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ON THE COMMON–NEIGHBORHOOD ENERGY OF A GRAPH

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A b s t r a c t. *We introduce the concept of common–neighborhood energy E_{CN} of a graph G and obtain an upper bound for E_{CN} when G is strongly regular. We also show that E_{CN} of several classes of graphs is less than the common–neighborhood energy of the complete graph K_n .*

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1. *Introduction*

Let G be a simple graph with n vertices, and let $\mathbf{A} = ||\mathbf{a_{ij}}||$ be its adjacency matrix. The eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of $\mathbf{A}$ are the (ordinary) eigenvalues of the graph G [3]. Since $\mathbf{A}$ is a symmetric matrix with zero trace, these eigenvalues are real with sum equal to zero.

The energy of the graph G is defined [5] as the sum of the absolute values of its eigenvalues:

$$E(G) = \sum_{i=1}^{n} |\lambda_i| \ .$$

Details on the theory of graph energy can be found in the reviews [7, 10, 9], whereas details on its chemical applications in the book [11] and in the review [8].

The energy of the complete graph K_n is equal to $2(n-1)$. An n-vertex graph G is said to be *hyperenergetic* [6, 15] if $E(G) > E(K_n)$. Details on hyperenergetic graphs can be found in the review [9]. The most important achievement in this area is Nikiforov's:

Theorem 1.1 [16] For almost all n-vertex graphs

$$E(G) = \left(\frac{4}{3\pi} + o(1)\right) n^{3/2} .$$

Theorem 1.1 immediately implies that almost all graphs are hyperenergetic, making any further search for them pointless.

In what follows we shall need a few auxiliary results.

Lemma 1.2 [3] *Let G be a connected k-regular graph with n vertices and $k \geq 3$. Let $k, \lambda_2, \ldots, \lambda_n$ be its eigenvalues. Then the eigenvalues of the line graph of G are $2k-2, \lambda_2 + k - 2, \ldots, \lambda_n + k - 2$, and -2 with multiplicity $n(k-2)/2$.*

We also mention here some results pertaining to graphs with greatest energy (see [12]). For this we need some preparations.

Definition 1.3 *A strongly regular graph with parameters (n, k, λ, μ) is a k-regular graph with n vertices, such that any two adjacent vertices have λ common neighbors, and any two non-adjacent vertices have μ common neighbors.*

Theorem 1.4 [13] *If G is a graph on n vertices, then $E(G) \leq n(1 + \sqrt{n})/2$. Equality holds if and only if G is strongly regular with parameters*

$$\left(n\,,\ \frac{n+\sqrt{n}}{2}\,,\ \frac{n+2\sqrt{n}}{4}\,,\ \frac{n+2\sqrt{n}}{4}\right).$$

* * * * *

In this paper we introduce a new kind of graph energy, called *common–neighborhood energy* and denoted by $E_{CN}(G)$. We first determine a few basic properties of E_{CN}, and then focus our interest on finding "CN-hyperenergetic" graphs, namely those satisfying the condition $E_{CN}(G) > E_{CN}(K_n)$. As explained above, there exist countless graphs for which the inequality $E(G) > E(K_n)$ holds. In contrast to this, "CN-hyperenergetic" graphs appear to be less numerous or, maybe, do not exist at all. In what follows we point out a few negative results along these lines.

Definition 1.5 *Let G be simple graph with vertex set $\mathcal{V}(G) = \{v_1, v_2, \ldots, v_n\}$. For $i \neq j$, the common neighborhood of the the vertices v_i and v_j, denoted by $\Gamma(v_i, v_j)$, is the set of vertices, different from v_i and v_j, which are adjacent to both v_i and v_j. The common–neighborhood matrix of G is then* $\mathbf{CN} = \mathbf{CN(G)} = ||\gamma_{\mathbf{ij}}||$, *where*

$$\gamma_{ij} = \begin{cases} |\Gamma\{v_i, v_j\}| & \textit{if } \ i \neq j \\ \\ 0 & \textit{otherwise.} \end{cases}$$

According to the above definition, the common–neighborhood matrix is a real symmetric $n \times n$ matrix. Therefore its eigenvalues $\gamma_1, \gamma_2, \ldots, \gamma_n$ are real numbers. Since the trace of $\mathbf{CN(G)}$ is zero, the sum of its eigenvalues is also equal to zero.

