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$(\alpha, \beta, \theta, \partial, \mathcal{I})$-Continuous Mappings and their Decomposition

Aplicaciones $(\alpha, \beta, \theta, \partial, \mathcal{I})$-Continuas y su Descomposición

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Abstract

In this paper we introduce the concept of $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continuous mappings and prove that if α, β are operators on the topological space (X, τ) and θ, θ^* , ∂ are operators on the topological space (Y, φ) and $\mathcal{I}$ a proper ideal on X, then a function $f : X \to Y$ is $(\alpha, \beta, \theta \wedge \theta^*, \partial, \mathcal{I})$-continuous if and only if it is both $(\alpha, \beta, \theta, \partial, \mathcal{I})$ -continuous and $(\alpha, \beta, \theta^*, \partial, \mathcal{I})$-continuous, generalizing a result of J. Tong. Additional results on $(\alpha, Int, \theta, \partial, \{\emptyset\})$-continuous maps are given.
Key words and phrases: P-continuous, mutually dual expansions, expansion continuous

Resumen

En este artículo se introduce el concepto de aplicación $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continua y se prueba que si α, β son operadores en el espacio topológico (X, τ) y θ, θ^*, ∂ son operadores en el espacio topol'ogico (Y, φ) y $\mathcal{I}$ es un ideal propio en X, entonces una función $f : X \to Y$ es $(\alpha, \beta, \theta \wedge \theta^*, \partial, \mathcal{I})$-continua si y s'olo si es $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continua y $(\alpha, \beta, \theta^*, \partial, \mathcal{I})$-continua, generalizanso un resultado de J. Tong. Se dan resultados adicionales sobre aplicaciones $(\alpha, Int, \theta, \partial, \{\emptyset\})$-continuas.
Palabras y frases clave: P-continuas, expansiones mutuamente duales, expansión continua.

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1 Introduction

In [17] Kasahara introduced the concept of an operation associated with a topology τ on set X as a map $\alpha : \tau \to P(X)$ such that $U \subset \alpha(U)$ for every $U \in \tau$. In [30] J. Tong called this kind of maps, expansions on X. In [24] Vielma and Rosas modified the above definition by allowing the operator α to be defined on $P(X)$; they are called operators on (X, τ).

Preliminaries

First of all let us introduce a concept of continuity in a very general setting: In fact, let (X, τ) and (Y, φ) be two topological spaces , α and β be operators on (X, τ), θ and ∂ be operators in (Y, φ) respectively. Also let $\mathcal{I}$ be a proper ideal on X.

Definition 1. A mapping $f : X \to Y$ is said to be $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continuous if for every open set $V \in \varphi$, $\alpha\left(f^{-1}(\partial V)\right) \setminus \beta\ f^{-1}(\theta V) \in \mathcal{I}$.

We can see that the above definition generalizes the concept of continuity, when we choose: α = identity operator, β=interior operator, ∂ = identity operator, θ =identity operator and $\mathcal{I} = \{\emptyset\}$.

Also, if we ask the operator α to satisfy the additional condition that $\alpha(\emptyset) = \emptyset$, $\partial \leq \theta$, then the constant maps are always $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continuous for any ideal $\mathcal{I}$ on X.

1. In fact, let $f : X \to Y$ be a map such that $f(x) = y_0 \ \ \forall x \in X$. Let V be on open set in (Y, φ)

   - If $y_0 \in V$, then $f^{-1}(\partial V) = X$, $\alpha\left(f^{-1}(\partial V)\right) = X$, $f^{-1}(\theta V) = X$, $\beta\left(f^{-1}(\theta V)\right) = X$ Then $\alpha\left(f^{-1}(\partial V)\right) \setminus\ \beta f^{-1}(\theta V) = \emptyset \in \mathcal{I}$
   - If $y_0 \notin V$ but $y_0 \in \partial V$ and $y_0 \in \theta V$ then

$$\begin{aligned} f^{-1}(\partial V) &= X \qquad & f^{-1}(\theta V) &= X \\ \alpha\left(f^{-1}(\partial V)\right) &= X \qquad & \beta\ f^{-1}(\theta V) &= X \end{aligned}$$

   and $\alpha\left(f^{-1}(\partial V)\right) \setminus \beta\ f^{-1}(\theta V) = \emptyset \in \mathcal{I}$

