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PERIODIC SOLUTIONS FOR NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH IMPULSES ON TIME SCALES

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Abstract. Let $\mathbb{T}$ be a periodic time scale. We use Krasnoselskii's fixed point theorem to show that the neutral functional differential equation with impulses

$$\begin{gathered} x^{\Delta}(t) = -A(t)x^{\sigma}(t) + g^{\Delta}(t, x(t-h(t))) + f(t, x(t), x(t-h(t))), \\ t \neq t_j,\ t \in \mathbb{T}, \\ x(t_j^+) = x(t_j^-) + I_j(x(t_j)), \quad j \in \mathbb{Z}^+ \end{gathered}$$

has a periodic solution. Under a slightly more stringent conditions we show that the periodic solution is unique using the contraction mapping principle.

1. Introduction

The study of differential equations on time scales, which has been created in order to unify the study of differential and difference equations, is an area of mathematics that has recently gained a lot of attention, moreover, many results on this issue have been well documented in the monographs [1, 2, 6].

Recently Kaufmann and Raffoul [3] investigated the existence of periodic solutions for the neutral dynamical equation on time scale

$$x^{\Delta}(t) = -a(t)x^{\sigma}(t) + c(t)x^{\Delta}(t-k) + q(t, x(t), x(t-k)), \quad t \in \mathbb{T}, \tag{1.1}$$

where k is a fixed constant if $\mathbb{T} = \mathbb{R}$ and is a multiple of the period of $\mathbb{T}$ if $\mathbb{T} \neq \mathbb{R}$.

Differential equations with impulses provide an adequate mathematical model of many evolutionary process that suddenly change their state at certain moments. Therefore, the study of this class of dynamical system has gained prominence and it is rapidly growing field. See, for instance the monographs [4, 5, 8, 9, 10].

In this article, we are concerned with the system

$$\begin{gathered} x^{\Delta}(t) = -A(t)x^{\sigma}(t) + g^{\Delta}(t, x(t-h(t))) + f(t, x(t), x(t-h(t))), \\ t \neq t_j,\ t \in \mathbb{T}, \\ x(t_j^+) = x(t_j^-) + I_j(x(t_j)), \quad j \in \mathbb{Z}^+, \end{gathered} \tag{1.2}$$

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where $\mathbb{T}$ is an ω-periodic time scale and $0 \in \mathbb{T}$. For each interval U of $\mathbb{R}$, we denote by $U_{\mathbb{T}} = U \cap \mathbb{T}$, $x(t_j^+)$ and $x(t_j^-)$ represent the right and the left limit of $x(t_j)$ in the sense of time scales, in addition, if t_j is left-scattered, then $x(t_j^-) = x(t_j)$, $A(t) = \operatorname{diag}(a_i(t))_{n\times n}(a_i \in C(\mathbb{T}, \mathcal{R}^+)$ is a diagonal matrix with continuous real-valued functions as its elements, $\mathcal{R}^+ = \{a(t) \in C(\mathbb{T}, \mathbb{R}) : 1 + \mu(t)a(t) > 0\}$, $h \in C(\mathbb{T}, \mathbb{T})$, $g = (g_1, g_2, \dots, g_n) \in C(\mathbb{T}\times\mathbb{R}_0^n, \mathbb{R}_0^n)$, $f = (f_1, f_2, \dots, f_n) \in C(\mathbb{T}\times\mathbb{R}_0^n\times\mathbb{R}_0^n, \mathbb{R}_0^n)$, $I_j = (I_j^{(1)}, I_j^{(2)}, \dots, I_j^{(n)}) \in C(\mathbb{R}_0^n, \mathbb{R}_0^n)$, $\mathbb{R}_0^n = \{(t_1, t_2, \dots, t_n) : t_i \in \mathbb{R}, t_i \geq 0, i = 1, 2, \dots\}$ and $A(t), h(t), g(t, u(t - h(t))), f(t, u(t), u(t - h(t)))$ are all ω-periodic functions respect to t, $\omega > 0$ is a constant. There exists a positive integer p such that $t_{j+p} = t_j + \omega$, $I_{j+p} = I_j$, $j \in \mathbb{Z}^+$, without loss of generality, we also assume that $[0, \omega)_{\mathbb{T}} \cap \{t_j, j \in \mathbb{Z}^+\} = \{t_1, t_2, \dots, t_p\}$.

Our main purpose in this paper is using Krasnoselskii's fixed point theorem to study the existence of positive periodic solutions to system (1.2).

The organization of this paper is as follows. In Section 2, we introduce some notations and definitions, and state some preliminary results needed in later sections, then we give the Green's function of (1.2), which plays an important role in this paper. In Section 3, we establish our main results for positive periodic solutions by applying Krasnoselskii's fixed point theorem, and provide an example to illustrate the effectiveness of our results obtained in the previous sections.

2. Preliminaries

In this section, we shall recall some basic definitions and lemmas which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T} : s < t\}, \quad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense points and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$. The set of continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C(\mathbb{T})$.

For $x : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $x(t)$, $x^{\Delta}(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood $U_{\mathbb{T}}$ of t such that

$$|[x(\sigma(t)) - x(s)] - x^{\Delta}(t)[\sigma(t) - s]| < \varepsilon|\sigma(t) - s|$$

for all $s \in U_{\mathbb{T}}$.

If x is continuous, then x is right-dense continuous, and if x is delta differentiable at t, then x is continuous at t.

Remark 2.1. $x : \mathbb{T} \to \mathbb{R}^n$ is delta derivable or right-dense continuous or continuous if each entry of x is delta derivable or right-dense continuous or continuous.

Let x be right-dense continuous. If $X^{\Delta}(t) = x(t)$, then we define the delta integral by

$$\int_a^t x(s)\Delta s = X(t) - X(a).$$

Definition 2.2 ([3]). We say that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale.

Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the smallest positive number such that $f(t + \omega) = f(t)$.

