Electron. J. Math. Phys. Sci., **2002**, *Sem. 1*, 1-8

Electronic Journal of
Mathematical
and Physical Sciences
EJMAPS
ISSN: 1538-3318
www.ejmaps.org

Bayesian Economic Cost Plans I. Comparison to Classical Plans

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Received: 12 December 2001 / Accepted: 2 January 2002/Published: 15 January 2002

Abstract: The primary focus of this work is to specify the basic parameters in terms of prior distributions and finding the appropriate conditional posterior distributions to affect the transformation. The principle parameters include the upper and lower limit for the process's quality characteristic X, its mean μ, and the standard deviation, the materials fraction defective p, the sample size n, and the lot size N. The Bayesian model's posterior distributions are derived using known priors as functions of these parameters.

Keywords: Bayesian, cost model, comparison, lot size, fraction defective
AMS Mathematical Subject Classification: 46N30, 62-06,62P30

Introduction

Most of the previous research efforts in Bayesian sampling concentrated on attribute sampling plans. These plans employ statistical rather than economical cost parameters. In the literature there has been little or no work done in the area of destructive variable sampling plans for fraction defective. This deficiency has led to a lack of quality engineering (QE) tools to specify QC test procedures.

In this note our objective was to compare the set of tools for equivalent Bayesian and variable sampling plans for the fraction defective, p. These tools allow the analysis of a product's quality characteristic. The tools selected were the upper and lower limits of the sample mean, the material fraction defective, and the two-sided OC (Operational Condition) cure for an inspected lot. For specifying a plan, the limits allow one to draw a conclusion concerning lot disposition (i.e. accept or reject). The OC curve permits a quantitative estimate of selected quality characteristic's true mean.

The following systematic analytical approach was used to estimate these limits and construct the corresponding OC curve: (1) evaluate the expected values of p given the population mean, μ, the sample mean, $\bar{x}_{n,}$ and the number of defectives, s; (2) estimates from these expected values the decision points which are the mean's upper and lower limits based on a sample size; (3) find the optimal sample size by minimizing the total posterior cost and using these expected values and a set of defined economic cost parameters; (4) formulated a decision on lot disposition from the sample mean's upper and lower limits and the optimal sample size; (5) evaluate the costs relative to each decision and the total posterior profit; (6) deduce an estimate of the quality characteristic's population mean by constructing an OC curve.

Equivalent classical attribute and variable sampling acceptance plans for fraction defective can be used on the same process. This is done by specifying the average outgoing quality (AOQ) and the average total inspection parameters for both types of plans. In addition, the variance for the process with variable sampling must be a known. The underlying sampling distribution governing the variable plan is assumed to be a normal. The binomial is assumed for the attribute plan. In both plans the fraction defective is given in terms of the AOQ and ATI. To make the transition from the classical attribute plan to the variable one, Romig supplies a table to obtain the number in the sample, s, and the acceptance number, c, for this variable plan.

The approach presented above by Duncan and Romig laid the groundwork for a more sophisticated equivalence methodology put forward by Hamaker (1979) Hamaker's approach calls for additional criteria of constructing equivalent OC curves for both plans. Here, the equivalent OC curves possess the same slope at the fraction defective level p, corresponding to the 0.5 probability of acceptance value. Unlike the Duncan and Romig result, however, Hamaker provides approximation techniques to consider an unknown process variance for the variable acceptance plan. In summary, the classical acceptance sampling plans prescribe a procedure to specify the risk of accepting lots of a certain quality.

An outgrowth of the above equivalence methodologies for classical plans is to develop a similar set of criteria and framework for Bayesian type attribute and variable sampling plan. This technical note details the parameter requirements to go from a Bayesian attribute plan to the equivalent variable plan. The implication of the work is being currently developed for practical use. It should be emphasized that the procedures outlined above for the classical plans do not consider the economic impact of producing an item. Bayesian attribute and variable sampling plans fall into two major modeling categories:

(1) Not Economic

(2) Economic

For this note, the approach considered is a Bayesian economic attribute and variable sampling plan. The transformation procedure of going from a Bayesian variable plan to an attribute plan requires the specification distributions for the process characteristic X and its mean value to be found. In this case, the distribution for the fraction defective parameter, p, this ω(p), and its corresponding conditional expectations E(p|μ) and E(p|x) can be derived. However, if the distribution for p is beta as in the case for attribute sampling, the distribution for μ and E (p|$\bar{x}$) are derived. The distributions, conditional expectations, and the economic parameters are employed to define the economic profit and cost and evaluate a set of decision points for a variable sampling plan for fraction defective. The migration from classical plans to the Bayesian plan is summarized in table 1. A complete list of all the parameters and functional definitions used in table 1, are in **Appendix A**. The example, integrates the total cost and profit with Bayesian sampling into the transformation procedure. [1-6]

Summary

We have thus compared the three models of Bayesian noneconomic and economic models with the classical results. This provides the starting model for further industrial advancements in which Bayesian analysis is to be used for quality control testing. [7-9]

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Table 1. Classical Versus Bayesian Economic Plan.

