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Existence of almost periodic solution for SICNN with a neutral delay

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Abstract

In this paper, a kind of shunting inhibitory cellular neural network with a neutral delay was considered. By using the Banach fixed point theorem, we established a result about the existence and uniqueness of the almost periodic solution for the shunting inhibitory cellular neural network.

Keywords: shunting inhibitory cellular neural network, neutral delay, almost periodic solution, fixed point theorem.

1. Introduction

Shunting inhibitory cellular neural network (SICNN) is a kind of very important model and has been investigated by many authors (see [1, 2, 3, 4] and the reference therein) due to its wide applications in practical fields such as robotics, adaptive pattern recognition and image processing. In [1], Ding studied the following SICNN

$$x'_{ij} = -a_{ij}x_{ij} - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} f[x_{kl}(t-\tau(t))]x_{ij}(t) + L_{ij}(t),$$

Most of the existing SICNN models are concerns with the delays in the state. However, it is not enough for it can not describe the phenomenon precisely. It is natural and useful to consider the model with a neutral delay, it means that the system describing the model depends on not only the past information of the state but also the information of the derivative of the state, i.e., the decay rate of the cells. This kind of model is described by a differential equation with a neutral delay. The neutral type differential equations have many applications, for more details we refer to [6]. Some authors have considered the Hopfield neural networks with neutral delays, see [7, 8]. To the best of our knowledge, there is few consideration about the shunting inhibitory cellular neural network with a neutral delay. In this paper, we consider the following

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shunting inhibitory cellular neural network with a neutral delay:

$$
\begin{aligned}
x'_{ij} = -a_{ij}(t)x_{ij} \quad &- \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f[x_{kl}(t-\tau(t))]x_{ij}(t) \\
&- \sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(t) g[x'_{kl}(t-\sigma(t))]x_{ij}(t) + I_{ij}(t).
\end{aligned} \tag{1.1}
$$

In this model, C_{ij} represents the cell at the (i,j) position of the lattice, the $r-$neighborhood $N_r(i,j)$ of C_{ij} is defined as follows

$$N_r(i,j) = \Big\{C_{kl}: \ \max\{| \ k-i \ |, | \ l-j \ |\} \le r, \quad 1 \le k \le m, 1 \le l \le n\Big\},$$

$x_{ij}(t)$ describes the state of the cell C_{ij}, the coefficient $a_{ij}(t) > 0$ is the passive decay rate of the cell activity, $C_{ij}^{kl}(t)$, $D_{ij}^{kl}(t)$ are connection weights or coupling strength of postsynaptic activity of the cell C_{kl} transmitted to the cell C_{ij}, and f, g are continuous activity functions, representing the output or firing rate of the cell C_{kl} , and $\tau(t)$, $\sigma(t)$ correspond to the transmission delays.

In the following, we give some basic knowledge about the almost periodic functions and almost periodic solutions of differential equations, please refer to [9, 10] for more details.

Definition 1.1 (See [10]) Let $u : \mathbb{R} \to \mathbb{R}^n$ be continuous in t. u is said to be almost periodic on R if, for any $\epsilon > 0$, the set $T(u,\epsilon) = \{\delta :| \ u(t+\delta) - u(t) \ |< \epsilon, \forall \ t \in \mathbb{R}\}$ is relatively dense, i.e., for $\forall \epsilon > 0$, it is possible to find a real number $l = l(\epsilon) > 0$, for any interval with length $l(\epsilon)$, there exists a number $\delta = \delta(\epsilon)$ in this interval such that $| \ u(t+\delta) - u(t) \ |< \epsilon$, for all $t \in \mathbb{R}$.

Definition 1.2 ([9, 10]) If $u : \mathbb{R} \to \mathbb{R}^n$ is continuously differentiable in t, $u(t)$ and $u'(t)$ are almost periodic on $\mathbb{R}$, then $u(t)$ is said to be a continuously differentiable almost periodic function.