   If $y_0 \notin \theta V$ then

$$\begin{aligned} f^{-1}(\partial V) &= \emptyset \qquad & f^{-1}(\theta V) &= \emptyset \\ \alpha\left(f^{-1}(\partial V)\right) &= \emptyset, \qquad & \beta\ f^{-1}(\theta V) &\subset X \end{aligned}$$

   and $\alpha\left(f^{-1}(\partial V)\right) \setminus \beta\ f^{-1}(\theta V) = \emptyset \in \mathcal{I}$

If $y_0 \notin \partial V$ and $y_0 \in \theta V$ then

$$\begin{array}{ll} f^{-1}(\partial V) = \emptyset & f^{-1}(\theta V) = X \\ \alpha\left(f^{-1}(\partial V)\right) = \emptyset & \beta\ f^{-1}(\theta V) = X \end{array}$$

and $\alpha\left(f^{-1}(\partial V)\right) \setminus \beta\ f^{-1}(\theta V) = \emptyset \in \mathcal{I}$

Let us give a historical justification of the above definition:

1. In 1922, H. Blumberg [5] defined the concept of *densely approached* maps: For every open set V in $Y, f^{-1}(V) \subset Intcl\ f^{-1}(V)$. Here $\alpha =$ identity operator, $\beta =$ Interior closure operator, $\partial =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \{\emptyset\}$.

2. In 1932, S. Kempisty [14] defined *quasi-continuous* mappings: For every open set V in $Y, f^{-1}(V)$ is semi open. Here $\alpha =$ identity operator, $\beta =$ Interior operator, $\partial =$ identity operator, $\theta =$ identity operator and $\mathcal{I} =$ nowhere dense sets of X.

3. In 1961, Levine [18] defined *weakly continuous* mappings: For every open set V in Y, $f^{-1}(V) \subset Intf^{-1}(clV)$. Here $\alpha =$ identity operator, $\beta =$ Interior operator, $\partial =$ identity operator, $\theta =$ closure operator and $\mathcal{I} = \{\emptyset\}$.

4. In 1966, Singal and Singal [27] defined *almost continuous* mappings: For every open set V in Y, $f^{-1}(V) \subset Intf^{-1}(IntclV)$. Here $\alpha =$ identity operator, $\beta =$ Interior operator, $\partial =$ identity operator, $\theta =$ interior closure operator and $\mathcal{I} = \{\emptyset\}$.

5. In 1972, S. G. Crossley and S. K. Hildebrand [8] defined *irresolute* maps: For every semi open set V in Y, $f^{-1}(V)$ is semi open. Here $\alpha =$ identity operator, $\beta =$ Interior operator, $\partial =$ identity operator, $\theta =$ identity operator and $\mathcal{I} =$ nowhere dense sets of X.

6. In 1973, Carnahan [6] defined *R-maps*: For every regular open set V in Y, $f^{-1}(V)$ is regularly open. Here $\alpha =$ Interior closure operator, $\beta =$ Interior closure operator, $\partial =$ identity operator, $\theta =$ Interior closure operator and $\mathcal{I} = \{\emptyset\}$.

7. In 1982, J. Tong [29] defined *weak almost continuous* mappings: For every open set V in Y, $f^{-1}(V) \subset Intf^{-1}(IntKerclV)$. Here $\alpha =$ identity operator, $\beta =$ Interior operator, $\partial =$ identity operator, $\theta =$ Interior Kernel closure operator and $\mathcal{I} = \{\emptyset\}$.

8. In 1982, J. Tong [29] defined*very weakly continuous* maps. For every open set V in Y, $f^{-1}(V) \subset Intf^{-1}(KerclV)$. Here α = identity operator, β = Interior operator, ∂ = identity operator, θ = Kernel closure operator and $\mathcal{I} = \{\emptyset\}$.