Factor	Classical Plan		Bayesian Plan (Not Economic)		Bayesian Plan (Economic)	
	Attribute	**Variable**	**Attribute**	**Variable**	**Attribute**	**Variable**
Parameters	α, β, c, s AOQ, ATI	α, β, X, L, U	$\alpha, \beta, c, s, \sigma_\mu^2$ p Has Known Distribution	$\alpha, \beta, X, L, U, \sigma_\mu^2$ μ Has Known Distribution And x, $f(x\|\mu)$	$\alpha, \beta, c, s, \sigma_\mu^2$, Cost Parameters, $\omega(p)$ Assumed	$\alpha, \beta, X, L, U, \sigma_\mu^2$, Cost Parameters, $h(\mu)$ Assumed And Deduce $\omega(p)$ Or $\omega(p)$ Assumed And Deduce $h(\mu)$
Compute	p'	p'	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$
Decision	If $s < c$ Accept Lot	If $L < x < U$ Accept Lot	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$ Control Limits	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$ Control Limits	$E(p\|x), E(p\|\mu)$, $E(p\|\overline{x})$ Esimated For Each Decision To Dispose Lot	If $\overline{x} < \overline{x}_{n1}$ Reject Lot If $\overline{x} > \overline{x}_{n2}$ Reject Lot

α = Producer's Risk β = Consumer's Risk s = Sample Size c = Acceptance Number
X = Quality Characteristic L = Lower Control Limit U = Upper Control Limit
AOQ = Average Outgoing Quality ATI = Average Total Inspection p' = Fraction Defective

Appendix A: Statistical and Economic Terms

X – Product Quality Characteristic

L – Lower Specification Limit Of X

U – Upper Specification Limit Of X

N – Lot Size

n – Sample Size

$\overline{x}$ – Sample Mean

μ – Mean Of The Quality Characteristic X

σ^2 – Variance Of The Quality Characteristic X

m – Mean Of The Mean μ

c – Acceptance Number Under Attribute Sampling

s – Number Of Defectives In A Sample Under Attribute Sampling

K_R – Sales Price Of An Item

K_p – Production Cost Of An Item

K_J – Junk Value Of A Scrapping Item = 0 In This Model

K_A – Cost Of Accepting A Defective Item Delivered To The Consumer

p – Fraction Of Items Defective

p' – Minimum Variance Unbiased Estimate Of The Fraction Defective p

$1-\alpha$ – Minimum Probability Of Accepting A Lot Given A Lot Of Acceptable Quality

$1-\beta$ – Maximum Probability Of Rejecting A Lot Given A Lot Of Rejectable Quality

C_1 – Prior Cost Function Associated With The Decision To Accept Outright

C_2 – Prior Cost Function Associated With The Decision To Reject Outright and scrap

p_1 – Profit Per Item To Accept The Lot Without Sampling

p_2 – Profit Per Item To Reject The Lot Outright

p_3 – Expected Posterior Profit Per Item For Accepting The Remainder Of The Lot

p_4 – Expected Posterior Profit Per Item For Rejecting And Scrapping The Remainder Of The Lot

p_5 – Profit Per Item Resulting From Sampling And Scrapping n Units

K_1' – Posterior Cost Function Associated With Acceptance

K_2' – Posterior Cost Function Associated With Rejection

$E(K_1'|\Phi)$ – Expected Posterior Cost Associated With Acceptance

$E(K_2'|\Phi)$ – Expected Posterior Cost Associated With Rejection

$K(n, \Phi_n^{01}, \Phi_n^{02})$ – Cost Equation In Terms Of The Sample Size n And An Upper And Lower Limits Of The Parameter Φ_n.

σ_μ^2 – Variance Of The Mean μ (In This Work σ_μ Is Assumed Known)

$f(x|\mu)$ – Conditional Probability Density Function Of The Quality Characteristic (X) Given μ

$h(\mu)$ Probability Density Function Of The Mean μ

$T(\overline{x}|\mu)$ Conditional Probability Of $\overline{x}$ Given μ

$t(x|\overline{x}, \mu)$ Conditional Probability Density Function Of Individual Observations Given The Sample Mean $\overline{x}$ And Population Mean μ

k_s – Sampling Cost Per Unit

k_r – Rejection Cost Per Unit