If $\mathbb{T} = \mathbb{R}$, we say that f is periodic with period $\omega > 0$ if ω is the smallest positive number such that $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$.

Remark 2.3. According to [3], if $\mathbb{T}$ is a periodic time scale with period p, then $\sigma(t + np) = \sigma(t) + np$ and the graininess function μ is a periodic function with period p.

Definition 2.4 ([2]). An $n \times n$-matrix-valued function A on time scale $\mathbb{T}$ is called regressive (respect to $\mathbb{T}$) provided

$$I + \mu(t)A(t)$$

is invertible for all $t \in \mathbb{T}^k$.

Let $A, B : \mathbb{T} \to \mathbb{R}^{n\times n}$ be two $n \times n$-matrix-valued regressive functions on $\mathbb{T}$, we define

$$\begin{gathered}
(A \oplus B)(t) := A(t) + B(t) + \mu(t)A(t)B(t), \\
(\ominus A)(t) := -[I + \mu(t)A(t)]^{-1}A(t) = -A(t)[I + \mu(t)A(t)]^{-1}, \\
(A(t)) \ominus (B(t)) := (A(t)) \oplus (\ominus(B(t))),
\end{gathered}$$

for all $t \in \mathbb{T}^k$.

Theorem 2.5 ([2]). *Let A be an regressive and rd-continuous $n \times n$-matrix-valued function on $\mathbb{T}$ and suppose that $f : \mathbb{T} \to \mathbb{R}^n$ is rd-continuous. Let $t_0 \in \mathbb{T}$ and $y_0 \in \mathbb{R}^n$. Then the initial value problem*

$$y^{\Delta} = A(t)y + f(t), \quad y(t_0) = y_0$$

has a unique solution $y : \mathbb{T} \to \mathbb{R}^n$.

Definition 2.6 ([2]). Let $t_0 \in \mathbb{T}$ and assume that A is an regressive and rd-continuous $n \times n$-matrix-valued function. The unique matrix-valued solution of the initial value problem

$$x^{\Delta}(t) = A(t)x(t), \quad x(t_0) = I,$$

where I denotes as usual the $n \times n$-identity matrix, is called the matrix exponential function (at t_0), and it is denoted by $e_A(\cdot, t_0)$.

Remark 2.7. Assume that A is a constant $n \times n$-matrix. If $\mathbb{T} = \mathbb{R}$, then

$$e_A(t, t_0) = e^{A(t-t_0)},$$

while if $\mathbb{T} = \mathbb{Z}$ and $I + A$ is invertible, then

$$e_A(t, t_0) = (I + A)^{(t-t_0)}.$$

In the following lemma, we give some properties of the matrix exponential function.

Lemma 2.8 ([2]). *Assume that $A, B : \mathbb{T} \to \mathbb{R}^{n\times n}$ are regressive and rd-continuous matrix-valued functions on $\mathbb{T}$. Then*

(i) $e_0(t,s) \equiv I$ *and* $e_A(t,t) \equiv I$;
(ii) $e_A(\sigma(t), s) = (I + \mu(t)A(t))e_A(t,s)$;
(iii) $e_A^{-1}(t,s) = e^*_{\ominus A^*}(t,s)$;
(iv) $e_A(t,s) = e_A^{-1}(s,t) = e^*_{\ominus A^*}(s,t)$;
(v) $e_A(t,s)e_A(s,r) =_A (t,r)$;
(vi) $e_A(t,s)e_B(t,s) = e_{A\oplus B}(t,s)$, *if* $e_A(t,s)$ *and* $B(t)$ *commute,*

where A^* denotes the conjugate transpose of A.

Lemma 2.9 ([2]). *Suppose A and B are regressive matrix-valued functions, then*

(i) A^* *is regressive;*
(ii) $\ominus A^* = (\ominus A)^*$;
(iii) $(A^*)^\Delta = (A^\Delta)^*$ *holds for any differential matrix-valued function A.*

Next, we state Krasnoselskii's fixed point theorem which enables us to prove the existence of a periodic solution of (1.2). For its proof we refer the reader to [7] .

Theorem 2.10 (Krasnoselskii). *Let $\mathbb{M}$ be a closed convex nonempty subset of Banach space $(\mathbb{B}, \|\cdot\|)$. Suppose that Φ and Ψ map $\mathbb{M}$ into $\mathbb{B}$ such that*

(i) $x, y \in \mathbb{M}$ *imply* $\Phi x + \Psi y \in \mathbb{M}$;
(ii) Ψ *is compact and continuous;*
(iii) Φ *is a contraction mapping.*

Then there exists $z \in \mathbb{M}$ with $z = \Phi z + \Psi z$.

Lemma 2.11. *A function $x(t)$ is an ω-periodic solution of* (1.2) *if and only if $x(t)$ is an ω-periodic solution of the equation*

$$
\begin{aligned}
x(t) = g(t, x(t-h(t))) &+ \int_t^{t+\omega} G(t,s)[f(s, x(s), x(s-h(s))) \\
&- (\ominus A(t)) g^\sigma(s, x(s-h(s)))]\Delta s + \sum_{j:t_j\in[t,t+\omega)} G(t,t_j) I_j(x(t_j)),
\end{aligned}
$$

where

$$
\begin{gathered}
G(t,s) = \operatorname{diag}(G_i(t,s))_{n\times n}, G_i(t,s) = \big(1 - e_{\ominus a_i}(\omega, 0)\big)^{-1} e_{\ominus a_i}(t,s) \\
\ominus A(t) = \operatorname{diag}(\ominus a_i(t))_{n\times n}.
\end{gathered}
$$

Proof. If x is an ω-periodic solution of (1.2). For any $t \in \mathbb{T}$, there exists $j \in \mathbb{Z}$ such that t_j is the first impulsive point after t. Then for $i = 1, 2, \dots, n$, x_i is an ω-periodic solution of the equation

$$
x_i^\Delta(t) + a_i(t) x_i^\sigma(t) = g_i^\Delta(t, x_i(t-h(t))) + f_i(t, x_i(t), x_i(t-h(t))). \tag{2.1}
$$