9. In 1984, T. Noiri [22] defined *perfectly continuous* maps: For every open set V in Y, $f^{-1}(V)$ is clopen. Here α = Closure operator, β = Interior operator, ∂ = identity operator, θ = identity operator and $\mathcal{I} = \{\emptyset\}$

10. In 1985, D. S.Jankovic [13], defined *almost weakly continuous* maps: For every open set V in Y, $f^{-1}(V) \subset Intclf^{-1}(clV)$. Here α = Identity operator, β = Interior closure operator, ∂ = identity operator, θ = closure operator and $\mathcal{I} = \{\emptyset\}$.

In order to continue the justification of the above definition, let us consider a certain property P that is satisfied by a collection of open sets in Y.

Definition 2. A map $f: X \to Y$ is said to be P-continuous if $f^{-1}(U)$ is open for each open set U in Y satisfiying property P.

Let $\theta_P : P(Y) \to P(Y)$ be a operator in (Y, φ) defined as follows

$$\theta_P(A) = \begin{cases} A & \text{if } A \text{ is open and satisfies property } P \\ Y & \text{otherwise} \end{cases}$$

Theorem 1. *A map $f: X \to Y$ is P-continuous if and only if it is $(id, int, \theta_P, id, \{\emptyset\})$-continuous.*

Proof. In fact, suppose that f is P-continuous and let V an open set in (Y, φ).

Case 1. If V satisfies property P, $\theta_P(V) = V$, then by hypothesis $f^{-1}(V)$ is open and then $f^{-1}(V) \subset Intf^{-1}(\theta_P(V)) = Intf^{-1}(V)$.

Case 2. If V does not satisfies property P, $\theta_P(V) = Y$, then clearly $f^{-1}(V) \subset Intf^{-1}(\theta_P(V)) = Y$.

Conversely, suppose that $f^{-1}(V) \subset Intf^{-1}(\theta_P(V))$ for each open set V in (Y, φ). Take V an open set satisfying property P, then $\theta_P(V) = V$ and since $f^{-1}(V) \subset Intf^{-1}(\theta_P(V)) = Intf^{-1}(V)$. We conclude that $f^{-1}(V)$ is open and then f is P-continuous. □

11. In 1970, K. R. Gentry and H. B. Hoyle [12] defined *C-continuous* functions: For every open set V in Y with compact complement, $f^{-1}(V)$ is open.

12. In 1971, Y. S. Park [23] defined $\mathbf{C}^*$-*continuous* function: For every open set V in Y with countably compact complement, $f^{-1}(V)$ is open.

13. In 1978, J. K. Kohli [15] defined *S-continuous* functions: For every open set V in Y with connected complement, $f^{-1}(V)$ is open.

14. In 1981, J. K. Kohli [16] defined *L-continuous* functions: For every open set V in Y with Lindelof complement, $f^{-1}(V)$ is open.

15. In 1981, P. E. Long and L. L. Herringtong [20] defined *para-continuous* functions: For every open set V in Y with paracompact complement, $f^{-1}(V)$ is open.

16. In 1984, S. R. Malgan and V. V. Hanchinamani [21] defined *N-continuous* functions: For every open set V in Y with nearly compact complement, $f^{-1}(V)$ is open.

17. In 1987, F. Cammaroto and T. Noiri [9] defined *WC-continuous* functions: For every open set V in Y with weakly compact complement, $f^{-1}(V)$ is open.

18. In 1992, M. K. Singal and S. B. Niemse [26] defined *Z-continuous* functions: For every open set V in Y with Zero set complement, $f^{-1}(V)$ is open.

Definition 3. If β and β^* are operators on (X, τ), the intersection operator $\beta \wedge \beta^*$ is defined as follows

$$(\beta \wedge \beta^*)(A) = \beta(A) \cap \beta^*(A)$$

The operators β and β^* are said to be mutually dual if $\beta \wedge \beta^*$ is the identity operator.

