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NECESSARY AND SUFFICIENT CONDITIONS FOR THE EXISTENCE OF PERIODIC SOLUTIONS IN A PREDATOR-PREY MODEL ON TIME SCALES

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Abstract. This article explores the existence of periodic solutions for non-autonomous impulsive semi-ratio-dependent predator-prey systems on time scales. Based on a continuous theorem in coincidence degree theory, sharp sufficient and necessary conditions are derived in which most popular monotonic, non-monotonic and predator functional responses are applicable. This article extends the work in [6, 10, 12, 13, 14, 18, 25].

1. Introduction

It is well known that time scales were introduced by Hilger[17] in his Doctoral degree thesis to unify the continuous and discrete analysis. Today, it has become a new important branch for its tremendous potential applications in many mathematical models of real process and phenomena such as population dynamics, biotechnology, economics, neural networks and social science; see, e.g., Agarwal[1, 2], Aulbach[3], Bohner[7, 8], Erbe[11], Lakshmikantham[19] and the reference therein. In the way of time scales, not only are the results related to the set of real numbers or to the set of integers, but also pertaining to more general time scales are obtained. On the other hand, impulsive effects always occur in the simulation of process and phenomena observed in control theory, chemistry, population dynamics, industrial robotics etc. To incorporate it into those models, impulsive differential/difference equations are an adequate mathematical apparatus. The interesting in impulsive systems has grown because of the importance of both theoretical and practice need, and more richer dynamics are observed, see, e.g. [9, 4, 20].

Let $\mathbb{T}$ be a time scale; i.e., $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$(see Definition 2.1–2.5 in Section 2), $\{t_k\}_{k\in\mathbb{N}} \subset \mathbb{T}$ ($\mathbb{N}$ is the set of positive integers) is the impulsive moment sequence with $t_0 = \min\{[0,\infty)\cap\mathbb{T}\} < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty, k \in \mathbb{N}$. In present paper, we consider following impulsive semi-ratio-dependent

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predator-prey model on time scale $\mathbb{T}$

$$
\begin{gathered}
x_1^{\Delta}(t) = a(t) - b(t)\mathrm{e}^{x_1(t)} - \varphi\big(t, \mathrm{e}^{x_1(t)}, \mathrm{e}^{x_2(t)}\big)\mathrm{e}^{x_2(t)-x_1(t)}, \quad t \in \mathbb{T}\backslash t_k, \\
x_2^{\Delta}(t) = d(t) - \beta(t)\mathrm{e}^{x_2(t)-x_1(t)}, \quad t \in \mathbb{T}\backslash t_k, \\
\Delta x_1(t) = \ln(1 + c_{1k}), \quad t = t_k,\ k \in \mathbb{N}, \\
\Delta x_2(t) = \ln(1 + c_{2k}), \quad t = t_k,\ k \in \mathbb{N},
\end{gathered}
\tag{1.1}
$$

where $x_1(t)$ and $x_2(t)$ stand for the population (or density) of the prey and the predator, respectively. The same symbol Δ in (1.1) in differential positions has different meanings, we think, which are easily distinguished by readers, that is, $x_i^{\Delta}(t)$ is the delta-derivative of x_i at t, and $\Delta x_i(t) = x_i(t^+) - x_i(t^-) = \lim_{s\to t^+} x_i(s) - \lim_{s\to t^-} x_i(s)$, $i = 1, 2$ are impulsive perturbations. A natural constraint is $1+c_{ik} > 0$, $k \in \mathbb{N}$, $i = 1, 2$. In (1.1), it has been assumed that the prey grows logistically with growth rate a and carrying capacity a/b in the absence of predation. The predator consumes the prey according to the function response $\varphi(t, x, y)$ and grow logistically with growth rate d and carrying capacity x/β proportional to the population size of prey (or prey abundance). The parameter β is a measure of the food quality that the prey provides for conversion into predator birth.

As mentioned above, time scales can unify continuous and discrete analysis. If $\mathbb{T} = \mathbb{R}$, (1.1) reduces the following impulsive differential equations

$$
\begin{gathered}
x_1'(t) = a(t) - b(t)\mathrm{e}^{x_1(t)} - \varphi(t, \mathrm{e}^{x_1(t)}, \mathrm{e}^{x_2(t)})\mathrm{e}^{x_2(t)-x_1(t)}, \quad t \in \mathbb{R}\backslash t_k, \\
x_2'(t) = d(t) - \beta(t)\mathrm{e}^{x_2(t)-x_1(t)}, \quad t \in \mathbb{R}\backslash t_k, \\
\Delta x_1(t) = \ln(1 + c_{1k}), \quad t = t_k, k \in \mathbb{N}, \\
\Delta x_2(t) = \ln(1 + c_{2k}), \quad t = t_k, k \in \mathbb{N},
\end{gathered}
\tag{1.2}
$$

or its equivalent form

$$
\begin{gathered}
x'(t) = x(t)[a(t) - b(t)x(t)] - \varphi(t, x(t), y(t))y(t), \quad t \in \mathbb{R}\backslash t_k, \\
y'(t) = y(t)[d(t) - \frac{\beta(t)y(t)}{x(t)}], \quad t \in \mathbb{R}\backslash t_k, \\
\Delta x(t) = c_{1k}x(t), \quad t = t_k, k \in \mathbb{N}, \\
\Delta y(t) = c_{2k}y(t), \quad t = t_k, k \in \mathbb{N}.
\end{gathered}
\tag{1.3}
$$

If $\mathbb{T} = \mathbb{Z}$, then $\{t_k\} \subset \mathbb{Z}$ and system (1.1) may turn into the following impulsive difference equations

$$
\begin{gathered}
x(t + 1) = x(t) \exp(a(t) - b(t)x(t) - \varphi(t, x(t), y(t))\frac{y(t)}{x(t)}), \quad t \in \mathbb{Z}\backslash t_k, \\
y(t + 1) = y(t) \exp(d(t) - \beta(t)\frac{y(t)}{x(t)}), \quad t \in \mathbb{Z}\backslash t_k, \\
\Delta x(t_k + 1) = (1 + c_{1k})x(t_k), \quad k \in \mathbb{N}, \\
\Delta y(t_k + 1) = (1 + c_{2k})y(t_k), \quad k \in \mathbb{N}.
\end{gathered}
\tag{1.4}
$$