Multiply both sides of (2.1) by $e_{a_i}(t,0)$ and then integrate from t to $s \in [t, t_j]_{\mathbb{T}}$, we obtain

$$
\begin{aligned}
&\int_t^s [e_{a_i}(\tau,0) x_i(\tau)]^\Delta \Delta\tau \\
&= \int_t^s e_{a_i}(\tau,0)[g_i^\Delta(\tau, x_i(\tau-h(\tau))) + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau,
\end{aligned}
$$

or

$$\begin{aligned} e_{a_i}(s,0)x_i(s) = e_{a_i}(t,0)x_i(t) + \int_t^s e_{a_i}(\tau,0)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau, \end{aligned}$$

then

$$\begin{aligned} x_i(s) = e_{\ominus a_i}(s,t)x_i(t) + \int_t^s e_{\ominus a_i}(s,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau, \quad i=1,2,\dots,n, \end{aligned}$$

hence

$$\begin{aligned} x_i(t_j) = e_{\ominus a_i}(t_j,t)x_i(t) + \int_t^{t_j} e_{\ominus a_i}(t_j,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau, \quad i=1,2,\dots,n. \end{aligned} \tag{2.2}$$

Similarly, for $s \in (t_j, t_{j+1}]$, we have

$$\begin{aligned} x_i(s) &= e_{\ominus a_i}(s,t_j)x_i(t_j^+) + \int_{t_j}^s e_{\ominus a_i}(s,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ &\quad + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau \\ &= e_{\ominus a_i}(s,t_j)x_i(t_j^-) + \int_{t_j}^s e_{\ominus a_i}(s,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ &\quad + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau + e_{\ominus a_i}(s,t_j)I_{ij}(x_i(t_j)) \\ &= e_{\ominus a_i}(s,t_j)x_i(t_j) + \int_{t_j}^s e_{\ominus a_i}(s,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ &\quad + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau + e_{\ominus a_i}(s,t_j)I_{ij}(x_i(t_j)), \end{aligned}$$

for $i=1,2,\dots,n$. Substituting (2.2) in the above equality, we obtain

$$\begin{aligned} x_i(s) &= e_{\ominus a_i}(s,t)x_i(t) + \int_t^s e_{\ominus a_i}(s,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ &\quad + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau + e_{\ominus a_i}(s,t_j)I_{ij}(x_i(t_j)). \end{aligned}$$

Repeating the above process for $s \in [t, t+\omega]_{\mathbb{T}}$, we have

$$\begin{aligned} x_i(s) &= e_{\ominus a_i}(s,t)x_i(t) + \int_t^s e_{\ominus a_i}(s,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ &\quad + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau + \sum_{j:t_j\in[t,t+\omega)} e_{\ominus a_i}(s,t_j)I_{ij}(x_i(t_j)), \end{aligned}$$

for $i=1,2,\dots,n$. Let $s=t+\omega$ in the above equality, we have

$$\begin{aligned} x_i(t+\omega) &= e_{\ominus a_i}(t+\omega,t)x_i(t) + \int_t^{t+\omega} e_{\ominus a_i}(t+\omega,\tau)[g_i^{\Delta}(\tau, x_i(\tau-h(\tau))) \\ &\quad + f_i(\tau, x_i(\tau), x_i(\tau-h(\tau)))]\Delta\tau + \sum_{j:t_j\in[t,t+\omega)} e_{\ominus a_i}(t+\omega,t_j)I_{ij}(x_i(t_j)), \end{aligned}$$

$i = 1, 2, \dots, n$. Noticing that $x_i(t+\omega) = x_i(t)$ and $e_{\ominus a_i}(t+\omega, t) = e_{\ominus a_i}(\omega, 0)$, we obtain

$$\begin{aligned}(1 - e_{\ominus a_i}(\omega, 0))x_i(t) = \int_t^{t+\omega} & e_{\ominus a_i}(t+\omega, \tau)[g_i^{\Delta}(\tau, x_i(\tau - h(\tau))) \\ & + f_i(\tau, x_i(\tau), x_i(\tau - h(\tau)))]\Delta\tau \\ & + \sum_{j:t_j \in [t, t+\omega)} e_{\ominus a_i}(t+\omega, t_j) I_{ij}(x_i(t_j)),\end{aligned} \tag{2.3}$$

for $i = 1, 2, \dots, n$. Notice that

$$\begin{aligned}&\int_t^{t+\omega} e_{\ominus a_i}(t, \tau) g_i^{\Delta}(\tau, x_i(\tau - h(\tau)))\Delta\tau \\ &= e_{\ominus a_i}(t, t+\omega) g_i(t+\omega, x_i(t+\omega - h(t+\omega))) - e_{\ominus a_i}(t, t) g_i(t, x_i(t - h(t))) \\ &\quad - \int_t^{t+\omega} e_{\ominus a_i}(t, \tau)(\ominus a_i(t)) g_i^{\sigma}(\tau, x_i(\tau - h(\tau)))\Delta\tau \\ &= [e_{\ominus a_i}(0, \omega) - 1] g_i(t, x_i(t - h(t))) \\ &\quad - \int_t^{t+\omega} e_{\ominus a_i}(t, \tau)(\ominus a_i(t)) g_i^{\sigma}(\tau, x_i(\tau - h(\tau)))\Delta\tau, \quad i = 1, 2, \dots, n.\end{aligned} \tag{2.4}$$