Let $AP(\mathbb{R}, \mathbb{R}^{m\times n})$ and $AP^1(\mathbb{R}, \mathbb{R}^{m\times n})$ be the set of continuous almost periodic functions, and continuously differentiable almost periodic functions from $\mathbb{R}$ to $\mathbb{R}^{m\times n}$, respectively. For each $\varphi \in AP^1(\mathbb{R}, \mathbb{R}^{m\times n})$, define

$$\|\varphi\|_0 = \sup_{t\in\mathbb{R}} \max_{i,j}\{| \ \varphi_{i,j} \ |\},$$

$$\|\varphi\| = \max\{\|\varphi\|_0, \|\varphi'\|_0\}.$$

It is easy to check that $(AP(\mathbb{R}, \mathbb{R}^{m\times n}), \|\cdot\|_0)$ and $(AP^1(\mathbb{R}, \mathbb{R}^{m\times n}), \|\cdot\|)$ are all Banach spaces.

Definition 1.3 ([9, 10]) Let $x \in \mathbb{R}^n$ and $Q(t)$ be an $n \times n$ continuous matrix defined on $\mathbb{R}$. The linear system

$$x'(t) = Q(t)x(t) \tag{1.2}$$

is said to admit an exponential dichotomy on $\mathbb{R}$ if there exist positive constants k, α, projection P and the fundamental solution matrix $X(t)$ of (1.2) satisfying

$$\| X(t)PX^{-1}(s) \| < ke^{-\alpha(t-s)}, \quad t \geq s,$$

$$\| X(t)(I-P)X^{-1}(s) \| < ke^{-\alpha(s-t)}, \quad t \leq s.$$

Lemma 1.1 ([9, 10]) If the linear system (1.2) admits an exponential dichotomy, then almost periodic system

$$x'(t) = Q(t)x(t) + g(t) \tag{1.3}$$

has a unique almost periodic solution $x(t)$, and

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)ds - \int_{t}^{+\infty} X(t)(I-P)X(s)ds. \tag{1.4}$$

Lemma 1.2 ([9, 10]) Let $c(t)$ be an almost periodic function on $\mathbb{R}$ and

$$M[c_i] = \lim_{T\to+\infty} \frac{1}{T} \int_{t}^{t+T} c_i(s)ds > 0, \quad i = 1, 2, \cdots n.$$

Then the linear system

$$x'(t) = diag(-c_1(t), -c_2(t), \cdots, c_n(t))x(t)$$

admits an exponential dichotomy on $\mathbb{R}$.

2. Main results

Firstly, We give some assumptions.

(H$_1$) $a_{ij}(t),\ C_{ij}^{kl}(t),\ D_{ij}^{kl}(t),\ I_{ij}(t),\ \tau(t),\ \sigma(t)$ are all almost periodic functions, $i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, m$;

(H$_2$) the activity functions f and g are Lipschtiz functions, i.e., there exist $L > 0$, $l > 0$ such that

$$|f(x) - f(y)| \leq L|x - y|, \quad \forall x, y \in \mathbb{R},$$

$$|g(x) - g(y)| \leq l|x - y|, \quad \forall x, y \in \mathbb{R};$$

(H$_3$) $\sup_{t\in\mathbb{R}} |C_{ij}^{kl}(t)| = \overline{C_{ij}^{kl}} < +\infty,\ \sup_{t\in\mathbb{R}} |D_{ij}^{kl}(t)| = \overline{D_{ij}^{kl}} < +\infty,\ \sup_{t\in\mathbb{R}} |a_{ij}(t)| = a_{ij}^{+} < +\infty,\ a_{ij}(u) \geq a_{ij*} > 0$.

Let

$$\varphi_0 = \Big\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} I_{ij}(s)ds \Big\}, \quad \|\varphi_0\| = R_0$$

$$\theta_1 = \max_{i,j} \Big\{ \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big(2LR + |f(0)|\Big) + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}} \Big(2lR + |g(0)|\Big) \Big\};$$

$$\theta_2 = \max_{i,j} \Big\{ \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big(4LR + |f(0)|\Big) + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}} \Big(4lR + |g(0)|\Big) \Big\}.$$

where R is a constant with $R \geq R_0$.

Theorem 2.1 If $(\mathbf{H_1})$, $(\mathbf{H_2})$, $(\mathbf{H_3})$ and the following conditions are satisfied

(1) $R_0 \leq R < +\infty$;

(2) $\theta_1 \cdot \max_{i,j} \Big\{ \frac{1}{a_{ij*}}, 1 + \frac{a_{ij}^+}{a_{ij*}} \Big\} \leq \frac{1}{2}$;

(3) $\theta = \theta_2 \cdot \max_{i,j} \Big\{ \frac{1}{a_{ij*}}, 1 + \frac{a_{ij}^+}{a_{ij*}} \Big\} < 1$,

then Eq.(1.1) has a unique almost periodic solution.