Theorem 2. *Let (X, τ) and (Y, φ) be two topological spaces and $\mathcal{I}$ a proper ideal on X. Let α, β be operators on (X, τ) and ∂, θ and θ^* be operators on (Y, φ). Then a function $f : X \to Y$ is $(\alpha, \beta, \theta \wedge \theta^*, \partial, \mathcal{I})$-continuous if and only if it is both $(\alpha, \beta, \theta, \partial, \mathcal{I})$ and $(\alpha, \beta, \theta^*, \partial, \mathcal{I})$-continuous, provided that $\beta(A \cap B) = \beta(A) \cap \beta(B)$.*

Proof. If f is both $(\alpha, \beta, \theta, \partial, \mathcal{I})$ and $(\alpha, \beta, \theta^*, \partial, \mathcal{I})$-continuous, then for every open set V in (Y, φ)

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta V) \in \mathcal{I}$$

and

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta^* V) \in \mathcal{I},$$

then

$$\left[\alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}(\theta V)\right]\cup\left[\alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}(\theta^{*}V)\right]\in\mathcal{I}.$$

But

$$\begin{aligned}
&\left[\alpha\left(f^{-1}\left(\partial V\right)\right)\setminus\beta f^{-1}\left(\theta V\right)\right]\cup\left[\alpha\left(f^{-1}\left(\partial V\right)\right)\setminus\beta f^{-1}\left(\theta^{*}V\right)\right]\\
= \ &\alpha\left(f^{-1}\left(\partial V\right)\right)\setminus\beta f^{-1}\left(\left(\theta V\right)\cap\beta f^{-1}\left(\theta^{*}V\right)\right)\\
= \ &\alpha\left(f^{-1}\left(\partial V\right)\right)\setminus\beta f^{-1}\left(\theta V\cap\theta^{*}V\right)
\end{aligned}$$

then f is $(\alpha,\beta,\theta\wedge\theta^{*},\partial,\mathcal{I})$-continuous.

Conversely, if f is $(\alpha,\beta,\theta\wedge\theta^{*},\partial,\mathcal{I})$-continuous, then

$$\alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}\left((\theta\wedge\theta^{*})V\right)\in\mathcal{I}.$$

Now, by the above equalities we get that

$$\left[\alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}(\theta V)\right]\cup\left[\alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}(\theta^{*}V)\right]\in\mathcal{I}$$

which implies

$$\alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}(\theta V)\in\mathcal{I}\ \text{ and }\ \alpha\left(f^{-1}(\partial V)\right)\setminus\beta f^{-1}(\theta^{*}V)\in\mathcal{I}$$

which means that f is both $(\alpha,\beta,\theta,\partial,\mathcal{I})$ and $(\alpha,\beta,\theta^{*},\partial,\mathcal{I})$-continuous. □

Corollary 1 (Theorem 1 in [30]). *Let (X,τ) and (Y,φ) be two topological spaces and A and B be two mutually dual expansions on Y. Then a mapping $f: X\to Y$ is continuous if and only if f is A expansion continuous and B expansion continuous.*

Proof. Take α = identity operator, $\beta = \text{Int}$,$\theta = A$, $\theta^{*} = B$, ∂ = identity operator and $\mathcal{I}=\{\emptyset\}$, then the result follows from Theorem 2. □

Corollary 2 (Corollary 28 in [10]). *Let (X,τ) and (Y,φ) be two topological spaces. A mapping $f: X\to Y$ is continuous if and only of f is almost continuous and $f^{-1}(V)\subset Intf^{-1}(\partial_{s}V)^{c}$ for each open set $V\in\varphi$*

Proof. Almost continuous equals $(id, Int, Int\ closure, id, \{\emptyset\})$-continuous. Since the operator $\Lambda: P(X)\to P(X)$ where

$$\Lambda(A) = (\partial_{s}A)^{c} = A\cup(Int\ closure\ A)^{c}$$

is mutually dual with the $Int\ closure\ A$ operator, the result follows from Theorem 2. □

In the set Φ of all operators on a topological space (X,τ) a partial order can be defined by the relation $\alpha<\beta$ if and only if $\alpha(A)\subset\beta(A)$ for any $A\in P(X)$.