It follows from (2.3) and (2.4) that

$$\begin{aligned}x_i(t) &= g_i(t, x_i(t - h(t))) + \int_t^{t+\omega} [1 - e_{\ominus a_i}(\omega, 0)]^{-1} e_{\ominus a_i}(t, \tau) \\ &\quad \times [f_i(\tau, x_i(\tau), x_i(\tau - h(\tau))) - (\ominus a_i(t)) g_i^{\sigma}(\tau, x_i(\tau - h(\tau)))]\Delta\tau \\ &\quad + \sum_{j:t_j \in [t, t+\omega)} [1 - e_{\ominus a_i}(\omega, 0)]^{-1} e_{\ominus a_i}(t, t_j) I_{ij}(x_i(t_j)) \\ &= g_i(t, x_i(t - h(t))) + \int_t^{t+\omega} G_i(t, \tau)[f_i(\tau, x_i(\tau), x_i(\tau - h(\tau))) \\ &\quad - (\ominus a_i(t)) g_i^{\sigma}(\tau, x_i(s\tau - h(\tau)))]\Delta\tau + \sum_{j:t_j \in [t, t+\omega)} G_i(t, t_j) I_{ij}(x_i(t_j)),\end{aligned}$$

for $i = 1, 2, \dots, n$. Next, we prove the converse. Let

$$\begin{aligned}x_i(t) = g_i(t, x_i(t - h(t))) &+ \int_t^{t+\omega} G_i(t, s)[f_i(s, x_i(s), x_i(s - h(s))) \\ &- (\ominus a_i(t)) g_i^{\sigma}(s, x_i(s - h(s)))]\Delta s + \sum_{j:t_j \in [t, t+\omega)} G_i(t, t_j) I_{ij}(x_i(t_j)),\end{aligned}$$

where

$$G_i(t, s) = (1 - e_{\ominus a_i}(\omega, 0))^{-1} e_{\ominus a_i}(t, s), \quad i = 1, 2, \dots, n.$$

Then if $t \neq t_i, i \in \mathbb{Z}^+$, we have

$$\begin{aligned}&x_i^{\Delta}(t) \\ &= g_i^{\Delta}(t, x_i(t - h(t))) \\ &\quad + \int_t^{t+\omega} \{G_i(t, s)[f_i(s, x_i(s), x_i(s - h(s))) - (\ominus a_i(t)) g_i^{\sigma}(s, x_i(s - h(s)))]\}^{\Delta}\Delta s \\ &\quad + G_i(t, t+\omega)[f_i(t+\omega, x_i(t+\omega), x_i(t+\omega - h(t+\omega))) \\ &\quad - (\ominus a_i(t)) g_i^{\sigma}(t+\omega, x_i(t+\omega - h(t+\omega)))]\end{aligned}$$

$$
\begin{aligned}
&\quad - G_i(t,t)[f_i(t,x_i(t),x_i(t-h(t))) - (\ominus a_i(t))g_i^\sigma(t,x_i(t-h(t)))] \\
&= g_i^\Delta(t,x_i(t-h(t))) + f(t,x_i(t),x_i(t-h(t))) \\
&\quad + \int_t^{t+\omega} \{G_i(t,s)[f_i(s,x_i(s),x_i(s-h(s))) \\
&\quad - (\ominus a_i(t))g_i^\sigma(s,x_i(s-h(s)))]\}^\Delta \Delta s - (\ominus a_i(t))g_i^\sigma(t,x_i(t-h(t))) \\
&= g_i^\Delta(t,x_i(t-h(t))) + f_i(t,x_i(t),x_i(t-h(t))) - a_i(t)x_i^\sigma(t) \\
&= -a_i(t)x_i^\sigma(t) + g_i^\Delta(t,x_i(t-h(t))) + f(t,x_i(t),x_i(t-h(t))),\ i=1,2,\dots,n.
\end{aligned}
$$

If $t=t_i, i\in\mathbb{Z}^+$, we obtain

$$
\begin{aligned}
&x_i(t_i^+) - x(t_i^-) \\
&= \sum_{j:t_j\in[t_i^+,t_i^++\omega)} G_i(t_i,t_j)I_{ij}(x_i(t_j)) - \sum_{j:t_j\in[t_i^-,t_i^-+\omega)} G_i(t_i,t_j)I_{ij}(x_i(t_j)) \\
&= G_i(t_i,t_i+\omega)I_i(x_i(t_i+\omega)) - G_i(t_i,t_i)I_i(x_i(t_i)) \\
&= I_i(x_i(t_i)),\ i=1,2,\dots,n.
\end{aligned}
$$

So we know that, x is also an ω-periodic solution of (1.2). This completes the proof. □

Throughout this paper, we make the following assumptions:

(H1) The function $g=(g_1,g_2,\dots,g_n)$ satisfies a Lipschitz condition in x. That is, for $i\in\{1,2,\dots,n\}$, there exists a positive constant L_i such that
$$|g_i(t,x)-g_i(t,y)| \le L_i\|x-y\|, \quad \text{for all } t\in\mathbb{T}, x,y\in\mathbb{R}_0^n.$$

(H2) The function $f=(f_1,f_2,\dots,f_n)$ satisfies a Lipschitz condition in x and y. That is, for $i\in\{1,2,\dots,n\}$, there exist positive constants M_i and N_i such that
$$|f_i(t,x,y)-f_i(t,\xi,\zeta)| \le M_i\|x-\xi\|+N_i\|y-\zeta\|, \text{ for all } t\in\mathbb{T},(x,y),(\xi,\zeta)\in\mathbb{R}_0^n\times\mathbb{R}_0^n.$$

(H3) For $j\in Z$, $I_j=(I_j^{(1)},I_j^{(2)},\dots,I_j^{(n)})$ satisfies Lipschitz condition. That is, for $i\in\{1,2,\dots,n\}$ there exists a positive constant $P_j^{(i)}$ such that
$$|I_j^{(i)}(x)-I_j^{(i)}(y)| \le P_j^{(i)}\|x-y\|, \quad \text{for all } x,y\in\mathbb{R}_0^n.$$

To apply Theorem 2.10 to (1.2), we define
$$PC(\mathbb{T}) = \{x:\mathbb{T}\to\mathbb{R}^n : x|_{(t_j,t_{j+1})_\mathbb{T}} \in C(t_j,t_{j+1})_\mathbb{T}, \exists x(t_j^-)=x(t_j), x(t_j^+), j\in Z^+\}.$$

Consider the Banach space
$$X=\{x(t)\in PC(\mathrm{T}) : x(t+\omega)=x(t)\}$$
with the norm $\|x\| = \max_{t\in[0,\omega]_\mathbb{T}}|x(t)|_0$, where $|x(t)|_0 = \max_{1\le i\le n}|x_i(t)|$.