The key term $\varphi(t, x, y)$ in (1.1) is called functional response, which is the rate of prey consumption by an average predator and can be classified as prey-dependence and predator-dependence. The response is a function of prey alone in prey-dependence while both predator and prey density have an effect on the response in predator-dependence. After the classical work of Lotka[21] and Volterra[22], various prey-dependent responses are presented. For example, $\varphi_1(t, x) = r(t)x, \varphi_2(t, x) =$

$r(t)x/(A(t)+x), \varphi_3(t,x) = r(t)x^2/(A(t)+x^2)$ and $\varphi_4(t,x) = r(t)x/(A(t)+B(t)x+C(t)x^2)$ are well known as Holling type I, II, III and IV respectively. Particularly, φ_4 is non-monotone and declines at high prey densities, while φ_1-φ_3 are monotone, and in more general case is $\varphi_5(t,x) = r(t)x^\theta/(A(t)+x^\theta)$, $\theta > 2$ which is known as the sigmoidal response. Similar monotone responses as $\varphi_6(t,x) = r(t)x^2/((A(t)+x)(D(t)+x))$ and $\varphi_7(t,x) = r(t)(1-\mathrm{e}^{-A(t)x})$ can be found Freedman[15].

On the other hand, there are evidences to show predator density also has an effect on functional response. A typical predator-dependent response is proposed by Beddington and DeAngelis, now, popular referred to as Beddington-DeAngelis functional response taking the form $\varphi_8(t,x,y) = r(t)x/(A(t)+B(t)x+C(t)y)$. Recently, D. Miller etc[24] proposed the following modified Holling type II and III response $\varphi_9(t,x,y) = r(t)x/((A(t)+x)(D(t)+y))$, $\varphi_{10}(t,x,y) = r(t)x^2/((A(t)+x^2)(D(t)+y))$. This dynamical relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance[5].

It is an interesting topic to explore the existence of periodic solutions in nonautonomous semi-ratio-dependent predator-prey dynamical systems; see, e.g. [6, 10, 12, 13, 14, 18, 25] and the reference therein. For the case without impulses (i.e, $c_{ik} = 0$, $i = 1,2, k \in \mathbb{N}$), the existence of periodic solutions for system (1.1)–(1.4) has been studied by many authors. For example, Huo and Li[18] considered system (1.3) with $\varphi(t,x,y) = \varphi_1$, which is called Leslie-Gower system. For more general monotone functional responses in (1.3), some criteria of existence are presented by Wang et al[25]. Ding et al[10] also establish a criterion for (1.3) with non-monotone functional response φ_4. The discrete analogue (1.4) was then explored by Fazly and Hesaaraki[13], Fan and Wang[12]. Recently, Bohner et al. [6], Fazly and Hesaaraki [14] investigate the dynamical system (1.1) on time scales with monotone functional responses $\varphi_1 - \varphi_3$ and $\varphi_5 - \varphi_7$. Especially, for some widely recognized functional responses which are not monotone such as φ_4, some sufficient conditions are derived in [14].

In this paper, our approach is based on continuation theorem developed by Gaines and Mawhin[16] and also used by many authors. However, by the invariance property of homotopy and analysis technique, we establish some new sufficient and necessary results where the exponential or monotone conditions are not necessary, which improves and extends many previous work in the literature [6, 10, 12, 13, 14, 18, 25]; see, Remark 3.5, Remark 4.2, Remark 4.4, and Proposition 4.5.

The rest of the paper is arranged as follows. In Section 2, we introduce some notation and concepts for time scales and continuous theorem of coincidence, at the same time we give some necessary lemmas. In Section 3, we establish new sharp conditions for the existence of periodic solutions for system (1.1). Its applications then are illustrated in Section 4.

2. Preliminaries

Denote $\mathbb{R}, \mathbb{R}_+, \mathbb{Z}, \mathbb{Z}_+$ and $\mathbb{N}$ are real numbers set, non-negative real numbers set, integer numbers set, non-negative integer numbers set and positive integer numbers set respectively. For the convenience of the reader, we list some definitions and notations on the time scale calculus as follows. These definitions and notations are common in the related literature.

Definition 2.1. A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of $\mathbb{R}$. The set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.

Let $\omega > 0$, throughout this paper, the time scale $\mathbb{T}$, impulsive sequence and impulsive functions are assumed to be ω-periodic; i.e., $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$ and there exists an integer $p \geq 1$ such that $I_\omega \cap \{t_k\} = \{t_1, t_2, \dots, t_p\}$, $t_{k+p} = t_k + \omega$, $c_{ik} = c_{i(k+p)}$, $i = 1, 2,\ k \in \mathbb{N}$. Some examples of such time scales are

$$\mathbb{R},\ \mathbb{Z},\ \cup_{k\in\mathbb{Z}}[2k, 2k+1].$$

Definition 2.2. For $t \in \mathbb{T}$, the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ and the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ are defined by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T} : s < t\},$$

respectively.

In this definition, a point $t \in \mathbb{T}$ is said to be left-dense if $\rho(t) = t$, left-scattered if $\rho(t) < t$, left-dense if $\sigma(t) = t$ and right-scattered if $\sigma(t) > t$. The graininess μ of the time scale is defined by $\mu(t) = \sigma(t) - t$.

