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SOME PROPERTIES OF $m-$PROJECTIVE CURVATURE TENSOR IN KENMOTSU MANIFOLDS

(COMMUNICATED BY PROFESSOR U. C. DE)

S. K. CHAUBEY*, SHASHI PRAKASH** AND R. NIVAS***

Abstract. In this paper, some properties of $m-$projective curvature tensor in Kenmotsu manifolds are studied.

1. Introduction

The study of odd dimensional manifolds with contact and almost contact structures was initiated by Boothby and Wong [1] in 1958 rather from topological point of view. Sasaki and Hatakeyama [2] re-investigated them using tensor calculus in 1961. In 1972, K. Kenmotsu studied a class of almost contact metric manifolds and call them Kenmotsu manifold [3]. He proved that if a Kenmotsu manifold satisfies the condition $R(X,Y).R = 0$, then the manifold is of negative curvature -1, where R is the Riemannian curvature tensor of type $(1,3)$ and $R(X,Y)$ denotes the derivation of the tensor algebra at each point of the tangent space. Recently first author with Ojha [4] studied the properties of the $m-$projective curvature tensor in Riemannian and Kenmotsu manifolds. They proved that an $n-$dimensional Kenmotsu manifold M_n is $m-$projectively flat if and only if it is either locally isometric to the hyperbolic space $H^n(-1)$ or M_n has constant scalar curvature $-n(n-1)$. They also shown that the $m-$projective curvature tensor in an η-Einstein Kenmotsu manifold M_n is irrotational if and only if it is locally isometric to the hyperbolic space $H^n(-1)$. The properties of Kenmotsu manifolds have been studied by several authors such as De, Yildiz and Yaliniz [5], De and Pathak [6], Jun, De and Pathak [7], Sinha and Srivastava [8], De [9], Bhattacharya [16], Yildiz and De [17] and many others.

In 1971, Pokhariyal and Mishra [10] defined a tensor field W^* on a Riemannian manifold as

$$W^*(X,Y)Z = R(X,Y)Z - \frac{1}{4m}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \quad (1)$$

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so that

$$'W^*(X,Y,Z,U) \overset{\text{def}}{=} g(W^*(X,Y)Z,U) =' W^*(Z,U,X,Y) \tag{2}$$

and $'W^*_{ijkl}w^{ij}w^{kl} =' W_{ijkl}w^{ij}w^{kl}$, where $'W^*_{ijkl}$ and $'W_{ijkl}$ are components of $'W^*$ and $'W$, w^{kl} is a skew-symmetric tensor [11], [19], [21], Q is the Ricci operator, defined by

$$S(X,Y) \overset{\text{def}}{=} g(QX,Y) \tag{3}$$

and S is the Ricci tensor for arbitrary vector fields X, Y, Z. Such a tensor field W^* is known as $m-$projective curvature tensor. Ojha [12], [13] defined and studied an $m-$projective curvature tensor in a Kähler as well as in Sasakian manifolds.

The purpose of this paper is to study the properties of $m-$projective curvature tensor in Kenmotsu manifolds. Section 2 contains some preliminaries. Section 3 is the study of $m-$projectively flat (that is $W^* = 0$) Kenmotsu manifolds satisfying $R(X,Y).S = 0$ and it has shown that the symmetric endomorphism Q of the tangent space corresponding to S has three different non-zero eigen values and the corresponding manifolds have no flat points. It has also shown that if $m-$projectively flat Kenmotsu manifolds satisfy $R(X,Y).S = 0$, then $\theta.\theta = 0$, where θ denotes the Kulkarni-Nomizu product of g and S. In section 4, we proved that an $m-$projectively semi-symmetric Kenmotsu manifold is an Einstein manifold. Also an $n-$dimensional Kenmotsu manifold is $m-$projectively semi-symmetric if and only if it is locally isometric to the hyperbolic space $H^n(-1)$ or it is $m-$projectively flat. Section 5 deals with Kenmotsu manifolds satisfying the condition $W(X,Y).W^* = 0$. In the last section, we find certain geometrical results if the Kenmotsu manifolds satisfying the condition $C(X,Y).W^* = 0$.