Lemma 2.12 ([3]). *Let $x\in X$. Then there exists $\|x^\sigma\|$, and $\|x^\sigma\|=\|x\|$.*

Noticing that
$$G_i(t,s) \le (1-e_{\ominus a_i}(\omega,0))^{-1} := \eta_i,$$
for convenience, we introduce the notation
$$\bar\eta := \max_{1\le i\le n}\eta_i, \quad \gamma := \max_{1\le i\le n}\max_{t\in[0,\omega]_\mathbb{T}}|\ominus a_i(t)|, \quad L:=\max_{1\le i\le n}L_i, \quad M:=\max_{1\le i\le n}M_i,$$

$$N := \max_{1\le i\le n} N_i, \quad P_j := \max_{1\le i\le n} P_j^{(i)}, \quad P := \max_{1\le j\le p} P_j.$$

Define the mapping $H : X \to X$ by

$$\begin{aligned}(H\varphi)(t) = g(t, \varphi(t-h(t))) + \int_t^{t+\omega} G(t,s)[f(s, \varphi(s), \varphi(s-h(s))) \\ - (\ominus A(t))g^\sigma(s, \varphi(s-h(s)))]\Delta s + \sum_{j:t_j\in[t,t+\omega)} G(t,t_j)I_j(x(t_j)).\end{aligned} \tag{2.5}$$

To apply Theorem 2.10, we need to construct two mappings: one is a contraction and the other is continuous and compact. We express (2.5) as

$$(H\varphi)(t) = (\Phi\varphi)(t) + (\Psi\varphi)(t),$$

where

$$(\Phi\varphi)(t) = g(t, \varphi(t-h(t))), \tag{2.6}$$

$$\begin{aligned}(\Psi\varphi) = \int_t^{t+\omega} G(t,s)[f(s, \varphi(s), \varphi(s-h(s))) - (\ominus A(t))g^\sigma(s, \varphi(s-h(s)))]\Delta s \\ + \sum_{j:t_j\in[t,t+\omega)} G(t,t_j)I_j(\varphi(t_j)).\end{aligned} \tag{2.7}$$

Lemma 2.13. *Suppose* (H1) *holds and* $L < 1$*, then* $\Phi : X \to X$*, as defined by* (2.6)*, is a contraction.*

Proof. Trivially, $\Phi : X \to X$. For $\varphi, \psi \in X$, we have

$$\begin{aligned}\|\Phi(\varphi) - \Phi(\psi)\| &= \max_{t\in[0,\omega]_{\mathbb{T}}} \max_{1\le i\le n} |g_i(t, \varphi(t-h(t))) - g_i(t, \psi(t-h(t)))| \\ &\le L\|\varphi - \psi\|.\end{aligned} \tag{2.8}$$

Hence Φ defines a contraction mapping with contraction constant L. $\square$

Lemma 2.14. *Suppose* (H1)–(H3) *hold, then* $\Psi : X \to X$*, as defined by* (2.7)*, is continuous and compact.*

Proof.

$$\begin{aligned}&(\Psi\varphi)(t+\omega) \\ &= \int_{t+\omega}^{t+2\omega} G(t+\omega, s)[f(s, \varphi(s), \varphi(s-h(s))) - (\ominus A(t+\omega))g^\sigma(s, \varphi(s-h(s)))]\Delta s \\ &\quad + \sum_{j:t_j\in[t+\omega,t+2\omega)} G(t+\omega, t_j)I_j(\varphi(t_j)). \\ &= \int_t^{t+\omega} G(t, u+\omega)[f(u+\omega, \varphi(u+\omega), \varphi(u+\omega-h(u+\omega))) \\ &\quad - (\ominus A(t+\omega))g^\sigma(u+\omega, \varphi(u+\omega-h(u+\omega)))]\Delta u + \sum_{k:t_k\in[t,t+\omega)} G(t,t_k)I_j(\varphi(t_k)) \\ &= \int_t^{t+\omega} G(t,u)[f(u, \varphi(u), \varphi(u-h(u))) \\ &\quad - (\ominus A(t))g^\sigma(u, \varphi(u-h(u)))]\Delta u + \sum_{k:t_k\in[t,t+\omega)} G(t,t_k)I_j(\varphi(t_k))\end{aligned}$$

$$= (\Psi\varphi)(t).$$

That is, $\Psi : X \to X$.

Now, we show that Ψ is continuous. Let $\varphi, \psi \in X$, given $\varepsilon > 0$, take

$$\delta = \frac{\varepsilon}{\overline{\eta}[\omega(M+N+L\gamma)+P]}$$

such that for $\|\varphi - \psi\| \le \delta$. By using the Lipschitz condition, we obtain