Proof. For any given $\varphi = \{\varphi_{ij}\} \in AP^1(R, \mathbb{R}^{m\times n})$, we consider the almost periodic solution of the following differential equation

$$\begin{aligned} x'_{ij} = -a_{ij}(t)x_{ij} \quad &- \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f(\varphi_{kl}(t-\tau(t)))\varphi_{ij}(t) \\ &- \sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(t) g(\varphi'_{kl}(t-\sigma(t)))\varphi_{ij}(t) + I_{ij}(t). \end{aligned} \tag{2.1}$$

Since $\varphi_{ij}(t)$, $a_{ij}(t)$, $C_{ij}^{kl}(t)$, $D_{ij}^{kl}(t)$, $\tau(t)$, $\sigma(t)$ and $I_{ij}(t)$ are all almost periodic functions, and $M[a_{ij}] > 0$, according to Lemma 1 and lemma 2, we know that Eq.(2.1) has a unique almost periodic solution $x^\varphi = \{x_{ij}^\varphi\}$, which can be expressed as follows

$$\begin{aligned} x_{ij}^\varphi = \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big[\quad &- \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f(\varphi_{kl}(s-\tau(s)))\varphi_{ij}(s) \\ &- \sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s) g(\varphi'_{kl}(s-\sigma(s)))\varphi_{ij}(s) + I_{ij}(s)\Big] ds. \end{aligned}$$

We define a nonlinear operator on $AP^1(\mathbb{R}, \mathbb{R}^{m\times n})$ as follows

$$T(\varphi)(t) = x^\varphi(t), \quad \forall \varphi \in AP^1(\mathbb{R}, \mathbb{R}^{m\times n}).$$

It is obvious that the fixed point of T is a solution of Eq.(1.1). In the following we will show that T is a contract mapping, thus the Banach fixed point theorem assert that T has a fixed point.

Let E be defined as follows

$$E = \{\varphi \in AP^1(\mathbb{R}, \mathbb{R}^{m\times n}) |\ \|\varphi - \varphi_0\| \le R\}.$$

Firstly, we show that $T(E) \subseteq E$. For each $\varphi \in E$, we have $\|\varphi - \varphi_0\| \le R$, $\|\varphi\| \le \|\varphi - \varphi_0\| + \|\varphi_0\| \le R + R \le 2R$. Thus

$$\begin{aligned}
&\|T\varphi - \varphi_0\|_0 \\
= &\sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \Big| \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big[-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f(\varphi_{kl}(s-\tau(s)))\varphi_{ij}(s) \\
&\qquad - \sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s) g(\varphi'_{kl}(s-\sigma(s)))\varphi_{ij}(s) \Big] ds \Big| \Big\} \\
\le &\sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big| -\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f(\varphi_{kl}(s-\tau(s)))\varphi_{ij}(s) \\
&\qquad - \sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s) g(\varphi'_{kl}(s-\sigma(s)))\varphi_{ij}(s) \Big| ds \Big\} \\
\le &\sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big[\sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(s)| |f(\varphi_{kl}(s-\tau(s)))| |\varphi_{ij}(s)| \\
&\qquad + \sum_{C_{kl}\in N_s(i,j)} |D_{ij}^{kl}(s)| |g(\varphi'_{kl}(s-\sigma(s)))| |\varphi_{ij}(s)| \Big] ds \Big\} \\
\le &\sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big[\sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big(|f(\varphi_{kl}(s-\tau(s))) - f(0)| + |f(0)| \Big) \\
&\qquad + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}} \Big(|g(\varphi'_{kl}(s-\sigma(s)) - g(0)| + |g(0)|) \Big) \Big] ds \Big\} 2R \\
\le &\sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big[\sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} (L|\varphi_{kl}(s-\tau(s))| + |f(0)|) \\
&\qquad + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}} (l|\varphi'_{kl}(s-\sigma(s))| + |g(0)|) \Big] ds \Big\} 2R \\
\le &\sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du} \Big[\sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} (2LR + |f(0)|)
\end{aligned}$$

$$
\begin{aligned}
&\quad + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}}(2lR+|g(0)|)\Big]ds\Big\}2R \\
&\le \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t} e^{-a_{ij*}(t-s)}ds\Big\}2R\theta_1 \\
&\le 2R\theta_1 \max_{i,j}\Big\{\frac{1}{a_{ij*}}\Big\} \\
&\le R \qquad (2.2)
\end{aligned}
$$