Definition 2.3. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sides limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions is shown by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.4. For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}$ we define $f^\Delta(t)$, the delta-derivative of f at t, to be the number (provided it exists) with the property that, given any $\epsilon > 0$, there is a neighborhood U of t in $\mathbb{T}$ such that for any $s \in U$ it holds that

$$|[f(\sigma(t)) - f(s)] - f^\Delta(t)[\sigma(t) - s]| \leq \epsilon|\sigma(t) - s|.$$

Thus, f is said to be delta-differential if its delta-derivative exists. The set of function $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative are rd-continuous functions is denoted by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.5. A function $F : \mathbb{T} \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta(t) = f(t)$, for all $t \in \mathbb{T}$. Then for all $a, b \in \mathbb{T}$ we write

$$\int_a^b f(s)\Delta s = F(b) - F(a).$$

Lemma 2.6 (Existence of Antiderivatives)**.** *Every rd-continuous function has an antiderivative. In particular if $t_0 \in \mathbb{T}$, then F defined by*

$$F(t) = \int_{t_0}^t f(\tau)\Delta\tau, \quad \textit{for } t \in \mathbb{T}$$

is an antiderivative of f.

In fact, for the usual time scale $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$ we have

$$\sigma(t) = \rho(t) = t, \quad \mu(t) = 0, \quad f^\Delta(t) = f'(t),$$

$$\int_a^b f(t)\Delta t = \int_a^b f(t)\,\mathrm{d}t, \quad \sigma(t) = t + 1, \quad \rho(t) = t - 1,$$

$$\mu(t) = 1, \quad f^\Delta(t) = f(t+1) - f(t), \quad \int_a^b f(t)\Delta t = \sum_{t=a}^{b-1} f(t),$$

respectively. For more information about the above definitions and their related concepts, the reader is referred to [7, 8, 19].

The method to be used in this paper involves the application of the continuous theorem of coincidence degree. It is necessary to introduce some concepts and results from Gaines and Mawhin[16].

Let X, Z be two Banach spaces, $L : X \cap \mathrm{Dom}L \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\ker L = \operatorname{codim}\operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$, $\operatorname{Im} L = \ker Q = \operatorname{Im}(I-Q)$, then it follows that $L|_{\mathrm{Dom}\, L\cap\ker P} : (I-P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_p. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N : \overline{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \operatorname{Im} Q \to \ker L$.

Lemma 2.7. *Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose that*

1. *$Lx \neq \lambda Nx$, for any $x \in \partial\Omega$ and $\lambda \in (0, 1)$;*
2. *$QNx \neq 0$, for any $x \in \partial\Omega \cap \ker L$;*
3. *$\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.*

Then the operator equation $Lx = Nx$ has at least one solution in $\mathrm{Dom}\, L \cap \bar{\Omega}$.

In what follows in this section, we will translate (1.1) into its equivalent operator equations. Firstly of all, the following notation are introduced

$$I_\omega = [t_0, t_0 + \omega] \cap \mathbb{T}, \quad \widehat{f} = \frac{1}{\omega}\int_{I_\omega} f(t)\Delta t,$$

where $f \in C_{rd}(\mathbb{T})$ is an ω-periodic real function. Moreover, we denote

$$PC_\omega = \Big\{\phi : \mathbb{T} \to \mathbb{R} : \text{(i) } \phi(t) \text{ is rd-continuous for } t \in \mathbb{T}\backslash t_k \text{ and } \omega\text{-periodic;} \\ \text{(ii) } \lim_{s\to t_k-0}\phi(s) = \phi(t_k^-) \text{ and } \lim_{s\to t_k+0}\phi(s) = \phi(t_k^+) \text{ exist.}\Big\},$$

and

$$PC^1_\omega = \{\phi \in PC_\omega : \ \phi^\Delta \in PC_\omega\}.$$

Let

$$X = \{x = (x_1, x_2)^{\mathrm{T}} : x_i \in PC_\omega, i = 1, 2\},\ Z = X \times \mathbb{R}^{2p}$$

with the norms

$$\|x\|_X = \sum_{i=1}^{2}\sup_{t\in I_\omega} |\ x_i(t), \quad x = (x_1, x_2)^{\mathrm{T}} \in X,$$
$$\|z\|_Z = \|x\|_X + \|y\|, \quad z = (x, y) \in Z,$$

where $\|\cdot\|$ is the Euclidean norm of $\mathbb{R}^{2p}$. Then X and Z are Banach spaces when they are endowed with above norms. Set

$$L : \mathrm{Dom}\, L \cap X \to Z, \quad Lx = (x^\Delta, \Delta x(t_1), \dots, \Delta x(t_p)), \quad x = (x_1, x_2)^{\mathrm{T}} \in X$$

with $\mathrm{Dom}\, L = \{x = (x_1, x_2)^{\mathrm{T}} \mid x_i \in PC^1_\omega, i = 1, 2\}$ and $N : X \to Z$ as

$$Nx = \begin{bmatrix} x_1^\Delta(t), & \ln(1 + c_{11}), & \dots, & \ln(1 + c_{1p}) \\ x_2^\Delta(t), & \ln(1 + c_{21}), & \dots, & \ln(1 + c_{2p}) \end{bmatrix} \quad x \in X.$$

Using this notation we may rewrite (1.1) in the equivalent form $Lx = Nx$, $x \in X$. So, $\ker L = \mathbb{R}^2$, $\operatorname{Im} L = \{z = (\phi, \gamma_1, \dots, \gamma_p) \in Z : \int_{I_\omega} \phi(t)\Delta t + \sum_{k=1}^{p} \gamma_k = 0\}$ is closed in Z, and $\dim\ker L = \operatorname{codim}\operatorname{Im} L = 2$. Thus, L is a Fredholm mapping of index zero. Define two projections $P : X \to X$ and $Q : Z \to Z$ as

$$Px = \widehat{x}, \quad x \in X,$$
$$Qz = Q(x, \gamma_1, \dots, \gamma_p) = (\widehat{x} + \frac{1}{\omega}\sum_{k=1}^{p} \gamma_k, 0, \dots, 0).$$