$$\begin{aligned}
&\|\Psi\varphi - \Psi\psi\| \\
&\le \max_{t\in[0,\omega]_{\mathbb{T}}} |\int_t^{t+\omega} G(t,s)[f(s,\varphi(s),\varphi(s-h(s))) - f(s,\psi(s)-\psi(s-(h(s))))]\Delta s|_0 \\
&\quad + \max_{t\in[0,\omega]_{\mathbb{T}}} |\int_t^{t+\omega} G(t,s)(\ominus A(t))[g(s,\varphi(s-h(s))) - g(s,\psi(s-h(s)))]\Delta s|_0 \\
&\quad + \max_{t\in[0,\omega]_{\mathbb{T}}} \sum_{j:t_j\in[t,t+\omega)} |G(t,t_j)[I_j(\varphi(t_j)) - I_j(\psi(t_j))]|_0 \\
&\le \overline{\eta}\int_0^{\omega} |f(s,\varphi(s),\varphi(s-h(s))) - f(s,\psi(s),\psi(s-h(s)))|_0\Delta s \\
&\quad + \overline{\eta}\gamma\int_0^{\omega} |g(s,\varphi(s-h(s))) - g(s,\psi(s-h(s)))|_0\Delta s \\
&\quad + \overline{\eta}\max_{1\le j\le p} |I_j(\varphi(t_j)) - I_j(\psi(t_j))|_0 \\
&\le \overline{\eta}[\omega(M+N+L\gamma)+P]\|\varphi-\psi\| < \varepsilon.
\end{aligned}$$

This proves Ψ is continuous. Next, we need to show that Ψ is compact. Consider the sequence of periodic functions $\{\varphi_n\} \subset X$ and assume that the sequence is uniformly bounded. Let $\Theta > 0$ be such that $\|\varphi_n\| \le \Theta$, for all $n \in N$. In view of (H1)–(H3), we arrive at

$$\begin{aligned}
\|g^{\sigma}(t,x)\| &\le \|g^{\sigma}(t,x) - g^{\sigma}(t,0)\| + \|g^{\sigma}(t,0)\| \\
&= \max_{t\in[0,\omega]_{\mathbb{T}}}\max_{1\le i\le n} |g_i^{\sigma}(t,x) - g_i^{\sigma}(t,0)| + \alpha_g \\
&\le L\|x\| + \alpha_g;
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
\|f(t,x,y)\| &\le \|f(t,x,y) - f(t,0,0)\| + \|f(t,0,0)\| \\
&= \max_{t\in[0,\omega]_{\mathbb{T}}}\max_{1\le i\le n} |f_i(t,x,y) - f_i(t,0,0)| + \alpha_f \\
&\le (M+N)\|x\| + \alpha_f;
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
\|I_j(x)\| &\le \|I_j(x) - I_j(0)\| + \|I_j(0)\| \\
&= \max_{1\le i\le n} n|I_j^{(i)}(x) - I_j^{(i)}(0)| + \alpha_{I_j} \\
&\le P_j\|x\| + \alpha_{I_j}, \quad \text{for} \quad j \in \mathbb{Z}^+,
\end{aligned} \tag{2.11}$$

where $\alpha_g = \|g^\sigma(t,0,0)\|$, $\alpha_f = \|f(t,0,0)\|$ and $\alpha_{I_j} = \|I_j(0)\|$. Hence,

$$
\begin{aligned}
&\|\Psi\varphi_n\| \\
&\le \max_{t\in[0,\omega]_{\mathbb{T}}} \Big| \int_t^{t+\omega} G(t,s)[f(s,\varphi_n(s),\varphi_n(s-h(s))) \\
&\quad - (\ominus A(t))g^\sigma(s,\varphi_n(s-h(s)))]\Delta s\Big|_0 + \max_{t\in[0,\omega]_{\mathbb{T}}} \sum_{j:t_j\in[t,t+\omega)} |G(t,t_j)I_j(\varphi_n(t_j))|_0 \\
&\le \overline{\eta}\int_0^\omega |f(s,\varphi_n(s),\varphi_n(s-h(s)))|_0\Delta s + \overline{\eta}\gamma\int_0^\omega |g^\sigma(s,\varphi_n(s-h(s)))|_0\Delta s \\
&\quad + \overline{\eta}\sum_{j=1}^p |I_j(\varphi_n(t_j))|_0 \\
&\le \overline{\eta}\omega(M\|\varphi_n\| + N\|\varphi_n\| + \alpha_f) \\
&\quad + \overline{\eta}\gamma\omega(L\|\varphi_n\| + \alpha_g) + \overline{\eta}(\max_{1\le j\le p}(P_j\|\varphi_n\| + \alpha_{I_j})) \\
&\le \overline{\eta}\omega\Theta(M+N+\gamma L) + \overline{\eta}(\omega\alpha_f + \gamma\omega\alpha_g + P\Theta + \alpha) := D,
\end{aligned}
\tag{2.12}
$$

where $\alpha = \max_{1\le j\le p}\alpha_{I_j}$. Thus the sequence $\{\Psi\varphi_n\}$ is uniformly bounded. Now, it can be easily checked that

$$
\begin{aligned}
(\Psi\varphi_n)^\Delta(t) &= -A(t)(\Psi\varphi_n)^\sigma(t) + f(t,\varphi_n(t),\varphi_n(t-h(t))) \\
&\quad + \sum_{j:t_j\in[t,t+\omega)} G^\Delta(t,t_j)I_j(\varphi_n(t_j)).
\end{aligned}
$$

Consequently, it follows from (2.10), (2.11), (2.12) and Lemma 2.12 that

$$
\begin{aligned}
|(\Psi\varphi_n)^\Delta(t)|_0 &\le \|A\|\|(\Psi\varphi_n)^\sigma\| + \|f(t,\varphi_n(t),\varphi_n(t-h(t)))\| \\
&\quad + |\ominus A(t)|_0 \sum_{j:t_j\in[t,t+\omega)} |G(t,t_j)I_j(\varphi_n(t_j))|_0 \\
&\le \|A\|\|(\Psi\varphi_n)\| + (M+N)\|\varphi_n\| + \alpha_f + \gamma\overline{\eta}\sum_{j=1}^p |I_j(\varphi_n(t_j))|_0 \\
&\le \|A\|D + (M+N)\|\varphi_n\| + \alpha_f + \gamma\overline{\eta}\sum_{j=1}^p \|I_j(\varphi_n)\| \\
&\le \|A\|D + (M+N)\|\varphi_n\| + \alpha_f + \gamma\overline{\eta}\sum_{j=1}^p (P_j\|\varphi_n\| + \alpha_{I_j}) \\
&\le \|A\|D + (M+N)\Theta + \alpha_f + \gamma\overline{\eta}(P\Theta + \alpha),
\end{aligned}
$$

for all n. That is, $\|(\Psi\varphi_n)^\Delta\| \le \|A\|D + (M+N)\Theta + \alpha_f + \gamma\overline{\eta}(P\Theta+\alpha)$, thus the sequence $\{\Psi\varphi_n\}$ is uniformly bounded and equi-continuous. The Arzela-Ascoli theorem implies that Ψ is compact. $\square$

3. Main Result

Our main result reads as follows.