Definition 1.6 *The common–neighborhood energy (or, shorter, CN-energy) of the graph G is*

$$E_{CN} = E_{CN}(G) = \sum_{i=1}^{n} |\gamma_i| \ .$$

Definition 1.7 *If G is a graph with n vertices, and if $E_{CN}(G) > E_{CN}(K_n)$, then G is said to be CN-hyperenergetic.*

2. *Elementary results*

Denote by $\mathbf{I_n}$ the unit matrix of order n, and by $\mathbf{J_n}$ the square matrix of order n whose all elements are equal to unity. Let further $\mathbf{0}$ stand for a matrix (or pertinent dimension) whose all elements are equal to zero.

Example 2.1 *$E_{CN}(K_n) = 2(n-1)(n-2)$, where K_n is the complete graph of order n.*

P r o o f. Observing that $\mathbf{CN(K_n)} = \mathbf{(n-2)}\,\mathbf{A(K_n)}$, we get $E_{CN}(K_n) = (n-2)\,E(K_n)$. □

Example 2.2 *$E_{CN}(K_{a,b}) = 2(2ab-a-b)$, where $K_{a,b}$ is the complete bipartite graph of order $a+b$.*

P r o o f. First observe that if the vertices of $K_{a,b}$ are labeled so that all vertices $v_1, \ldots, v_a$ are adjacent to all vertices $v_{a+1}, \ldots, v_{a+b}$, then

$$\mathbf{CN(K_{a,b})} = \begin{pmatrix} \mathbf{0} & b(\mathbf{J_a} - \mathbf{I_a}) \\ a(\mathbf{J_b} - \mathbf{I_b}) & \mathbf{0} \end{pmatrix}.$$

Observing that $\mathbf{J_a} - \mathbf{I_a} = \mathbf{A(K_a)}$ and $\mathbf{J_b} - \mathbf{I_b} = \mathbf{A(K_b)}$, we have

$$\mathbf{CN(K_{a,b})} = \begin{pmatrix} \mathbf{0} & b\,\mathbf{A(K_a)} \\ a\,\mathbf{A(K_b)} & \mathbf{0} \end{pmatrix}$$

which implies $E_{CN}(K_{a,b}) = b\,E(K_a) + a\,E(K_b) = b[2(a-1)] + a[2(b-1)]$. □

Example 2.3 *$E_{CN}(K_{a,a}) = n(n-2)$, where $K_{a,a}$ is the complete bipartite graph of order $a+a=n$.*

Directly from Definition 1.6 follows:

Proposition 2.4 *If the graph G consists of (disconnected) components $G_1, G_2, \ldots, G_p$, then*

$$E_{CN}(G) = E_{CN}(G_1) + E_{CN}(G_2) + \cdots + E_{CN}(G_p)\ .$$

Proposition 2.5 *$E_{CN}(G) = 0$ if and only if no component of G possesses more than two vertices.*

P r o o f. Evidently, $\mathbf{CN(K_1)} = \mathbf{0}$ and $\mathbf{CN(K_2)} = \mathbf{0}$. Therefore, $E_{CN}(K_1) = E_{CN}(K_2) = 0$. If all components of G are K_1 and/or K_2, then by Proposition 2.4, $E(G) = 0$. In this case, $\mathbf{CN(G)} = \mathbf{0}$. If at least one component of G has at least three vertices, then there exists a pair of vertices v_i, v_j possessing a non-empty common neighborhood. Then $\mathbf{CN(G)} \neq \mathbf{0}$ and therefore not all of its eigenvalues are equal to zero. □