It is trivial to show that P, Q are continuous projections such that

$$\operatorname{Im} P = \ker L, \quad \operatorname{Im} L = \ker Q = \operatorname{Im}(I - Q),$$

and hence, the generalized inverse K_p exists. For $x \in \operatorname{Dom} L \subset X$, it is not difficult to get

$$QNx = \frac{1}{\omega}\begin{pmatrix} \int_{I_\omega} x_1^\Delta(t)\Delta t + \sum_{k=1}^{p} \ln(1 + c_{1k}), & 0, & \dots, & 0 \\ \int_{I_\omega} x_2^\Delta(t)\Delta t + \sum_{k=1}^{p} \ln(1 + c_{2k}), & 0, & \dots, & 0 \end{pmatrix},$$

and

$$\begin{aligned}
&K_p(I - Q)Nx \\
&= \begin{pmatrix} \int_0^t x_1^\Delta(s)\Delta s + \sum_{k=i}^{p} \ln(1 + c_{1k}) \\ \int_0^t x_2^\Delta(s)\Delta s + \sum_{k=i}^{p} \ln(1 + c_{2k}) \end{pmatrix} \\
&\quad - (\frac{t}{\omega} - \frac{1}{2})\begin{pmatrix} \int_{I_\omega} x_1^\Delta(t)\Delta t + \sum_{k=i}^{p} \ln(1 + c_{1k}) \\ \int_{I_\omega} x_2^\Delta(t)\Delta t + \sum_{k=i}^{p} \ln(1 + c_{2k}) \end{pmatrix} \\
&\quad - \frac{1}{\omega}\begin{pmatrix} \int_{I_\omega}\int_0^t x_1^\Delta(s)\Delta s\Delta t + \omega\sum_{k=1}^{p} \ln(1 + c_{1k}) - \sum_{k=1}^{p} \ln(1 + c_{1k})t_k \\ \int_{I_\omega}\int_0^t x_2^\Delta(s)\Delta s\Delta t + \omega\sum_{k=1}^{p} \ln(1 + c_{2k}) - \sum_{k=1}^{p} \ln(1 + c_{2k})t_k \end{pmatrix}.
\end{aligned}$$

Clearly, QN and $K_p(I - Q)$ are continuous. It is not difficult to show that $\overline{K_p(I - Q)N(\overline{\Omega})}$ is compact for any open-bounded set $\Omega \subset X$. In addition, $QN(\overline{\Omega})$ is bounded. Therefore, N is L compact on $\overline{\Omega}$ with any open-bounded set $\Omega \subset X$.

3. Main Results

Theorem 3.1. *Assume that he following conditions hold.*

- (H1) *$a(t), b(t), d(t)$ and $\beta(t)$ are non-negative ω-periodic rd-continuous real functions and $\hat{a} > 0$, $\hat{d} > 0$;*
- (H2) *The functional response $\varphi : \mathbb{T} \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is rd-continuous and ω-periodic with respect to t, $\varphi(t, 0, y) = 0$ for any $t \in \mathbb{T}$, $y \geq 0$. In addition, there exist $m \in \mathbb{N}$ and ω-periodic rd-continuous functions $\alpha_i : \mathbb{T} \to \mathbb{R}_+$, $i = 0, \dots, m - 1$ such that*
$$\varphi(t, x, y) \leq \alpha_0(t)x^m + \dots + \alpha_{m-1}(t)x \tag{3.1}$$
for $t \in \mathbb{T}$, $x \geq 0$, $y \geq 0$.

Then, system (1.1) *has at least one ω-periodic solution if and only if*

$$\widehat{a}\omega + \sum_{k=1}^{p} \ln(1 + c_{1k}) > 0, \quad \widehat{d}\omega + \sum_{k=1}^{p} \ln(1 + c_{2k}) > 0. \tag{3.2}$$

Proof. First, suppose $(x(t), y(t))$ is a periodic solution of (1.1), integrating both sides of the first two equations in (1.1) on I_ω, we have

$$\begin{aligned}\widehat{a}\omega + \sum_{k=1}^{p} \ln(1+c_{1k}) &= \int_{I_\omega} b(t)\exp(x_1(t))\Delta t \\ &\quad + \int_{I_\omega} \varphi(t, \exp(x_1(t)), \exp(x_2(t)))\exp(x_2(t)-x_1(t))\Delta t > 0, \\ \widehat{d}\omega + \sum_{k=1}^{p} \ln(1+c_{2k}) &= \int_{I_\omega} \beta(t)\exp(x_2(t)-x_1(t))\Delta t > 0,\end{aligned}$$

which shows the condition is necessary.

Next, we show it is sufficient. By Lemma 2.7, it suffices to search for an appropriate open bounded subset $\Omega \subset X$. For some $\lambda \in (0,1)$, suppose that $(x_1, x_2)^{\mathrm{T}} \in X$ is a solution of

$$\begin{gathered} x_1^{\Delta}(t) = \lambda\Big[a(t) - b(t)\mathrm{e}^{x_1(t)} - \varphi\big(t, \mathrm{e}^{x_1(t)}, \mathrm{e}^{x_2(t)}\big)\mathrm{e}^{x_2(t)-x_1(t)}\Big], \quad t \in \mathbb{T}\backslash t_k, \\ x_2^{\Delta}(t) = \lambda\Big[d(t) - \beta(t)\mathrm{e}^{x_2(t)-x_1(t)}\Big], \quad t \in \mathbb{T}\backslash t_k, \\ \Delta x_1(t) = \lambda\ln(1+c_{1k}), \quad t = t_k,\ k \in \mathbb{N}, \\ \Delta x_2(t) = \lambda\ln(1+c_{2k}), \quad t = t_k,\ k \in \mathbb{N}. \end{gathered} \tag{3.3}$$

Integrating both sides of the first and second equation of (3.3) over I_ω, one has

$$\begin{aligned} &\widehat{a}\omega + \sum_{k=1}^{p} \ln(1+c_{1k}) \\ &= \int_{I_\omega} b(t)\exp(x_1(t))\Delta t \\ &\quad + \int_{I_\omega} \varphi(t, \exp(x_1(t)), \exp(x_2(t)))\exp(x_2(t)-x_1(t))\Delta t, \end{aligned} \tag{3.4}$$

$$\widehat{d}\omega + \sum_{k=1}^{p} \ln(1+c_{2k}) = \int_{I_\omega} \beta(t)\exp(x_2(t)-x_1(t))\Delta t. \tag{3.5}$$

