



## BOUNDEDNESS OF INTRINSIC LITTLEWOOD–PALEY FUNCTIONS ON MUSIELAK–ORLICZ MORREY AND CAMPANATO SPACES

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**ABSTRACT.** Let  $\varphi : \mathbb{R}^n \times [0, \infty) \rightarrow [0, \infty)$  be such that  $\varphi(x, \cdot)$  is nondecreasing,  $\varphi(x, 0) = 0$ ,  $\varphi(x, t) > 0$  when  $t > 0$ ,  $\lim_{t \rightarrow \infty} \varphi(x, t) = \infty$  and  $\varphi(\cdot, t)$  is a Muckenhoupt  $A_\infty(\mathbb{R}^n)$  weight uniformly in  $t$ . Let  $\phi : [0, \infty) \rightarrow [0, \infty)$  be nondecreasing. In this article, the authors introduce the Musielak–Orlicz Morrey space  $\mathcal{M}^{\varphi, \phi}(\mathbb{R}^n)$  and obtain the boundedness on  $\mathcal{M}^{\varphi, \phi}(\mathbb{R}^n)$  of the intrinsic Lusin area function  $S_\alpha$ , the intrinsic  $g$ -function  $g_\alpha$ , the intrinsic  $g_\lambda^*$ -function  $g_{\lambda, \alpha}^*$  and their commutators with  $\text{BMO}(\mathbb{R}^n)$  functions, where  $\alpha \in (0, 1]$ ,  $\lambda \in (\min\{\max\{3, p_1\}, 3 + 2\alpha/n\}, \infty)$  and  $p_1$  denotes the uniformly upper type index of  $\varphi$ . Let  $\Phi : [0, \infty) \rightarrow [0, \infty)$  be nondecreasing,  $\Phi(0) = 0$ ,  $\Phi(t) > 0$  when  $t > 0$ , and  $\lim_{t \rightarrow \infty} \Phi(t) = \infty$ ,  $w \in A_\infty(\mathbb{R}^n)$  and  $\phi : (0, \infty) \rightarrow (0, \infty)$  be nonincreasing. The authors also introduce the weighted Orlicz–Morrey space  $M_w^{\Phi, \phi}(\mathbb{R}^n)$  and obtain the boundedness on  $M_w^{\Phi, \phi}(\mathbb{R}^n)$  of the aforementioned intrinsic Littlewood–Paley functions and their commutators with  $\text{BMO}(\mathbb{R}^n)$  functions. Finally, for  $q \in [1, \infty)$ , the boundedness of the aforementioned intrinsic Littlewood–Paley functions on the Musielak–Orlicz Campanato space  $\mathcal{L}^{\varphi, q}(\mathbb{R}^n)$  is also established.

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