Proposition 2.6 *If $a = b = 1$, then $K_{a,b} \cong K_{a+b}$ and therefore $E_{CN}(K_{a,b}) = E_{CN}(K_{a+b})$. For all other (positive, integer) values of a and b, $E_{CN}(K_{a,b}) < E_{CN}(K_{a+b})$, i. e., $K_{a,b}$ is not CN-hyperenergetic.*

P r o o f. Bearing in mind the formulas given in Examples 2.1 and 2.2, we have

$$E_{CN}(K_{a+b}) - E_{CN}(K_{a,b}) = 2a(a-2) + 2b(b-2) + 4$$

This difference is evidently positive-valued for $a, b \geq 2$. If $a = 1$, then the difference is equal to $2b(b-2)+2$, which, again, is positive-valued for $b \geq 2$. □

Proposition 2.7 *Let G be a graph on n vertices, and $\deg(v_i)$ be the degree (= number of first neighbors) of its vertex v_i. Let $\mathbf{D(G)} = \mathbf{diag}\,[\deg(\mathbf{v_1}), \deg(v_2), \ldots, \deg(v_n)]$. Then,*

$$\mathbf{CN(G)} = \mathbf{A(G)^2} - \mathbf{D(G)} \ .$$

P r o o f. $(\mathbf{A(G)^k})_{\mathbf{ij}}$ is equal to the number of walks of length k between the vertices v_i and v_j of the graph G. Therefore, for $i \neq j$, $(\mathbf{A(G)^2})_{\mathbf{ij}} = \gamma_{\mathbf{ij}}$, whereas $(\mathbf{A(G)^2})_{\mathbf{ii}} = \deg(\mathbf{v_i})$. □

Corollary 2.8 *If the graph G is k-regular, then*

$$\mathbf{CN(G)} = \mathbf{A(G)^2} - \mathbf{k}\,\mathbf{I_n} \ .$$

Theorem 2.9 *Let G be a k-regular graph with eigenvalues $k, \lambda_2, \ldots, \lambda_n$. Then the common–neighborhood eigenvalues of G are $k^2 - k, (\lambda_2)^2 - k, \ldots, (\lambda_n)^2 - k$.*

P r o o f. Theorem 2.9 follows from Lemma 1.2 and Corollary 2.8. □

Corollary 2.10 *Let G be a connected k-regular graph and let $k, \lambda_2, \ldots, \lambda_n$ be its eigenvalues.*
(i) *The common–neighborhood eigenvalues of the complement of G are*

$$(n-k-1)(n-k-2)\,,\ \lambda_2^2 + 2\lambda_2 - n + k + 2\,,\ \ldots\,,\ \lambda_n^2 + 2\lambda_n - n + k + 2\ .$$

(ii) *The common–neighborhood eigenvalues of the line graph $L(G)$ of G are*

$$4k^2-10k+6\,,\ \lambda_2^2+(2k-4)\lambda_2+k^2-6k+6\,,\ \ldots\,,\ \lambda_n^2+(2k-4)\lambda_n+k^2-6k+6\,,\ 6-2k$$

where the CN-eigenvalue $6-2k$ has multiplicity $n(k-2)/2$.

P r o o f. (i) Let G be a k-regular graph with ordinary eigenvalues $k, \lambda_2, \dots, \lambda_n$. Then the eigenvalues of the complement of G are $n-k-1, -1-\lambda_2, \dots, -1-\lambda_n$, see [3]. The complement of G is $(n-1-k)$-regular. Then by Theorem 2.9, the respective CN-eigenvalues are: $(n-k-1)(n-k-2), \lambda_2^2+2\lambda_2-n+k+2, \dots, \lambda_n^2+2\lambda_n-n+k+2$.
(ii) Apply Lemma 1.2 and recall that $L(G)$ is regular of degree $2k-2$. □

Definition 2.11 [1, 2] *If G is a graph with vertex set $\mathcal{V}(G)$, then the derived graph of G, denoted by $G^\dagger$, is the graph with vertex set $\mathcal{V}(G)$, such that two vertices of $G^\dagger$ are adjacent if and only if their distance in G is equal to two.*

It should be noted that if G is a (connected) bipartite graph, then its derived graph is disconnected, consisting of two components.