Theorem 3.1. *Assume that* (H1)–(H3) *hold and $L < 1$. Suppose that there is a positive constant G such that all solutions $x(t)$ of* (1.2)*, $x(t) \in X$, satisfy $\|x\| \le G$,* *and the inequality*

$$\frac{\gamma\omega\alpha_g + \omega\alpha_f + \alpha}{1/\overline{\eta} - \omega(\gamma L + M + N) - L/\overline{\eta} - P} < G \tag{3.1}$$

holds. Then (1.2) *has an ω-periodic solution.*

Proof. Define $\mathbb{M} = \{\varphi \in X : \|\varphi\| \le G\}$. Then Lemma 2.14 implies $\Psi : X \to X$ and Ψ is compact and continuous. Also, from Lemma 2.13, the mapping Φ is a contraction and $\Phi : X \to X$. We need to show that if $\varphi, \psi \in M$, then $\|\Phi\varphi + \Psi\psi\| \le G$. Let $\varphi, \psi \in M$ with $\|\varphi\|, \|\psi\| \le G$, from (2.9)-(2.11), we have

$$\begin{aligned}
\|\Phi\varphi + \Psi\psi\| &\le \|\Phi\varphi\| + \|\Psi\psi\| \\
&\le LG + \overline{\eta}\omega G(\gamma L + M + N) + \overline{\eta}(\gamma\omega\alpha_g + \omega\alpha_f + GP + \alpha) \le G.
\end{aligned}$$

Thus $\Phi\varphi + \Psi\psi \in \mathbb{M}$. We see that all the conditions of Krasnoselskii theorem are satisfied on the set $\mathbb{M}$. Hence there exists a fixed point z in $\mathbb{M}$ such that $z = \Phi z + \Psi z$. By Lemma 2.11, this fixed point is a solution of (1.2). □

Theorem 3.2. *Suppose that* (H1)–(H3) *hold. If*

$$\Upsilon := \overline{\eta}[\omega(\gamma L + M + N) + P] < 1,$$

then (1.2) *has an unique ω-periodic solution.*

Proof. For $\varphi, \psi \in X$, we have

$$\begin{aligned}
\|H\varphi - H\psi\| &\le \overline{\eta}\int_0^\omega |f(s, \varphi(s), \varphi(s - h(s))) - f(s, \psi(s), \psi(s - h(s)))|_0 \Delta s \\
&\quad + \overline{\eta}\gamma \int_0^\omega |g^\sigma(s, \varphi(s - h(s))) - g^\sigma(s, \psi(s - h(s)))|_0 \Delta s \\
&\quad + \overline{\eta}\sum_{j=1}^{p} |I_j(\varphi(t_j)) - I_j(\psi(t_j))|_0 \\
&\le \overline{\eta}\omega(M\|\varphi - \psi\| + N\|\varphi - \psi\|) + \overline{\eta}\gamma\omega L\|\varphi - \psi\| + \overline{\eta}P\|\varphi - \psi\| \\
&< \overline{\eta}[\omega(\gamma L + M + N) + P]\|\varphi - \psi\| \\
&= \Upsilon\|\varphi - \psi\|.
\end{aligned}$$

This completes the proof. □

The next corollary shows that G in Theorem 3.1 can be attained.

Corollary 3.3. *Consider* (1.2) *and suppose that* (H1)–(H3) *hold and $L < 1$. Set $\rho = \min_{t \in [0,\omega]_{\mathbb{T}}} \max_{1 \le i \le n} |a_i(t)|$, if*

$$\rho > \frac{M + N}{1 - \omega(\|A\| + L + M + N)}$$

holds and defined by

$$G = \frac{\alpha_f + \rho\omega(\alpha_f + \alpha_g)}{\rho - (M + N) - \rho\omega(\|A\| + L + M + N)}$$

satisfies (3.1), *then* (1.2) *has an ω-periodic solution.*

Proof. Let $x \in X$. Then, for $i = 1, 2, \dots, n$, integrating (1.2) from 0 to ω, we obtain

$$\begin{aligned} x_i(\omega) - x_i(0) = &- \int_0^\omega a_i(t) x_i^\sigma(t)\Delta t + \int_0^\omega g_i^\Delta(t, x_i(t))\Delta t \\ &+ \int_0^\omega f_i(t, x_i(t), x_i(t-h(t)))\Delta t. \end{aligned}$$

Then

$$\begin{aligned} 0 = &- \int_0^\omega a_i(t) x_i^\sigma(t)\Delta t + g_i(\omega, x_i(\omega - h(\omega))) - g_i(0, x_i(0 - h(0))) \\ &+ \int_0^\omega f_i(t, x_i(t), x_i(t-h(t)))\Delta t,\ i = 1, 2, \dots, n; \end{aligned}$$

that is,

$$\int_0^\omega a_i(t) x_i^\sigma(t)\Delta t = \int_0^\omega f_i(t, x_i(t), x_i(t-h(t)))\Delta t, \quad i = 1, 2, \dots, n.$$

Claim. There exists $t^* \in [0, \omega]$ such that

$$\omega a_i(t^*) x_i^\sigma(t^*) \le \int_0^\omega a_i(t) x_i^\sigma(t)\Delta t.$$

Suppose the Claim is false. Define $S_i := \int_0^\omega a_i(t) x_i^\sigma(t)\Delta t,\ i = 1, 2, \dots, n$. Then there exists $\varepsilon_i > 0$ such that

$$\omega a_i(t) x_i^\sigma(t) > S_i + \varepsilon_i$$

for all $t \in [0, \omega]$. So

$$S_i := \int_0^\omega a_i(t) x_i^\sigma(t)\Delta t > \frac{1}{\omega}\int_0^\omega (S_i + \varepsilon_i)\Delta t = S_i + \varepsilon_i, \quad i = 1, 2, \dots, n,$$

which is a contradiction.