2. Preliminaries

Let on an odd dimensional differentiable manifold M_n, $n = 2m+1$, of differentiability class C^{r+1}, there exist a vector valued linear function ϕ, a $1-$form η, the associated vector field ξ and the Riemannian metric g satisfying

$$\phi^2 X = -X + \eta(X)\xi, \tag{4}$$

$$\eta(\phi X) = 0, \tag{5}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{6}$$

for arbitrary vector fields X and Y, then (M_n, g) is said to be an almost contact metric manifold and the structure $\{\phi, \eta, \xi, g\}$ is called an almost contact metric structure to M_n [14].

In view of (4), (5) and (6), we find

$$\eta(\xi) = 1, \quad g(X,\xi) = \eta(X), \quad \phi(\xi) = 0. \tag{7}$$

If moreover,

$$(D_X\phi)(Y) = -g(X,\phi Y)\xi - \eta(Y)\phi X, \tag{8}$$

and

$$D_X\xi = X - \eta(X)\xi, \tag{9}$$

where D denotes the operator of covariant differentiation with respect to the Riemannian metric g, then $(M_n, \phi, \xi, \eta, g)$ is called a Kenmotsu manifold [3]. Also, the following relations hold in a Kenmotsu manifold [5], [6], [7]

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{10}$$

$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{11}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{12}$$

$$\eta(R(X,Y)Z) = \eta(Y)g(X,Z) - \eta(X)g(Y,Z), \tag{13}$$

for arbitrary vector fields X, Y, Z.

A Kenmotsu manifold (M_n, g) is said to be $\eta-$Einstein if its Ricci-tensor S takes the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{14}$$

for arbitrary vector fields X, Y; where a and b are smooth functions on (M_n, g) [3, 14]. If $b = 0$, then $\eta-$Einstein manifold becomes Einstein manifold. Kenmotsu [3] proved that if (M_n, g) is an $\eta-$Einstein manifold, then $a + b = -(n-1)$.

In consequence of (1), (3), (7), (10), (12) and (13), we find

Lemma 1. *In an $n-$dimensional Kenmotsu manifold, the following relation holds*

$$\begin{aligned}\eta(W^*(X,Y)Z) &= \frac{1}{2}\left\{\eta(Y)g(X,Z) - \eta(X)g(Y,Z)\right\}\\ &- \frac{1}{2(n-1)}\left\{\eta(X)S(Y,Z) - \eta(Y)S(X,Z)\right\}.\end{aligned}$$

The Weyl projective curvature tensor W and concircular curvature tensor C of the Riemannian connection D are given by

$$W(X,Y,Z) = R(X,Y,Z) - \frac{1}{(n-1)}\left\{S(Y,Z)X - S(X,Z)Y\right\}, \tag{15}$$

$$C(X,Y,Z) = R(X,Y,Z) - \frac{r}{n(n-1)}\left\{g(Y,Z)X - g(X,Z)Y\right\}, \tag{16}$$

where R and r are respectively the curvature tensor and scalar curvature of the Riemannian connection D [14].

3. $m-$projectively flat Kenmotsu manifolds satisfying $R(X,Y).S = 0$

In view of $W^* = 0$, (1) becomes

$$\begin{aligned}R(X,Y)Z &= \frac{1}{4m}[S(Y,Z)X - S(X,Z)Y\\ &+ g(Y,Z)QX - g(X,Z)QY].\end{aligned} \tag{17}$$

Contracting (17) with respect to X and using (3), we obtain

$$S(Y,Z) = \frac{r}{n}g(Y,Z).$$

Thus, an $m-$projectively flat Riemannian manifold is an Einstein manifold.

Now, $R(X,Y).S = 0$ gives

$$S(R(X,Y)Z,U) + S(Z,R(X,Y)U) = 0.$$

In consequence of (17), above relation becomes

$$\begin{aligned}\frac{1}{4m}[S(QX,U)g(Y,Z) &- S(QY,U)g(X,Z)\\ &+ g(Y,U)S(QX,Z) - g(X,U)S(QY,Z)] = 0.\end{aligned}$$

Putting $Y = Z = \xi$ in the last relation and then using (7), we obtain

$$\begin{aligned}S(QX,U) &- \eta(X)S(Q\xi,U)\\ &+ \eta(U)S(QX,\xi) - g(X,U)S(Q\xi,\xi) = 0.\end{aligned} \tag{18}$$

With the help of (7) and (12), (18) gives

$$S(QX,U) = -(n-1)^2 g(X,U), \tag{19}$$

where $S^2(X,U) \overset{def}{=} S(QX,U)$.