Proposition 2.12 *If G is a triangle– and quadrangle-free graph, then $E_{CN}(G) = E(G^\dagger)$.*

P r o o f. If G is triangle-free, then the common neighborhood of the vertices v_i and v_j is non-empty if and only if these vertices are at distance two. If, in addition, the graph is quadrangle-free, then these common neighborhoods contain a single vertex. Consequently, for triangle– and quadrangle-free graphs, $\gamma_{ij} = 1$ if the distance between the vertices v_i and v_j is two, and $\gamma_{ij} = 0$ otherwise. Then, $\mathbf{CN(G)} = \mathbf{A}(\mathbf{G}^\dagger)$. □

Corollary 2.13 *If T is a tree, then $E_{CN}(T) = E(T^\dagger)$.*

Corollary 2.14 *Let P_n be the n-vertex path. Then $E_{CN}(P_n) = E(P_{\lfloor n/2 \rfloor}) + E(P_{\lceil n/2 \rceil})$.*

P r o o f. $(P_n)^\dagger \cong P_{\lfloor n/2 \rfloor} \cup P_{\lceil n/2 \rceil}$. □

Corollary 2.15 *Let C_n be the n-vertex cycle. If n is odd and $n \geq 3$, then $E_{CN}(C_n) = E(C_n)$. If $n = 4$, then $E_{CN}(C_n) = 4\,E(K_2) = 8$. If n is even and $n \geq 6$, then $E_{CN}(C_n) = 2\,E(C_{n/2})$.*

P r o o f. $\mathbf{CN}(\mathbf{C_3}) = \mathbf{A}(\mathbf{C_3})$ and therefore $E_{CN}(C_3) = E(C_3)$. If n is odd and $n > 3$, then C_n is triangle– and quadrangle-free and thus $E_{CN}(C_n) = E((C_n)^\dagger)$. In this case, $(C_n)^\dagger \cong C_n$. If n is even and $n > 4$, then, again, C_n is triangle– and quadrangle-free and thus $E_{CN}(C_n) = E((C_n)^\dagger)$. In this case, $(C_n)^\dagger \cong C_{n/2} \cup C_{n/2}$. Finally, from $\mathbf{CN}(\mathbf{C_4}) = \mathbf{2}\,\mathbf{A}((\mathbf{C_4})^\dagger)$, $(C_4)^\dagger \cong K_2 \cup K_2$, and $E(K_2) = 2$ follows $E_{CN}(C_4) = 8$. □

3. *More results*

Lemma 3.1 *If G is a strongly regular graph with parameters (n, k, λ, μ), then*

$$\sum_{i=1}^{n} |\gamma_i|^2 = n\left[k\lambda^2 + (n-k-1)\mu^2\right] . \tag{1}$$

P r o o f. If v_i and v_j are adjacent vertices of G, then $\gamma_{ij} = \lambda$. If v_j and v_j are non-adjacent vertices of G, then $\gamma_{ij} = \mu$. Since G has $nk/2$ pairs of adjacent vertices, and $\binom{n}{2} - nk/2$ pairs of non-adjacent vertices,

$$\begin{aligned} \sum_{i=1}^{n} |\gamma_i|^2 &= Tr(\mathbf{CN(G)})^{\mathbf{2}} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \sum_{\mathbf{k=1}}^{\mathbf{n}} \gamma_{\mathbf{ik}}\, \gamma_{\mathbf{ki}} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \sum_{\mathbf{k=1}}^{\mathbf{n}} (\gamma_{\mathbf{ik}})^{\mathbf{2}} \\ &= 2\left[\frac{nk}{2}\right]\lambda^2 + 2\left[\binom{n}{2} - \frac{nk}{2}\right]\mu^2 \end{aligned}$$

from which Eq. (1) follows straightforwardly. □

Theorem 3.2 *If G is a strongly regular graph with parameters (n, k, λ, μ), then*

$$E_{CN}(G) \le k(k-1) + \sqrt{n(n-1)[k\lambda^2 + (n-k-1)\mu^2] - (n-1)k^2\,(k-1)^2} . \tag{2}$$