Thus, it follows that

$$\begin{gathered} \int_{I_\omega} |x_1^{\Delta}(t)|\Delta t \le 2\widehat{a}\omega + \sum_{k=1}^{p} \ln(1+c_{1k}), \\ \int_{I_\omega} |x_2^{\Delta}(t)|\Delta t \le 2\widehat{d}\omega + \sum_{k=1}^{p} \ln(1+c_{2k}). \end{gathered} \tag{3.6}$$

For any $(x_1, x_2)^{\mathrm{T}} \in X$, clearly there exist $\zeta_i,\ \eta_i \in I_\omega$, $i = 1, 2$, such that

$$x_i(\zeta_i)(\text{or } x_i(\zeta_i^+)) = \inf_{t\in I_\omega} x_i(t), \quad x_i(\eta_i)(\text{or } x_i(\eta_i^+)) = \sup_{t\in I_\omega} x_i(t), \quad i = 1, 2.$$

We only consider the following case (other cases are proved only by replacing $\zeta_i(\eta_i)$ by $\zeta_i^+(\eta_i^+)$)

$$x_i(\zeta_i) = \inf_{t\in I_\omega} x_i(t),\ x_i(\eta_i) = \sup_{t\in I_\omega} x_i(t),\ i = 1, 2. \tag{3.7}$$

For any $\xi \in I_\omega$, it holds that

$$
\begin{aligned}
x_i(t) &\ge x_i(\xi) - \int_{I_\omega} |x_i^\Delta(s)|\Delta s - \sum_{k=1}^p |\ln(1+c_{ik})|, \quad i=1,2,\\
x_i(t) &\le x_i(\xi) + \int_{I_\omega} |x_i^\Delta(s)|\Delta s + \sum_{k=1}^p |\ln(1+c_{ik})|, \quad i=1,2.
\end{aligned}
$$

Now we find the bound from above for solutions of (3.3). By (3.4) and the first equation in (3.6), we have

$$
x_1(\zeta_1) \le \ln \frac{\widehat{a}\omega + \sum_{k=1}^p \ln(1+c_{1k})}{\widehat{b}\omega}.
$$

Thus,

$$
x_1(t) \le \ln \frac{\widehat{a}\omega + \sum_{k=1}^p \ln(1+c_{1k})}{\widehat{b}\omega} + 2\widehat{a}\omega + 2\sum_{k=1}^p |\ln(1+c_{1k})| := M_1. \tag{3.8}
$$

By (3.5) and (3.7), one has

$$
\widehat{d}\omega + \sum_{k=1}^p \ln(1+c_{2k}) \ge \widehat{\beta}\omega \exp(x_2(\zeta_2) - x_1(\eta_1)), \tag{3.9}
$$

$$
\widehat{d}\omega + \sum_{k=1}^p \ln(1+c_{2k}) \le \widehat{\beta}\omega \exp(x_2(\eta_2) - x_1(\zeta_1)). \tag{3.10}
$$

It follows from (3.8) and (3.9) that

$$
x_2(\zeta_2) \le M_1 + \ln \frac{\widehat{d}\omega + \sum_{k=1}^p \ln(1+c_{2k})}{\widehat{\beta}\omega} := A
$$

which leads to

$$
x_2(t) \le A + 2\widehat{d}\omega + 2\sum_{k=1}^p |\ln(1+c_{2k})| := M_2.
$$

Next, we find a bound from below for solutions of (3.3). This technique is similar to that in [13]. By (3.4) and the condition (3.1) of Theorem 3.1 we have

$$
\begin{aligned}
\widehat{a} + \frac{1}{\omega}\sum_{k=1}^p \ln(1+c_{1k}) &\le \widehat{b}\exp(x_1(\eta_1)) + \big[\widehat{\alpha}_0(\exp(x_1(\eta_1)))^{m-1} \\
&\quad + \widehat{\alpha}_1(\exp(x_1(\eta_1)))^{m-2} + \dots + \widehat{\alpha}_{m-1}\big]\exp(x_2(\eta_2)).
\end{aligned} \tag{3.11}
$$

There are two cases: $x_1(\eta_1) \le x_2(\eta_2)$ and $x_1(\eta_1) \ge x_2(\eta_2)$.

Case (1): $x_1(\eta_1) \le x_2(\eta_2)$. By (3.11), we obtain

$$
x_2(\eta_2) \ge \ln\Big[\frac{\widehat{a} + \frac{1}{\omega}\sum_{k=1}^p \ln(1+c_{1k})}{(\widehat{b} + \widehat{\alpha}_{m-1}) + \widehat{\alpha}_{m-2}\exp(W_{M1}) + \dots + \widehat{\alpha}_0(\exp(W_{M1}))^{m-1}}\Big] := B,
$$

hence,

$$
x_2(t) \ge B - 2\widehat{d}\omega - 2\sum_{k=1}^p |\ln(1+c_{2k})| := l_2^{(1)}.
$$

Therefore, by (3.9), one has $x_1(\eta_1) \geq l_2^{(1)} + \ln[\frac{\widehat{\beta}\omega}{\widehat{d}\omega + \sum_{k=1}^{p}\ln(1+c_{2k})}]$ and then

$$x_1(t) \geq l_2^{(1)} + \ln[\frac{\widehat{\beta}\omega}{\widehat{d}\omega + \sum_{k=1}^{p}\ln(1+c_{2k})}] - 2\widehat{a}\omega - 2\sum_{k=1}^{p}|\ln(1+c_{1k})| := l_1^{(1)}.$$

Case (2): $x_1(\eta_1) \geq x_2(\eta_2)$. In this case, by (3.11) we may have

$$\begin{aligned}\widehat{a} + \frac{1}{\omega}\sum_{k=1}^{p}\ln(1+c_{1k}) &\leq \widehat{\alpha}_0(\exp(x_1(\eta_1)))^m + \dots \\ &\quad + \widehat{\alpha}_{m-2}(\exp(x_1(\eta_1)))^2 + (\widehat{b} + \widehat{\alpha}_{m-1})\exp(x_1(\eta_1)).\end{aligned} \tag{3.12}$$