and

$$
\begin{aligned}
& \|(T\varphi-\varphi_0)'\|_0 \\
= & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\Big|\int_{-\infty}^{t} -a_{ij}(t)e^{-\int_s^t a_{ij}(u)du}\cdot \\
& \Big[-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)f(\varphi_{kl}(s-\tau(s))) \\
& -\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s)g(\varphi'_{kl}(s-\sigma(s)))\Big]\varphi_{ij}(s)ds \\
& +\Big[-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(\varphi_{kl}(t-\tau(t))) \\
& -\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(t)g(\varphi'_{kl}(t-\sigma(t)))\Big]\varphi_{ij}(t)\Big|\Big\} \\
= & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\Big|\int_{-\infty}^{t} a_{ij}(t)e^{-\int_s^t a_{ij}(u)du}\cdot \\
& \Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)f(\varphi_{kl}(s-\tau(s))) \\
& +\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s)g(\varphi'_{kl}(s-\sigma(s)))\Big]\varphi_{ij}(s)ds \\
& -\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(\varphi_{kl}(t-\tau(t))) \\
& +\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(t)g(\varphi'_{kl}(t-\sigma(t)))\Big]\varphi_{ij}(t)\Big|\Big\} \\
\le & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t} |a_{ij}(t)|e^{-\int_s^t a_{ij}(u)du}\cdot \\
& \Big[\sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(s)|\ |f(\varphi_{kl}(s-\tau(s)))| \\
& +\sum_{C_{kl}\in N_s(i,j)} |D_{ij}^{kl}(s)|\ |g(\varphi'_{kl}(s-\sigma(s)))|\Big]\ |\varphi_{ij}(s)|ds
\end{aligned}
$$

$$
\begin{aligned}
& \quad + \Big[\sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(t)| \; |f(\varphi_{kl}(t-\tau(t)))| \\
& \qquad + \sum_{C_{kl}\in N_s(i,j)} |D_{ij}^{kl}(t)| \; |g(\varphi'_{kl}(t-\sigma(t)))| \Big] |\varphi_{ij}(t)| \Big\} \\
\leq & \quad \sup_{t\in\mathbb{R}} \max_{i,j} \Big\{ \int_{-\infty}^{t} a_{ij}^{+} e^{-a_{ij*}(t-s)} ds \cdot \\
& \qquad \Big[\sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big(2LR + |f(0)|\Big) \\
& \qquad + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}} \Big(2lR + |g(0)|\Big) \Big] \\
& \qquad + \Big[\sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big(2LR + |f(0)|\Big) \\
& \qquad + \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}} \Big(2lR + |g(0)|\Big) \Big] \Big\} \cdot 2R \\
\leq & \quad 2R\theta_1 \max_{i,j} \Big\{ 1 + \frac{a_{ij}^{+}}{a_{ij*}} \Big\} \\
\leq & \quad R. \qquad\qquad (2.3)
\end{aligned}
$$

From (2.2) and (2.3), we have $\|T\varphi - \varphi_0\| \leq R$, thus $T(E) \subseteq E$.

Let $\varphi,\ \psi \in E$, denote by

$$
\begin{aligned}
I_1(s) &= \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s) \Big[f\Big(\varphi_{kl}(s-\tau(s))\Big)\varphi_{ij}(s) - f\Big(\psi_{kl}(s-\sigma(s))\Big)\psi_{ij}(s) \Big], \\
I_2(s) &= \sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s) \Big[g\Big(\varphi'_{kl}(s-\sigma(s))\Big)\varphi_{ij}(s) - g\Big(\psi'_{kl}(s-\sigma(s))\Big)\psi_{ij}(s) \Big].
\end{aligned}
$$