P r o o f. Proof follows an idea first used by Koolen and Moulton [13, 14]. Let $\gamma_1, \gamma_2, \dots, \gamma_n$ be the common–neighborhood eigenvalues of G, and let γ_1 be the greatest eigenvalue. Because the greatest ordinary eigevalue of G is equal to k, by Theorem 2.9, $\gamma_1 = k^2 - k$.

The Cauchy–Schwarz inequality states that if $(a_1, a_2, \dots, a_n)$ and $(b_1, b_2, \dots, b_n)$ are n-vectors, then

$$\left(\sum_{i=1}^{n} a_i\, b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right).$$

Now, by setting $a_i = 1$ and $b_i = |\gamma_i|$, $i = 2, 3, \dots, n$, in the above inequality, we obtain

$$\left(\sum_{i=2}^{n} |\gamma_i|\right)^2 \le \left(\sum_{i=2}^{n} 1^2\right)\left(\sum_{i=2}^{n} |\gamma_i|^2\right).$$

Therefore

$$\sum_{i=2}^{n} |\gamma_i| \leq \sqrt{(n-1)\sum_{i=2}^{n} |\gamma_i|^2}$$

i. e.,

$$\sum_{i=1}^{n} |\gamma_i| - k(k-1) \leq \sqrt{(n-1)\left[\sum_{i=1}^{n} |\gamma_i|^2 - [k(k-1)]^2\right]}$$

i. e.,

$$E_{CN}(G) \leq k(k-1) + \sqrt{(n-1)\left[\sum_{i=1}^{n} |\gamma_i|^2 - [k(k-1)]^2\right]}.$$

By using Lemma 3.1,

$$E_{CN}(G) \leq k(k-1) + \sqrt{(n-1)\left[n\left[k\lambda^2 + (n-k-1)\mu^2\right] - [k(k-1)]^2\right]}$$

which immediately leads to inequality (2). $\square$

Theorem 3.3 *Let n be an even integer, $n \geq 4$. If the graph G is the complement of $(n/2)\,K_2$, also known as the cocktail party graph, then G is not CN-hyperenergetic.*

P r o o f. G is a regular graph of degree $n-2$ with n vertices. Suppose first that $n \geq 8$. By a lengthy but elementary calculation it can be shown that the characteristic polynomial of $\mathbf{CN}(\mathbf{G})$ is of the form $[x-(n-2)(n-3)](x+n-2)^{n/2}\,(x+n-6)^{n/2-1}$. Therefore,

$$\begin{aligned} E_{CN}(G) &= |(n-2)(n-3)| + \frac{n}{2}\,|-(n-2)| + \frac{n-2}{2}\,|-(n-6)| \\ &= 2(n-2)(n-6)\ . \end{aligned}$$

Hence $E_{CN}(G) = 2(n-2)(n-6)$ whereas by Example 2.1, $E_{CN}(K_n) = 2(n-2)(n-1)$. Evidently, $E_{CN}(G) < E_{CN}(K_n)$.

The fact that G is not CN-hyperenergetic also in the case $n = 4$ and $n = 6$ is verified by direct calculation. $\square$

The line graph of the complete graph K_n is referred to as the triangular graph $T(n)$. Its parameters are $(n(n-1)/2, 2(n-2), (n-2), 4)$, and $T(n)$ is unique for $n \neq 8$.