Consider the function

$$\varpi(t) = \widehat{\alpha}_0 t^m + \dots + (\widehat{b} + \widehat{\alpha}_{m-1})t,$$

which is increasing for $t \geq 0$ with $\lim_{t\to\infty}\varpi(t) = \infty$ and $\varpi(0) = 0$, so there exists $t^* > 0$ such that $\varpi(t^*) = \widehat{a} + \frac{1}{\omega}\sum_{k=1}^{p}\ln(1+c_{1k})]$. The inequality (3.12) implies $\exp(x_1(\eta_1)) \geq t^*$, namely, $x_1(\eta_1) \geq \ln t^*$, which yields

$$x_1(t) \geq \ln t^* - 2\widehat{a}\omega - 2\sum_{k=1}^{p}|\ln(1+c_{1k})| := l_1^{(2)}. \tag{3.13}$$

Thus, by (3.10) and (3.13), one has

$$x_2(\eta_2) \geq l_1^{(2)} + \ln[\frac{\widehat{d}\omega + \sum_{k=1}^{p}\ln(1+c_{2k})}{\widehat{\beta}\omega}]$$

and hence

$$x_2(t) \geq l_1^{(2)} + \ln[\frac{\widehat{d}\omega + \sum_{k=1}^{p}\ln(1+c_{2k})}{\widehat{\beta}\omega}] - 2\widehat{d}\omega - 2\sum_{k=1}^{p}|\ln(1+c_{2k})| := l_2^{(2)}.$$

Choose $l_i = \min\{l_i^{(1)}, l_i^{(2)}\}$, $i = 1,2$ such that any solution of (3.3) satisfies

$$|x_i(t)| \leq \max\{|M_i|, |l_i|\} := W_i, \quad i = 1,2. \tag{3.14}$$

Clearly, W_1, W_2 are independent of λ.

Consider the algebraic equations

$$\begin{gathered}\widehat{a} - \widehat{b}\exp(x_1) + \frac{1}{\omega}\sum_{k=1}^{p}\ln(1+c_{1k}) \\ -\frac{\exp(x_2 - x_1)}{\omega}\int_{I_\omega}\mu\varphi(t, \exp(x_1), \exp(x_2))\Delta t = 0, \\ \widehat{d} + \frac{1}{\omega}\sum_{k=1}^{p}\ln(1+c_{2k}) - \widehat{\beta}\exp(x_2 - x_1) = 0\end{gathered} \tag{3.15}$$

for $(x_1, x_2)^{\mathrm{T}} \in \mathbb{R}^2$, where $\mu \in [0,1]$. Replace φ with $\mu\varphi$ in (3.3), then the key inequality (3.11) still holds. So carry out similar arguments as above, any solution (x_1^*, x_2^*) of (3.15) with $\mu \in [0,1]$ also satisfies

$$|x_i^*| \leq \max\{|M_i|,\ |l_i|\} := W_i, \quad i = 1,2. \tag{3.16}$$

Define $\Omega = \{x \in X | \|x\|_X < W\}$ with $W > W_1 + W_2$, it is easy to see Ω satisfies the condition (1) of Lemma 2.7. Let $x \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^2$, then x is a constant vector in $\mathbb{R}^2$ with $\|x\|_X = W$. Hence from (3.15) and the definition of W, we see that $QNx \neq 0$.

Define a homotopy

$$H_\mu((x_1,x_2)^{\mathrm{T}}) = \mu QN((x_1,x_2)^{\mathrm{T}}) + (1-\mu)U((x_1,x_2)^{\mathrm{T}}), \quad \mu \in [0,1], \tag{3.17}$$

where

$$U((x_1,x_2)^{\mathrm{T}}) = \begin{pmatrix} \widehat{a} - \widehat{b}\exp(x_1) + \frac{1}{\omega}\sum_{k=1}^{p}\ln(1+c_{1k}) \\ \widehat{d} + \frac{1}{\omega}\sum_{k=1}^{p}\ln(1+c_{2k}) - \widehat{\beta}\exp(x_2 - x_1) \end{pmatrix}. \tag{3.18}$$

then it follows from (3.14) and (3.16) that $H_\mu(x) \neq 0$ for $x \in \partial\Omega \cap \ker L$ and $\mu \in [0,1]$. In addition, it is clear that the algebraic equation $U((x_1,x_2)^{\mathrm{T}}) = 0$ has a unique solution in $\mathbb{R}^2$. Choose the isomorphism J to be the identity mapping, by a direct computation and the invariance property of homotopy, so

$$\deg\{JQN, \Omega \cap \ker L, 0\} = \deg\{QN, \Omega \cap \ker L, 0\} = \deg\{U, \Omega \cap \ker L, 0\} \neq 0,$$

where $\deg(\cdot,\cdot,\cdot)$ is the Brouwer degree.

Therefore, Ω defined above satisfies all conditions of Lemma 2.7, then, the system (1.1) has at least one ω-periodic solution in $\mathrm{Dom}\, L \cap \overline{\Omega}$. This completes the proof. $\square$

For the time scale of non-negative real numbers; i.e., $\mathbb{T} = \mathbb{R}_+ = [0,\infty)$, Equaton (1.1) reduces to (1.2) or its equivalent form (1.3), and $\widehat{f} = \frac{1}{\omega}\int_0^\omega f(s)\,\mathrm{d}s$ for ω-periodic function f. Correspondingly, we have following result.

Theorem 3.2. *Suppose* (1.3) *satisfies the following assumptions:*

- (R1) *$a(t), b(t), d(t)$ and $\beta(t)$ are non-negative ω-periodic continuous real functions and $\hat{a} > 0$, $\hat{d} > 0$;*
- (R2) *The functional response $\varphi : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and ω-periodic with respect to t, $\varphi(t,0,y) = 0$ for any $t \in \mathbb{R}_+$, $y \geq 0$. In addition, there exists positive integer m and ω-periodic continuous functions $\alpha_i : \mathbb{R} \to \mathbb{R}_+$, $i = 0,\dots,m-1$ such that*
$$\varphi(t,x,y) \leq \alpha_0(t)x^m + \dots + \alpha_{m-1}(t)x \tag{3.19}$$
for $t \in \mathbb{R}_+, x \geq 0, y \geq 0$.