Theorem 3. *Let (X, τ) and (Y, φ) be two topological spaces, $\mathcal{I}$ an ideal on X, α and β operators on (X, τ) and ∂, θ and θ^* operators on (Y, φ) with $\theta < \theta^*$. If $f : X \to Y$ is $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continuous then it is $(\alpha, \beta, \theta^*, \partial, \mathcal{I})$-continuous, provided that β is a monotone operator.*

Proof. Since f is $(\alpha, \beta, \theta, \partial, \mathcal{I})$-continuous, then for every open set V in (Y, φ) it happens that

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta V) \in \mathcal{I}.$$

Now we know that $\theta < \theta^*$, then for every $V \in \varphi$, $\theta(V) \subset \theta^*(V)$ and then $f^{-1}(\theta V) \subset f^{-1}(\theta^* V)$ and

$$\beta f^{-1}(\theta V) \subset \beta f^{-1}(\theta^* V).$$

Therefore

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta^* V) \subset \alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta V) \in \mathcal{I},$$

then

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta^* V) \in \mathcal{I},$$

which means that f is $(\alpha, \beta, \theta^*, \partial, \mathcal{I})$-continuous. □

Definition 4. An operator β on the space (X, τ) induces another operator $Int\beta$ defined as follows

$$(Int\beta)(A) = Int(\beta(A))$$

Observe that $Int\beta < \beta$.

Definition 5. A function $f : X \to Y$ satisfies the openness condition with respect to the operator β on X if for every B in Y, $\beta f^{-1}(B) \subset \beta f^{-1}(IntB)$.

Remark. If β is the interior operator it is routine verification to prove that the openness condition with respect to β is equivalent to the condition of being open.

Theorem 4. *Let (X, τ) and (Y, φ) be two topological spaces. If $f : X \to Y$ is $(\alpha, \beta, \theta, \partial, \mathcal{I})$ continuous and satisfies the openness condition with respect to the operator β, then f is $(\alpha, \beta, Int\theta, \partial, \mathcal{I})$ continuous.*

Proof. Let V be an open set in (Y, φ) we have that

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta V) \in \mathcal{I}$$

since f satisfies the openness condition with respect to the operator β , then

$$\beta f^{-1}(\theta V) \subset \beta f^{-1}(Int\theta V).$$

since

$$\alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(Int\theta V) \subset \alpha\left(f^{-1}(\partial V)\right) \setminus \beta f^{-1}(\theta V) \in \mathcal{I}$$

it follows that f is $(\alpha, \beta, Int\theta, \partial, \mathcal{I})$ continuous. □

Corollary 3 (Theorem 2.3 [27]). *Let (X, τ) and (Y, φ) be two topological spaces. If $f : X \to Y$ is weakly continuous and open then it is almost continuous.*

Proof. Let $\mathcal{I} = \{\emptyset\}$. $\alpha =$ identity operator, $\beta = Int$, $\partial =$ identity operator and $\theta =$ closure operator then the result follows from Theorem 4. □

Corollary 4. *Let (X, τ) and (Y, φ) be two topological spaces. If $f : X \to Y$ is very weakly continuous and open, then it is weak almost continuous.*

Proof. Let $\mathcal{I} = \{\emptyset\}$, $\alpha =$ identity operator, $\beta = Int$, $\partial =$ identity operator and $\theta =$ ker closure operator, then the result follows from Theorem 3. □

2 Some results on $(\alpha, Int, \theta, \partial, \{\emptyset\})$-continuous maps

Definition 6. Let β be an operator in a topological space (X, τ). We say that (X, τ) is $\beta - T_1$ if for every pair of points $x, y \in X$, $x \neq y$ there exists open sets V and W such that $x \in V$ and $y \notin \beta V$ and $y \in W$ and $x \notin \beta W$.

Observe that if β is the closure operator Cl then a space (X, τ) is T_2 if and only if it is $Cl - T_1$.