Theorem 3.4 *If G is the triangular $T(n)$, $n > 4$, then $E_{CN}(G) = 2n(n-3)(n-4)$.*

P r o o f. The ordinary eigenvalues of K_n are $n-1$ and -1 with multiplicities 1 and $n-1$, respectively. Then by Corollary 2.10, the common–neighborhood eigenvalues of $T(n)$ are $4n^2-18n+20$, $n^2-10n+20$, and $-(2n-8)$ with multiplicities 1, $n-1$, and $n(n-3)/2$, respectively. Then

$$E_{CN}(T(n)) = |4n^2-18n+20|+(n-1)|n^2-10n+20|+\frac{n(n-3)}{2}\,|-(2n-8)|\,.$$

□

Corollary 3.5 $E_{CN}(L(K_n)) < E_{CN}(K_{n(n-1)/2})$, *i. e., the line graph of the complete graph is not CN-hyperenergetic.*

At this point we note that the $L(K_n)$, $n \geq 5$, was the first discovered class of hyperenergetic graphs [17].

The line graph of the complete bipartite graph $K_{a,a}$ is strongly regular with parameters $(a^2, 2a-2, a-2, 2)$. Also this graph, for $a \geq 4$ was found to be hyperenergetic [17].

Theorem 3.6 *For* $a \geq 4$, $E_{CN}(L(K_{a,a})) = 4(a-1)^2\,(a-3)$.

P r o o f. Proof is analogous to the proof of Theorem 3.4. □

Corollary 3.7 $E_{CN}(L(K_{a,a})) < E_{CN}(K_{a^2})$, *i. e., the line graph of the complete bipartite graph* $K_{a,a}$, $a > 2$, *is not CN-hyperenergetic.*

P r o o f. For $a \geq 4$, Corollary 3.7 follows from $4(a-1)^2\,(a-3) < 2(a^2-1)(a^2-2) = E_{CN}(K_{a^2})$. For $a = 2, 3$, the validity of the corollary is checked by direct calculation. □

Theorem 3.8 *Let* G *be the strongly regular graph specified in Theorem 1.4. Its common–neighborhood energy is equal to* $(n-1)(n+2\sqrt{n})/2$.

P r o o f. It is easy to show that the eigenvalues of G are $\frac{n+\sqrt{n}}{2}$, $\frac{\sqrt{n}}{2}$, $\frac{-\sqrt{n}}{2}$ with multiplicities 1, $\frac{n-\sqrt{n}}{2}-1$, $\frac{n+\sqrt{n}}{2}$, respectively. By Lemma 2.10, the CN-eigenvalues of G are $\frac{(n+\sqrt{n})(n+\sqrt{n}-2)}{4}$ and $-(\frac{n+2\sqrt{n}}{4})$, with multiplicities 1 and $n-1$, respectively. Then the CN-energy of G is

$$\begin{aligned} E_{CN}(G) &= \left|\frac{(n+\sqrt{n})(n+\sqrt{n}-2)}{4}\right| + (n-1)\left|-\left(\frac{n+2\sqrt{n}}{4}\right)\right| \\ &= \frac{1}{4}\,[(n+\sqrt{n})(n+\sqrt{n}-2)+(n-1)(n+2\sqrt{n})] \\ &= \frac{(n-1)(n+2\sqrt{n})}{2}\,. \end{aligned}$$

□

Corollary 3.9 *Let G be same as in Theorem 3.8. Then $E_{CN}(G) < E_{CN}(K_n)$.*

4. *Concluding remarks*

In this paper we established a few fundamental properties of a new member of the graph–energy family. The results obtained might be viewed as routine, with a single noteworthy exception. Namely, in all studied cases, we found that the considered graphs are **not** CN-hyperenergetic. The studied cases embrace classes of graphs known to have great (and greatest) energy, all known to be hyperenergetic. In spite of these findings, it may be too early to conjecture that CN-hyperenergetic graphs do not exist at all. Instead, we state the following:

Open problem 1. Find an n-vertex graph G, such that $E_{CN}(G) > E_{CN}(K_n)$ i. e., $E_{CN}(G) > 2(n-1)(n-2)$.

More subtle, and certainly more difficult, would be the:

Open problem 2. Let G be a graph and e its edge. Under which conditions $E_{CN}(G-e) < E_{CN}(G)$?

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