We have

$$
\begin{aligned}
& |I_1(s)| \\
= & \Big| \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s) \Big[f\Big(\varphi_{kl}(s-\tau(s))\Big)\varphi_{ij}(s) - f\Big(\psi_{kl}(s-\sigma(s))\Big)\psi_{ij}(s) \Big] \Big| \\
\leq & \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big[\Big| f\Big(\varphi_{kl}(s-\tau(s))\Big)\varphi_{ij}(s) - f\Big(\varphi_{kl}(s-\tau(s))\Big)\psi_{ij}(s) \Big| \\
& + \Big| f\Big(\varphi_{kl}(s-\tau(s))\Big)\psi_{ij}(s) - f\Big(\psi_{kl}(s-\sigma(s))\Big)\psi_{ij}(s) \Big| \Big] \\
= & \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}} \Big[\Big| f\Big(\varphi_{kl}(s-\tau(s))\Big) \Big| \cdot \Big| \varphi_{ij}(s) - \psi_{ij}(s) \Big|
\end{aligned}
$$

$$
\begin{aligned}
&\qquad +\Big|f\Big(\varphi_{kl}(s-\tau(s))\Big)-f\Big(\psi_{kl}(s-\sigma(s))\Big)\Big|\cdot\Big|\psi_{ij}(s)\Big|\Big]\\
\le\ & \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}}\Big[\Big|f\Big(\varphi_{kl}(s-\tau(s))\Big)\Big|\cdot\Big|\varphi_{ij}(s)-\psi_{ij}(s)\Big|\\
&\qquad +L\cdot\Big|\varphi_{kl}(s-\tau(s))-\psi_{kl}(s-\sigma(s))\Big|\cdot\Big|\psi_{ij}(s)\Big|\Big]\\
\le\ & \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}}\Big[\Big(\Big|f\Big(\varphi_{kl}(s-\tau(s))\Big)-f(0)\Big|+\Big|f(0)\Big|\Big)\cdot\\
&\qquad \Big|\varphi_{ij}(s)-\psi_{ij}(s)\Big|\\
&\qquad +L\cdot\Big|\varphi_{kl}(s-\tau(s))-\psi_{kl}(s-\sigma(s))\Big|\cdot\Big|\psi_{ij}(s)\Big|\Big]\\
\le\ & \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}}\Big[\Big(2LR+|f(0)|\Big)\cdot\Big|\varphi_{ij}(s)-\psi_{ij}(s)\Big|\\
&\qquad +2LR\cdot\Big|\varphi_{kl}(s-\tau(s))-\psi_{kl}(s-\sigma(s))\Big|\Big]\\
\le\ & \sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}}\Big(4LR+|f(0)|\Big)\cdot\|\varphi-\psi\|
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& |I_2(s)|\\
=\ & \Big|\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s)\Big[g\Big(\varphi'_{kl}(s-\tau(s))\Big)\varphi_{ij}(s)-g\Big(\psi'_{kl}(s-\sigma(s))\Big)\psi_{ij}(s)\Big]\Big|\\
\le\ & \sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}}\Big(4lR+|g(0)|\Big)\cdot\|\varphi-\psi\|
\end{aligned}
$$

$$
\begin{aligned}
& \|T\varphi-T\psi\|_0\\
=\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\Big|\int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du}\cdot\\
& \Big[-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\Big([f(\varphi_{kl}(s-\tau(s)))\varphi_{ij}(s)-f(\psi_{kl}(s-\sigma(s)))\psi_{ij}(s)\Big)\\
& -\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s)\Big(g(\varphi'_{kl}(s-\sigma(s)))\varphi_{ij}(s)-g(\psi'_{kl}(s-\sigma(s)))\psi_{ij}(s)\Big)\Big]ds\Big|\Big\}\\
\le\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du}\Big(|I_1(s)|+|I_2(s)|\Big)ds\Big\}\\
\le\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t} e^{-\int_s^t a_{ij}(u)du}ds\Big[\sum_{C_{kl}\in N_r(i,j)} \overline{C_{ij}^{kl}}\Big(4LR+|f(0)|\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{C_{kl}\in N_s(i,j)} \overline{D_{ij}^{kl}}\Big(4lR+|g(0)|\Big)\Big]\Big\}\cdot\|\varphi-\psi\| \\
\leq\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t} e^{-a_{ij*}(t-s)}ds\Big\}\cdot\theta_2\cdot\|\varphi-\psi\| \\
\leq\ & \theta_2\cdot\max_{i,j}\{\frac{1}{a_{ij*}}\}\cdot\|\varphi-\psi\| \qquad (2.4)
\end{aligned}
$$