Theorem 5. *Let (X, τ) and (Y, φ) be two topological spaces, α an operator on (X, τ), θ and ∂ operators on (Y, φ) and (Y, φ) a $\theta - T_1$ space. If $f : X \to Y$ is $(\alpha, Int, \theta, \partial, \{\emptyset\})$ continuous and $A \subset \alpha(A)$ for all $A \subset X$, then f has closed point inverses.*

Proof. Let $q \in Y$ and let $a \in A = \{x \in X : f(x) \neq q\}$. Then there exists open sets V and V' in (Y, φ) such that $f(a) \in V$ and $q \notin \theta V$. By hypothesis

$$\alpha\left(f^{-1}(\partial V)\right) \subset Int f^{-1}(\theta V)$$

so there exists an open set U in (X, τ) such that

$$\alpha\left(f^{-1}(\partial V)\right) \subset U \subset f^{-1}(\theta V)$$

so $f(U) \subset \theta V$. If $b \in U \cap A^c$ then $f(b) \in \theta V$ and $f(b) = q \notin \theta V$ therefore $a \in U$ and $U \subset A$, therefore $\{x \in X : f(x) \neq q\}$ is open. □

Corollary 5 (Theorem 6 in [31]). *Let (X, τ) and (Y, φ) be two topological spaces. Let $f : X \to Y$ be a weakly continuous function. If Y is Hausdorff then f has closed point inverses.*

Proof. Let $\alpha =$ identity operator, $\beta = Int, \partial =$ identity operator, $\theta =$ Closure operator and $\mathcal{I} = \{\emptyset\}$, then the result follows from Theorem 5. □

Theorem 6. *Let (X, τ) and (Y, φ) be two topological spaces. α an operator on (X, τ), θ and ∂ operators on (Y, φ), $A \subset \alpha(A)$ $\forall A$, $A \subset X$. If $f : X \to Y$ is $(\alpha, Int, \theta, \partial, \{\emptyset\})$ continuous and K is a compact subset of X, then $f(K)$ is θ compact on Y.*

Proof. Let $\mathcal{V}$ be an open cover of $f(K)$ and suppose without lost of generality that each $V \in \mathcal{V}$ satisfies $V \cap f(K) \neq \emptyset$. Then for each $k \in K$, $f(k) \in V_k$ for some $V_k \in \mathcal{V}$. Since f is $(\alpha, Int, \theta, \partial, \{\emptyset\})$-continuous, for each $k \in K$ there exists an open set W_k in X such that

$$\alpha\left(f^{-1}(\partial V_k)\right) \subset W_k \subset f^{-1}(\theta V_k).$$

Also since $f^{-1}(\partial V_k) \subset \alpha\left(f^{-1}(\partial V_k)\right)$ for every $k \in K$ we have that the collection $\{W_k : k \in K\}$ is an open cover of K, so there exists $k_1, ..., k_n$ such that $K \subset \bigcup_{i=1}^{n}(W_{k_i})$. Then $f(K) \subset \bigcup_{i=1}^{n} f(W_{k_i})$. Therefore

$$f(K) \subset \bigcup_{i=1}^{n} \theta V_{k_i}$$

which means that $f(K)$ is θ-compact. □

Corollary 6 (Theorem 7 in [31]). *Let (X, τ) and (Y, φ) be two topological spaces. Let $f : X \to Y$ be a weakly continuous map and K a compact subset of X then $f(K)$ is an almost compact subset of Y.*

Proof. Let $\alpha =$ identity operator on X, $\beta = Int$, $\theta =$ closure operator on Y, $\partial =$ identity operator and $\mathcal{I} = \{\emptyset\}$. □

Corollary 7 (Theorem 3.2 in [25]). *Let (X, τ) and (Y, φ) be two topological spaces. Let $f : X \rightarrow Y$ be an almost continuous map and K a compact subset of X, then $f(K)$ is nearly compact.*

Proof. Let $\alpha =$ identity operator on $X, \beta = Int, \theta =$ closure operator on $Y, \partial =$ identity operator and $\mathcal{I} = \{\emptyset\}$. □

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