It is well known that

Lemma 2. [18] *If $\theta = g \overline{\wedge} A$ be the Kulkarni-Nomizu product of g and A, where g being Riemannian metric and A be a symmetric tensor of type $(0,2)$ at point x of a semi-Riemannian manifold (M_n, g). Then the relation*

$$\theta.\theta = \alpha Q(g,\theta), \qquad \alpha \in R$$

is satisfied at x if and only if the condition

$$A^2 = \alpha A + \lambda g, \qquad \lambda \in R$$

holds at x.

In consequence of (19) and lemma (2), we state

Theorem 1. *If an $m-$projectively flat Kenmotsu manifold satisfies the condition $R(X,Y).S = 0$, then $\theta.\theta = 0$, where $\theta = g \overline{\wedge} S$ and $\alpha = 0$.*

Let λ be the eigen value of the endomorphism Q corresponding to an eigen vector X, then putting $QX = \lambda X$ in (18) and using (3), we find

$$\begin{aligned} \lambda^2 g(X,U) \quad &- \quad 4m^2 \eta(X)\eta(U) \\ &- \quad 2m\lambda\eta(X)\eta(U) - 4m^2 g(X,U) = 0. \end{aligned} \tag{20}$$

Again, putting $U = \xi$ in the equation (20) and then using (7), we have

$$[\lambda^2 - 2m\lambda - 8m^2]\eta(X) = 0.$$

If X is perpendicular to ξ, then (20) gives

$$\lambda^2 = 4m^2 \Longrightarrow \lambda = \pm 2m \tag{21}$$

and hence the corresponding eigen values of Q would be $\pm 2m$. Since $\eta(X)$ is not equal to zero, in general, therefore

$$\lambda^2 - 2m\lambda - 8m^2 = 0, \tag{22}$$

which follows that the symmetric endomorphism Q of the tangent space corresponding to S has three different non-zero eigen values namely $4m$ and $\pm 2m$.

Thus, we can state

Theorem 2. *If an $m-$projectively flat Kenmotsu manifold satisfies $R(X,Y).S = 0$, then the symmetric endomorphism Q of the tangent space corresponding to S has three different non-zero eigen values.*

Now, putting $Y = Z = \xi$ in (17) and using (3), (7), (10) and (12), we obtain

$$QX = -(n-1)X$$

which gives

$$r = -n(n-1). \tag{23}$$

If λ_1, λ_2 and λ_3 be the eigen values of the Ricci operator Q and let multiplicity of λ_1 and λ_2 be p and q respectively, then multiplicity of λ_3 is $n-p-q$. Since the scalar curvature is the trace of the Ricci operator Q, therefore

$$r = p\lambda_1 + q\lambda_2 + (n-p-q)\lambda_3. \tag{24}$$

In consequence of (21), (22), (23), (24) and theorem (2), we obtain

$$p\lambda_1 + q\lambda_2 + (n-p-q)\lambda_3 = -n(n-1)$$

and

$$\lambda_1 + \lambda_2 + \lambda_3 = 2(n-1),$$

which gives

$$3p + 2q = 0.$$

Next, if V_1, V_2 and V_3 denote the eigen subspaces corresponding to the eigen values λ_1, λ_2 and λ_3 respectively of the manifold, then the sectional curvature on V_1 for orthonormal eigen vectors X, Y is $\frac{\lambda_1}{n-1}$.

Similarly on V_2 and V_3, the sectional curvature for orthonormal eigen vectors X and Y is $\frac{\lambda_2}{n-1}$ and $\frac{\lambda_3}{n-1}$ respectively. Since $\lambda_1 = 2(n-1)$, which is not equal to zero, therefore we have

Theorem 3. *If an $m-$projectively flat Kenmotsu manifold M_n, $(n \geq 2)$, satisfies $R(X,Y).S = 0$, then the manifold has no flat points.*