And

$$
\begin{aligned}
&\|(T\varphi-T\psi)'\|_0 \\
=\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\Big|\int_{-\infty}^{t} -a_{ij}(t)e^{-\int_s^t a_{ij}(u)du}\cdot \\
&\qquad \Big\{\Big[-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)f(\varphi_{kl}(s-\tau(s))) \\
&\qquad\qquad -\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s)g(\varphi'_{kl}(s-\sigma(s)))\Big]\varphi_{ij}(s) \\
&\qquad +\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)f(\psi_{kl}(s-\tau(s))) \\
&\qquad\qquad +\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(s)g(\psi'_{kl}(s-\sigma(s)))\Big]\psi_{ij}(s)\Big\}ds \\
&\qquad +\Big\{\Big[-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(\varphi_{kl}(t-\tau(t))) \\
&\qquad\qquad -\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(t)g(\varphi'_{kl}(t-\sigma(t)))\Big]\varphi_{ij}(t) \\
&\qquad +\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(\psi_{kl}(t-\tau(t))) \\
&\qquad\qquad +\sum_{C_{kl}\in N_s(i,j)} D_{ij}^{kl}(t)g(\psi'_{kl}(t-\sigma(t)))\Big]\psi_{ij}(t)\Big\}\Big|\Big\} \\
\leq\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t}|a_{ij}(t)|e^{-\int_s^t a_{ij}(u)du}\Big(|I_1(s)|+|I_2(s)|\Big)ds \\
&\qquad +\Big(|I_1(t)|+|I_2(t)|\Big)\Big\} \\
\leq\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t}|a_{ij}(t)|e^{-\int_s^t a_{ij}(u)du}ds+1\Big\}\cdot\theta_2\cdot\|\varphi-\psi\| \\
\leq\ & \sup_{t\in\mathbb{R}}\max_{i,j}\Big\{\int_{-\infty}^{t}a_{ij}^{+}e^{-a_{ij*}(t-s)}ds+1\Big\}\cdot\theta_2\cdot\|\varphi-\psi\|
\end{aligned}
$$

$$\leq \quad \theta_2 \cdot \max_{i,j} \left\{ 1 + \frac{a_{ij}^+}{a_{ij*}} \right\} \cdot \|\varphi - \psi\| \tag{2.5}$$

From (2.4) and (2.5), we have $\|T\varphi - T\psi\| \leq \theta_2 \cdot \max_{i,j} \left\{ \frac{1}{a_{ij*}}, \quad 1 + \frac{a_{ij}^+}{a_{ij*}} \right\} \cdot \|\varphi - \psi\| = \theta \cdot \|\varphi - \psi\|$. According to the condition of this theorem we know $\theta < 1$, therefore T has a unique fixed point.

Example We consider the following SICCN with neutral a delay:

$$\begin{aligned} x_{ij}' = -a_{ij}(t)x_{ij} \quad &- \sum_{C_{kl} \in N_r(ij)} C_{ij}^{kl}(t) f[x_{kl}(t - \tau(t))] x_{ij}(t) \\ &+ \sum_{C_{kl} \in N_s(i,j)} D_{ij}^{kl}(t) g[x_{kl}'(t - \sigma(t))] x_{ij}(t) + I_{ij}(t), \end{aligned} \tag{2.6}$$

where $i = 1, 2, 3,\ j = 1, 2, 3,\ \tau(t) = \cos^2 t,\ \sigma(t) = \sin 2t,\ f(x) = \frac{4}{5} \sin x,\ g(x) = \frac{3}{4}|x|,$

$$\begin{pmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{pmatrix} = \begin{pmatrix} 5 + |\sin t| & 5 + |\sin 2t| & 9 + |\sin t| \\ 6 + |\cos t| & 6 + |\sin t| & 7 + |\cos t| \\ 8 + |\cos t| & 8 + |\sin t| & 5 + |\sin 2t| \end{pmatrix}$$

$$\begin{pmatrix} c_{11}(t) & c_{12}(t) & c_{13}(t) \\ c_{21}(t) & c_{22}(t) & c_{23}(t) \\ c_{31}(t) & c_{32}(t) & c_{33}(t) \end{pmatrix} = \begin{pmatrix} 0.004|\sin 3t| & 0.002|\sin 3t| & 0.001|\sin 3t| \\ 0.002|\sin 3t| & 0.001|\sin 3t| & 0.001|\sin 3t| \\ 0.001|\sin 3t| & 0.002|\sin 3t| & 0.001|\sin 3t| \end{pmatrix}$$