Then, system (1.3) *has at least one ω-periodic solution if and only if*

$$\int_0^\omega a(s)\,\mathrm{d}s + \sum_{k=1}^{p}\ln(1+c_{1k}) > 0, \quad \int_0^\omega d(s)\,\mathrm{d}s + \sum_{k=1}^{p}\ln(1+c_{2k}) > 0.$$

If $\mathbb{T}$ is another usual time scale $\mathbb{Z}_+ = \{0,1,\dots,n,\dots,\}$, Equation (1.1) reduces to impulsive difference system (1.4) and $\widehat{f} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} f(k)$. Similarly, we have following theorem.

Theorem 3.3. *Assume that in system* (1.4) *the following conditions hold.*

- (Z1) *$a(j), b(j), d(j)$ and $\beta(j)$ are non-negative ω-periodic real sequences and $\hat{a} > 0$, $\hat{d} > 0$;*
- (Z2) *The functional response $\varphi : \mathbb{Z}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is ω-periodic with respect to t, $\varphi(j,0,y) = 0$ for any $j \in \mathbb{Z}_+$, $y \geq 0$. In addition, there exists positive integer m and ω-periodic sequences $\alpha_i : \mathbb{Z}_+ \to \mathbb{R}_+$, $i = 0,\dots,m-1$ such that*
$$\varphi(j,x,y) \leq \alpha_0(j)x^m + \dots + \alpha_{m-1}(j)x \tag{3.20}$$

for $j \in \mathbb{Z}_+, x \geq 0, y \geq 0$.

Then, system (1.4) *has at least one positive* ω*-periodic solution if and only if*

$$\sum_{k=0}^{\omega-1} a(k) + \sum_{k=1}^{p} \ln(1 + c_{1k}) > 0, \quad \sum_{k=0}^{\omega-1} d(k) + \sum_{k=1}^{p} \ln(1 + c_{2k}) > 0.$$

In particular, when $c_{ik} = 0$, $i = 1, 2, k \in \mathbb{N}$, the impulses in (1.1) disappear. In this case, (3.2) holds naturally. By Theorem 3.1, we have the following corollary.

Corollary 3.4. *It must have at least one* ω*-periodic positive solution if* (H1)–(H2) *are satisfied in system* (1.1) *with* $c_{ik} = 0$, $i = 1, 2, k \in \mathbb{N}$.

Remark 3.5. When $c_{ik} = 0$, $k \in \mathbb{N}$, $i = 1, 2$, the existence of periodic solutions of dynamical system (1.1) on time scales with monotonic functional responses φ_1–φ_3 and φ_5–φ_7 is investigated by Bohner et al [6]. The following conditions (the notation used here are same as that in corresponding papers cited)

- (i) $c(t, x) \leq C_0(t)x$ and $\bar{b}\bar{e} > \bar{C}_0\bar{d}\exp\{(a + |\bar{a}| + d + |\bar{d}|)\omega\}$ in [6, Theorem 3.4];
- (ii) $c(t, x) \leq C_1(t)$ and $e^l\bar{a} > C_1^u\bar{d}$ in [6, Theorem 3.5]

can be eliminated.

Moreover, our results are applicable for both monotone and nonmonotone functional responses. By Corollary 3.4, the conditions

- (iii) global monotonicity of response function $f(t, x)$ with respect to x in [14, Theorem 1] ;
- (iv) $\bar{f}(\cdot, x)$ is monotone function for $0 < x < \bar{a}/\bar{b}$ in [14, Theorem 2];
- (v) $A^l > (\bar{a}/\bar{b})^2$ in [14, Corollary 3]

are not necessary. Thus, our results improve and extend their related work.

4. Applications

In this section, we apply Theorem 3.2, Theorem 3.3 and Corollary 3.4 to impulsive differential/difference systems (1.3)–(1.4) with all kinds of functional responses mentioned in Section 1. The functions r, A, B, C and D, appearing in the functional responses φ_1–φ_{10}, are all ω-periodic and $r(t) > 0$, $A(t) > 0$, $B(t) \geq 0$, $C(t) \geq 0$ and $D(t) > 0$. Then it holds that

$$\varphi_1(t, x) \leq r(t)x, \quad \varphi_2(t, x) \leq r(t)x/A(t), \quad \varphi_3(t, x) \leq r(t)x^2/A(t),$$
$$\varphi_4(t, x) \leq r(t)x/A(t), \quad \varphi_5(t, x) \leq r(t)x^\theta/A(t)(\theta > 2),$$
$$\varphi_6(t, x) \leq r(t)x^2/(A(t)D(t)), \quad \varphi_7(t, x) \leq r(t)A(t)x, \quad \varphi_8(t, x, y) \leq r(t)x/A(t),$$
$$\varphi_9(t, x, y) \leq r(t)x/(A(t)D(t)), \quad \varphi_{10}(t, x, y) \leq r(t)x^2/(A(t)D(t))$$

for $t \in \mathbb{T}, x \in \mathbb{R}^+$ and $y \in \mathbb{R}^+$. Consequently, we have the following results.

Proposition 4.1. *Impulsive differential / difference system* (1.3) *or* (1.4) *with monotonic functional response* φ_1–φ_3 *and* φ_5–φ_7 *has at least one positive periodic solution if* (3.2) *holds. Moreover, it must have at least one positive periodic solution if* $c_{ik} = 0$, $k \in \mathbb{N}$, $i = 1, 2$.

Remark 4.2. The case of $c_{ik} = 0$, $k \in \mathbb{N}$, $i = 1, 2$ in (1.3) or (1.4) with monotonic functional responses φ_1–φ_3 and φ_5–φ_7 is studied by Bohner et al[6], Huo and Li[18], Fan and Wang[12], Wang et al[25]. By Proposition 4.1, the conditions

- (vi) $r_2(t) \geq a_2(t)$ in [18, Theorem 2.1];

(vii) (A7): $C_0\bar{d}\exp\{2(\bar{a}+\bar{d})\omega\}/(\bar{b}\bar{e}) < 1$ in [25, Theorem 3.3];
(viii) $\bar{b}\bar{e} > \bar{p}_0\bar{d}\exp\{(\bar{A}+\bar{a}+\bar{D}+\bar{d})\omega\}$ in [12, Theorem 2.1];
(ix) $e^l\bar{a} > p_1^u\bar{d}$ in [12, Theorem 2.2]

may be redundant.