4. $m-$projectively semi-symmetric Kenmotsu manifolds

We suppose that W^* is semi-symmetric, i.e.,

$$R(X,Y).W^* = 0 \implies R(\xi,Y).W^* = 0$$

which is equivalent to

$$R(\xi,Y)W^*(Z,U)V - W^*(R(\xi,Y)Z,U)V - W^*(Z,R(\xi,Y)U)V - W^*(Z,U)R(\xi,Y)V = 0.$$

In view of (1) and (11), above equation becomes

$$\begin{aligned}
&R(\xi,Y)R(Z,U)V - \eta(Z)R(Y,U)V + g(Y,Z)R(\xi,U)V - \eta(U)R(Z,Y)V \\
&+g(Y,U)R(Z,\xi)V - \eta(V)R(Z,U)Y + g(Y,V)R(Z,U)\xi \\
&-\frac{1}{4m}[S(U,V)R(\xi,Y)Z - S(Z,V)R(\xi,Y)U + g(U,V)R(\xi,Y)QZ \\
&-g(Z,V)R(\xi,Y)QU - \eta(Z)S(U,V)Y + \eta(Z)S(Y,V)U - \eta(Z)g(U,V)QY \\
&+\eta(Z)g(Y,V)QU + g(Y,Z)S(U,V)\xi - g(Y,Z)S(\xi,V)U + g(Y,Z)g(U,V)Q\xi \\
&-\eta(V)g(Y,Z)QU - \eta(U)S(Y,V)Z + \eta(U)S(Z,V)Y - \eta(U)g(Y,V)QZ \\
&+\eta(U)g(Z,V)QY + g(Y,U)S(\xi,V)Z - g(Y,U)S(Z,V)\xi + \eta(V)g(Y,U)QZ \\
&-g(Y,U)g(Z,V)Q\xi - \eta(V)S(U,Y)Z + \eta(V)S(Z,Y)U - \eta(V)g(U,Y)QZ \\
&+\eta(V)g(Y,Z)QU + g(Y,V)S(U,\xi)Z - g(Y,V)S(Z,\xi)U \\
&+\eta(U)g(Y,V)QZ - \eta(Z)g(Y,V)QU] = 0.
\end{aligned}$$

Using (7), (10), (11), (12) and (13) in the above expression, we obtain

$$\begin{aligned}
&\eta(U)g(Z,V)Y-\eta(Z)g(U,V)Y-{}'R(Z,U,V,Y)\xi-\eta(Z)R(Y,U)V\\
&+\eta(V)g(Y,Z)U-g(Y,Z)g(U,V)\xi-\eta(U)R(Z,Y)V+g(Y,U)R(Z,\xi)V\\
&-\eta(V)R(Z,U)Y+\eta(Z)g(Y,V)U-\eta(U)g(Y,V)Z\\
&-\frac{1}{4m}[g(U,V)S(Z,\xi)Y-S(Y,Z)g(U,V)\xi-g(Z,V)S(U,\xi)Y\\
&+g(Z,V)S(Y,U)\xi+\eta(Z)S(Y,V)U-\eta(Z)g(U,V)QY-2m\eta(V)g(Y,U)Z\\
&+g(Y,Z)g(U,V)Q\xi-\eta(U)S(Y,V)Z+\eta(U)g(Z,V)QY+2m\eta(V)g(Y,Z)U\\
&-g(Y,U)g(Z,V)Q\xi-\eta(V)S(U,Y)Z+\eta(V)S(Z,Y)U\\
&-2m\eta(U)g(Y,V)Z+2m\eta(Z)g(Y,V)U]=0.
\end{aligned}$$

Putting $Z=\xi$ in the above relation and then using $\eta(R(V,Y)U)=-{}'R(V,Y,\xi,U)$, (10), (12) and (13), we find

$$\begin{aligned}
&-R(Y,U)V-g(U,V)Y+g(Y,V)U-\frac{1}{4m}[-2mg(U,V)Y\\
&+2m\eta(Y)g(U,V)\xi+2m\eta(U)\eta(V)Y+\eta(V)S(Y,U)\xi+S(Y,V)U\\
&-g(U,V)QY+\eta(Y)g(U,V)Q\xi-\eta(U)S(Y,V)\xi+\eta(U)\eta(V)QY\\
&-2m\eta(V)g(Y,U)\xi-\eta(V)g(Y,U)Q\xi-\eta(V)S(U,Y)\xi\\
&-2m\eta(U)g(Y,V)\xi+2mg(Y,V)U]=0. \qquad (25)
\end{aligned}$$

Contracting above with respect to Y, we get

$$S(U,V)=\left(\frac{r-(n-1)^2}{2n-1}\right)g(U,V)-\left(\frac{r+n(n-1)}{2n-1}\right)\eta(U)\eta(V). \qquad (26)$$

Hence, the manifold is an $\eta-$Einstein manifold.

