_2$ and $B_3XB_3^* = A_3$ to have a common Hermitian solution, as well as necessary and sufficient conditions for the two matrix equations $B_1XB_1^* = A_1$ and $B_4X = A_4$ to have a common Hermitian solution.

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