



Banach J. Math. Anal. 8 (2014), no. 2, 118–123

BANACH **J**OURNAL OF **M**ATHEMATICAL **A**NALYSIS

ISSN: 1735-8787 (electronic)

www.emis.de/journals/BJMA/

A CHARACTERIZATION OF CONVEX FUNCTIONS AND ITS APPLICATION TO OPERATOR MONOTONE FUNCTIONS

MASATOSHI FUJII^{1*}, YOUNG OK KIM² AND RITSUO NAKAMOTO³

Communicated by M. S. Moslehian

ABSTRACT. We give a characterization of convex functions in terms of difference among values of a function. As an application, we propose an estimation of operator monotone functions: If $A > B \geq 0$ and f is operator monotone on $(0, \infty)$, then $f(A) - f(B) \geq f(\|B\| + \epsilon) - f(\|B\|) > 0$, where $\epsilon = \|(A - B)^{-1}\|^{-1}$. Moreover it gives a simple proof to Furuta's theorem: If $\log A > \log B$ for $A, B > 0$ and f is operator monotone on $(0, \infty)$, then there exists a $\beta > 0$ such that $f(A^\alpha) > f(B^\alpha)$ for all $0 < \alpha \leq \beta$.

¹ DEPARTMENT OF MATHEMATICS, OSAKA KYOIKU UNIVERSITY, KASHIWARA, OSAKA 582-8582, JAPAN.

E-mail address: mfujii@cc.osaka-kyoiku.ac.jp

² DEPARTMENT OF MATHEMATICS, SUWON UNIVERSITY, BONGDAMOU, WHASUNGS, KYUNGKIDO 445-743, KOREA.

E-mail address: evergreen1317@gmail.com

³ 3-4-13, DAIHARA-CHO, HITACHI 316-0021, JAPAN.

E-mail address: r-naka@net1.jway.ne.jp

Date: Received: Sep. 15, 2013; Accepted: Oct. 7, 2013.

* Corresponding author.

2010 *Mathematics Subject Classification.* Primary 47A63; Secondary 47B10, 47BA30.

Key words and phrases. Convex function, operator monotone function, Lówner–Heinz inequality, chaotic order.