Again, from (3), (7) and (26), we obtain

$$QU=\left(\frac{r-(n-1)^2}{2n-1}\right)U-\left(\frac{r+n(n-1)}{2n-1}\right)\eta(U)\xi \qquad (27)$$

and

$$r=-n(n-1). \qquad (28)$$

In consequence of (28), (26) becomes

$$S(U,V)=-(n-1)g(U,V). \qquad (29)$$

Thus, we can state

Theorem 4. *An $m-$projectively semi-symmetric Kenmotsu manifold is an Einstein manifold.*

In view of (27), (28) and (29), (25) becomes

$$R(Y,U)V=-g(U,V)Y+g(Y,V)U. \qquad (30)$$

A space form (i.e., a complete simply connected Riemannian manifold of constant curvature) is said to be elliptic, hyperbolic or Euclidean according as the sectional curvature is positive, negative or zero [15]. Thus we have

Theorem 5. *An $n-$dimensional Kenmotsu manifold M_n is $m-$projectively semi-symmetric if and only if it is locally isometric to the hyperbolic space $H^n(-1)$.*

In view of (29) and (30), (1) becomes

$$W^*(X,Y)Z = 0.$$

Thus we state

Theorem 6. *An $n-$dimensional Kenmotsu manifold M_n is $m-$projectively semi-symmetric if and only if it is $m-$projectively flat.*

It is well known that

Lemma 3. [4] *In an $n-$dimensional Riemannian manifold M_n, the following are equivalent*
(i) M_n is an Einstein manifold,
(ii) $m-$projective and Weyl projective curvature tensors are linearly dependent.
(iii) $m-$projective and concircular curvature tensors are linearly dependent.
(iv) $m-$projective and conformal curvature tensors are linearly dependent.

In consequence of above equivalent relations and theorems (4), (5) and (6), we state

Corollary 1. *In an $n-$dimensional Kenmotsu manifold M_n, the following are equivalent*
(i) M_n is an $m-$projectively semi-symmetric manifold,
(ii) M_n is $m-$projectively flat,
(iii) M_n is Weyl projectively flat,
(iv) M_n is concircularly flat,
(v) M_n is conformally flat,
(vi) M_n is locally isometric to the hyperbolic space $H^n(-1)$.

5. Kenmotsu manifolds satisfying $W(X,Y).W^* = 0$

In consequence of $W(X,Y).W^* = 0$, we have

$$W(X,Y)W^*(Z,U)V - W^*(W(X,Y)Z,U)V \\ -W^*(Z,W(X,Y)U)V - W^*(Z,U)W(X,Y)V = 0. \tag{31}$$

Replacing X by ξ in (31), we find

$$W(\xi,Y)W^*(Z,U)V - W^*(W(\xi,Y)Z,U)V \\ -W^*(Z,W(\xi,Y)U)V - W^*(Z,U)W(\xi,Y)V = 0. \tag{32}$$

Using (11), (12) and (15) in (32), we obtain

$$g(Y,W^*(Z,U)V)\xi - g(Y,Z)W^*(\xi,U)V - g(Y,U)W^*(Z,\xi)V - g(Y,V)W^*(Z,U)\xi \\ +\frac{1}{n-1}[S(Y,W^*(Z,U)V)\xi - S(Y,Z)W^*(\xi,U)V - S(Y,U)W^*(Z,\xi)V - S(Y,V)W^*(Z,U)\xi] = 0.$$

Taking inner product of above equation with ξ and then using (1), (2), (7), (12) and (13), we obtain

$$\begin{aligned} 'W^*(Z,U,V,Y) \quad &+ \quad \frac{1}{n-1}[S(U,V)g(Y,Z) - S(Z,V)g(Y,U) + (n-1)(g(U,V)g(Y,Z) \\ &- \quad g(Y,U)g(Z,V)] + \frac{1}{n-1}[S(Y,W^*(Z,U)V) + \frac{1}{2(n-1)}(S(Y,Z)S(U,V) \\ &- \quad S(Y,U)S(Z,V)) + \frac{1}{2}(S(Y,Z)g(U,V) - S(Y,U)g(Z,V))] = 0. \end{aligned} \tag{33}$$

