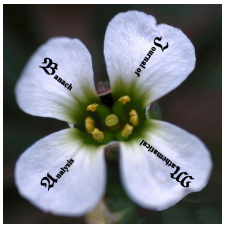

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# ON THE ESSENTIAL SPECTRUM OF THE SUM OF SELF-ADJOINT OPERATORS AND THE CLOSEDNESS OF THE SUM OF OPERATOR RANGES

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Abstract. Let $\mathcal{H}$ be a complex Hilbert space, and $A_1, \dots, A_N$ be bounded self-adjoint operators in $\mathcal{H}$ such that $A_iA_j$ is compact for any $i \neq j$. It is well-known that $\sigma_e(\sum_{i=1}^N A_i) \setminus \{0\} = (\cup_{i=1}^N \sigma_e(A_i)) \setminus \{0\}$, where $\sigma_e(B)$ stands for the essential spectrum of a bounded self-adjoint operator $B$.

In this paper we get necessary and sufficient conditions for $0 \in \sigma_e(\sum_{i=1}^N A_i)$. This conditions are formulated in terms of the projection valued spectral measures of $A_i$, $i = 1, \dots, N$. Using this result, we obtain necessary and sufficient conditions for the sum of ranges of $A_i$, $i = 1, \dots, N$ to be closed.

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