Proposition 4.3. *Impulsive differential/difference system* (1.3) *or* (1.4) *with non-monotonic functional response φ_4 has at least one positive periodic solution if* (3.2) *holds. Moreover, it must have at least one positive periodic solution if $c_{ik} = 0$, $k \in \mathbb{N}$, $i = 1, 2$.*

Remark 4.4. By Proposition 4.3, the following conditions are not necessary:
(x) $\bar{r}_1 - \bar{a}_{12}\exp\{H_2\}/m^2 > 0$ in [10, Theorem 2.1];
(xi) the monotonicity of response function with respect to prey in [12, condition (H2), P55] and in Fazly and Hesaaraki[13, Theorem 1].

For predator-dependent response functions we also have a result.

Proposition 4.5. *Impulsive differential / difference system* (1.3) *or* (1.4) *with predator-dependent function responses φ_8–φ_{10} has at least one positive periodic solution if* (3.2) *holds. Moreover, it must have at least one positive periodic solution if $c_{ik} = 0$, $k \in \mathbb{N}$, $i = 1, 2$.*

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References

[1] R. P. Agarwal, D. R. Anderson, A. Zafer; *Interval oscillation criteria for second-order forced delay dynamic equations with mixed nonlinearities*, Comput. Math. Appl., **59**(2010), 977-993.
[2] R. P. Agarwal, M. Bohner; *Quadratic functionals for second order matrix equations on time scales*, Nonlinear Anal. TMA, **33**(1998), 675-692.
[3] B. Aulbach, S. Hilger; *Linear dynamical processes with inhomogenous time scale*, Nonlinear Dynamics and Quantum Dynamical Systems, Akademie Verlage, Berlin, 1990.
[4] L. Berezansky, E. Braverman; *On integrable solutions of impulsive delay differential equations*, Diff. Eqns. Dynam. Syst: An Inter. J. Theo. Appl. **4** (1996), 21-42.
[5] A. A. Berryman; *The origins and evolution of predator-prey theory*, Ecology, **73**(1992), 1530-1535.
[6] M. Bohner, M. Fan, J. Zhang; *Existence of periodic solutions in predator-prey and competition dynamical systems*, Nonlinear Anal. RWA, **7**(2006), 1193-1204.
[7] M. Bohner, A. Peterson; *Dynamic Equations on Time Scales: An Introduction with Applicationsr*, Birkhäuse, Boston, 2001.
[8] M. Bohner, A. Peterson; *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, 2003.
[9] D. Bainov, P. Simeonov; *Impulsive Differential Equations: Periodic Solutions and Applications*, Longman, England, 1993.
[10] X. Ding, C. Lu, M. Liu; *Periodic solutions for a semi-ratio-dependent predator-prey system with nonmonotonic functional response and time delay*, Nonlinear Anal.: RWA, **9**(2008), 762-775.
[11] L. Erbe, A. Peterson, S. H. Saker; *Oscillation criteria for second order nonlinear dynamic equations on time scales*, J. London Math. Soci. **67**(2011), 701-714.
[12] M. Fan, Q. Wang; *Periodic solutions of a class of nonautonomous discrete time semi-ratio-dependent predator-prey systems*, Discr. Contin. Dyn. Syst. Ser. B, **4**(2004), 563-574.
[13] M. Fazly, H. Hesaaraki; *Periodic solutions for a discrete time predator-prey system with monotone functional responses*, C. R. Acad. Sci. Paris, Ser. I, **345**(2007), 199-202.

[14] M. Fazly, M. Hesaaraki; *Periodic solutions for a semi-ratio-dependent predator-prey dynamical system with a class of functional responses on time scales*, Discr. Contin. Dyn. Syst. B, **9**(2008), 267-279.

[15] H. I. Freedman; *Deterministic Mathematical Model in Population Ecology*, Monograph Textbooks Pure Applied Math. Vol 57, Marcel Dekker, New York, 1980.

[16] R. E. Gaines, J. L. Mawhin; *Coincidence Degree and Non-Linear Differential Equations*, Springer, Berlin, 1977.

[17] S. Hilger; *Analysis on measure chains-a unified approach to continuous and discrete calculus*, Results in Mathematics, **18**(1990), 18-56.

[18] H. Huo, W. Li; *Periodic solutions of delayed Leslie-Gower predator-prey models*, Appl. Math. Comput., **155**(2004), 591-605.

[19] V. Lakshmikantham, S. Sivasundaram, B. Kaymakcalan; *Dynamic system on measure chains, Mathematics and its applications*, 370, Kluwer Acad. Publ. Dordrecht. 1996.

[20] V. Lakshmikantham, D. D. Bainov, P. S. Simenov; *Theory of Impulsive Equations*, World scintific, Singapore, 1989.

[21] A. J. Lotka; *Elements of Physical Biology*, Williams and Wilkins, Baltimore, 1925.

[22] V. Volterra; *Fluctuations in the abundance of species considered mathematically*, Nature, **118** (1926), 558-560.

[23] Z. Lu, X, Liu; *Analysis of a predator-prey model with modified Holling-Tanner functional response and time delay*, Nonlinear Anal: RWA, **9**(2008), 641-650.

[24] D. A. Miller, J. B. Grand, T. F. Fondell, M. Anthony; *Predator functional response and prey survival: direct and indirect interactions affecting a marked prey population*, J. Anim. Ecol., **75**(2006), 101-110.

[25] Q. Wang, M. Fan, K. Wang; *Dynamical of a class of nonautonomous semi-ratio-dependent predator-prey systems with functional responses*, J. Math. Anal. Appl. **278**(2003), 443-471.

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