Again replacing Z and V by ξ in (33) and using (1), (7), (12) and (13), we find

$$S(QU,Y) = -2(n-1)S(U,Y) - (n-1)^2 g(U,Y), \tag{34}$$

where $S(QU,Y) \overset{def}{=} S^2(U,Y)$. Thus we state

Theorem 7. *If an $n-$dimensional $(n \geq 2)$ Kenmotsu manifold M_n satisfies the condition $W(X,Y).W^* = 0$, then the relation (34) holds on M_n.*

In consequence of lemma (2) and theorem (7), we state

Theorem 8. *If an $n-$dimensional Kenmotsu manifold (M_n, g) $(n \geq 2)$ satisfying the condition $W(X,Y).W^* = 0$, then $\theta.\theta = \alpha Q(g,\theta)$, where $\theta = g\overline{\wedge}S$ and $\alpha = -2(n-1)$.*

6. Kenmotsu manifolds satisfying $C(X,Y).W^* = 0$

We suppose $C(X,Y).W^* = 0$, then

$$\begin{aligned} &C(X,Y)W^*(Z,U)V - W^*(C(X,Y)Z,U)V \\ &-W^*(Z,C(X,Y)U)V - W^*(Z,U)C(X,Y)V = 0. \end{aligned} \tag{35}$$

Replacing X by ξ in (35), we find

$$\begin{aligned} &C(\xi,Y)W^*(Z,U)V - W^*(C(\xi,Y)Z,U)V \\ &-W^*(Z,C(\xi,Y)U)V - W^*(Z,U)C(\xi,Y)V = 0. \end{aligned} \tag{36}$$

In view of (16), (36) becomes

$$\begin{aligned} &(1+\frac{r}{n(n-1)})[-W^*(Z,U,V,Y)\xi + \eta(W^*(Z,U)V)Y \\ &-\eta(Z)W^*(Y,U)V + g(Y,U)W^*(Z,\xi)V - \eta(U)W^*(Z,Y)V \\ &+g(Y,V)W^*(Z,U)\xi - \eta(V)W^*(Z,U)Y + g(Y,Z)W^*(\xi,U)V] = 0. \end{aligned} \tag{37}$$

Taking inner product of (37) with ξ and then using lemma (1), we get

$$\begin{aligned} &(1+\frac{r}{n(n-1)})[-W^*(Z,U,V,Y) - \frac{1}{2(n-1)}(S(U,V)g(Y,Z) - S(Z,V)g(Y,U) \\ &+\eta(V)\eta(U)S(Y,Z) - \eta(V)\eta(Z)S(U,Y)) - \frac{1}{2}(g(U,V)g(Y,Z) \\ &-g(Y,U)g(Z,V) + \eta(V)\eta(U)g(Y,Z) - \eta(V)\eta(Z)g(U,Y))] = 0. \end{aligned} \tag{38}$$

Also replacing Z and V by ξ and using (7), (12) and lemma (1), we obtain

$$(1+\frac{r}{n(n-1)})[g(U,Y) + \frac{1}{n-1}S(U,Y)] = 0.$$

This equation implies

$$\textit{either} \quad r = -n(n-1) \quad \textit{or} \quad S(U,Y) = -(n-1)g(U,Y). \tag{39}$$

Thus we state

Theorem 9. *Let M_n be an $n-$dimensional Kenmotsu manifold. Then M_n satisfies the condition*

$$C(\xi,Y).W^* = 0$$

if and only if either M_n is an Einstein manifold or it has scalar curvature $r = -n(n-1)$.

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*Department of Mathematics,
GLA University, NH-2, Mathura-281406, India.
E-mail address: sk22_math@yahoo.co.in

**Department of Mathematics,
M.M.M. Engg. College, Gorakhpur-273010, India.
E-mail address: shashiprksh8491@gmail.com

***Department of Mathematics and Astronomy,
Lucknow University, 226007, India.
E-mail address: rnivas.lu@gmail.com