As a consequence of the claim, we have

$$\begin{aligned} \omega |A(t^*)|_0 |x^\sigma(t^*)|_0 &\le \int_0^\omega |f(t, x(t), x(t-h(t)))|_0 \Delta t \\ &\le \int_0^\omega (M\|x\| + N\|x\| + \alpha_f)\Delta t. \end{aligned}$$

So,

$$|A(t^*)|_0 |x^\sigma(t^*)|_0 \le (M+N)\|x\| + \alpha_f,$$

which implies

$$|x^\sigma(t^*)|_0 \le \frac{(M+N)\|x\| + \alpha_f}{|A(t^*)|_0} \le \frac{(M+N)\|x\| + \alpha_f}{\rho}.$$

Since for all $t \in [0, \omega]_{\mathbb{T}}$,

$$x^\sigma(t) = x^\sigma(t^*) + \int_{t^*}^t x^\Delta(\sigma(s))\Delta s,$$

it follows that

$$\begin{aligned} |x^\sigma(t)|_0 &\le |x^\sigma(t^*)|_0 + \int_0^t |x^\Delta(\sigma(s))|_0 \Delta s \\ &\le \frac{(M+N)\|x\| + \alpha_f}{\rho} + \omega\|x^\Delta\|. \end{aligned}$$

This implies

$$\|x\| \le \frac{1}{\rho}(M+N)\|x\| + \frac{\alpha_f}{\rho} + \omega\|x^{\Delta}\|. \tag{3.2}$$

From (1.2), we have

$$\|x^{\Delta}\| \le \|A\|\|x\| + L\|x\| + \alpha_g + (M+N)\|x\| + \alpha_f. \tag{3.3}$$

Substituting (3.3) in (3.2) yields

$$\|x\| \le \frac{1}{\rho}(M+N)\|x\| + \frac{\alpha_f}{\rho} + \omega\big[(\|A\| + L + M + N)\|x\| + \alpha_g + \alpha_f\big].$$

Then

$$\|x\| \le \frac{\alpha_f + \rho\omega(\alpha_g + \alpha_f)}{\rho - (M+N) - \rho\omega(\|A\| + L + M + N)} = G.$$

Thus, for all $x(t) \in X$, $\|x\| \le G$. Define $\mathbb{M} = \{\varphi \in X : \|\varphi\| \le G\}$. Then by Theorem 3.1, Equation (1.2) has an ω-periodic solution. The proof is complete. $\square$

4. Example

On time scale $\mathbb{T} = \bigcup_{k=-\infty}^{\infty}[k\pi, k\pi + \frac{\pi}{2}]$, consider the neutral dynamical equation, with period $\omega = \pi$,

$$\begin{gathered} x^{\Delta}(t) = -A(t)x^{\sigma}(t) + g^{\Delta}(t, x(t-h(t))) + f(t, x(t), x(t-h(t))), \quad t \ne t_j,\ t \in \mathbb{T}, \\ x(t_j^+) = x(t_j^-) + I_j(x(t_j)), \quad j \in \mathbb{Z}^+, \end{gathered} \tag{4.1}$$

where

$$A(t) = \begin{pmatrix} 0.001\sin 2t & 0 \\ 0 & 0.003\cos 2t \end{pmatrix}, \quad g_1(t,u) = g_2(t,u) = 0.0002|\sin t|\cos u,$$

$$f_1(t,u,v) = 0.0002|\sin t|(\sin u + \cos v), \quad f_2(t,u,v) = 0.0003|\cos t|(\sin u + \cos v),$$

$$I_j^1(u) = I_j^2(u) = 0.0009\begin{pmatrix} u \\ u \end{pmatrix}, \quad j \in \mathbb{Z}^+,$$

$$t_1 = \frac{\pi}{6}, \quad t_2 = \frac{\pi}{4}, \quad t_{j+2} = t_j + \pi, j \in \mathbb{Z}^+.$$

By simple calculation, we have $L = 0.0002$, $M = 0.0003$, $N = 0.0003$, $P = 0.0009$, $\alpha_f = 0.0003$, $\alpha_g = 0.0002$, $\alpha = 0, P = 0.0009$, $\|A\| = 0.003$. For $t \in \mathbb{T}$, if $t \ne k\pi + \frac{\pi}{2}$, we have $\mu(t) = 0$ and if $t = k\pi + \frac{\pi}{2}$, we have $\mu(t) = \pi$.

When $\mu(t) = 0$, we have $\ominus a_i = -a_i$, then

$$e_{\ominus a_i}(\omega, 0) = e_{-a_i}(\pi, 0) \le e_{0.003}(\pi, 0),\ i = 1, 2.$$

When $\mu(t) = \pi$, we have $\ominus a_i = -\frac{a_i}{1+\pi a_i}$, then

$$e_{\ominus a_i}(\omega, 0) \le e_{0.003}(\pi, 0), \quad i = 1, 2$$

and $\bar{\eta} = \frac{1}{0.0094}, \gamma \simeq 0.003$. It is easy to show that all conditions in Theorem 3.1 and Corollary 3.1 are satisfied. Therefore, (4.1) has a π-periodic solution.

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