$$\begin{pmatrix} d_{11}(t) & d_{12}(t) & d_{13}(t) \\ d_{21}(t) & d_{22}(t) & d_{23}(t) \\ d_{31}(t) & d_{32}(t) & d_{33}(t) \end{pmatrix} = \begin{pmatrix} 0.001|\cos 2t| & 0.001|\cos 2t| & 0.002|\cos 2t| \\ 0.001|\cos 2t| & 0.002|\cos 2t| & 0.003|\cos 2t| \\ 0.002|\cos 2t| & 0.002|\cos 2t| & 0.001|\cos 2t| \end{pmatrix}$$

$$\begin{pmatrix} I_{11}(t) & I_{12}(t) & I_{13}(t) \\ I_{21}(t) & I_{22}(t) & I_{23}(t) \\ I_{31}(t) & I_{32}(t) & I_{33}(t) \end{pmatrix} = \begin{pmatrix} \sin t & \sin t & \cos t \\ \sin t & \cos t & \cos t \\ \cos t & \cos t & \cos t \end{pmatrix}$$

In the following, we will check that all assumptions of the theorem are satisfied. By computing,we have

$$\begin{pmatrix} a_{11}^+ & a_{12}^+ & a_{13}^+ \\ a_{21}^+ & a_{22}^+ & a_{23}^+ \\ a_{31}^+ & a_{32}^+ & a_{33}^+ \end{pmatrix} = \begin{pmatrix} 6 & 6 & 10 \\ 7 & 7 & 8 \\ 9 & 9 & 6 \end{pmatrix}, \qquad \begin{pmatrix} a_{11*} & a_{12*} & a_{13*} \\ a_{21*} & a_{22*} & a_{23*} \\ a_{31*} & a_{32*} & a_{33*} \end{pmatrix} = \begin{pmatrix} 5 & 5 & 9 \\ 6 & 6 & 7 \\ 8 & 8 & 5 \end{pmatrix}$$

$$\begin{pmatrix} \overline{c_{11}} & \overline{c_{12}} & \overline{c_{13}} \\ \overline{c_{21}} & \overline{c_{22}} & \overline{c_{23}} \\ \overline{c_{31}} & \overline{c_{32}} & \overline{c_{33}} \end{pmatrix} = \begin{pmatrix} 0.004 & 0.002 & 0.001 \\ 0.002 & 0.001 & 0.001 \\ 0.001 & 0.002 & 0.001 \end{pmatrix},$$

$$\begin{pmatrix} \overline{d_{11}} & \overline{d_{12}} & \overline{d_{13}} \\ \overline{d_{21}} & \overline{d_{22}} & \overline{d_{23}} \\ \overline{d_{31}} & \overline{d_{32}} & \overline{d_{33}} \end{pmatrix} = \begin{pmatrix} 0.001 & 0.001 & 0.002 \\ 0.001 & 0.002 & 0.003 \\ 0.002 & 0.002 & 0.001 \end{pmatrix}.$$

Note that f and g are Lipschtz functions with $f(0) = g(0) = 0$, the Lipschtz constants of f and g, L, l, are less than 1, we take $L = l = 1$.

$$\begin{aligned}
&\sum_{c_{kl}\in N_1(1,1)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(1,1)} \overline{d_{kl}} = 0.014, \qquad \sum_{c_{kl}\in N_1(1,2)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(1,2)} \overline{d_{kl}} = 0.021,\\
&\sum_{c_{kl}\in N_1(1,3)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(1,3)} \overline{d_{kl}} = 0.013, \qquad \sum_{c_{kl}\in N_1(2,1)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(2,1)} \overline{d_{kl}} = 0.021,\\
&\sum_{c_{kl}\in N_1(2,2)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(2,2)} \overline{d_{kl}} = 0.030, \qquad \sum_{c_{kl}\in N_1(2,3)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(2,3)} \overline{d_{kl}} = 0.019,\\
&\sum_{c_{kl}\in N_1(3,1)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(3,1)} \overline{d_{kl}} = 0.013, \qquad \sum_{c_{kl}\in N_1(3,2)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(3,2)} \overline{d_{kl}} = 0.019,\\
&\sum_{c_{kl}\in N_1(3,3)} \overline{c_{kl}} + \sum_{c_{kl}\in N_1(3,3)} \overline{d_{kl}} = 0.013,
\end{aligned}$$

From computing we know $\|\varphi_0\| \leq \frac{11}{5}$, we choose $R = 3$. Obviously $\max\limits_{i,j}\left\{\frac{1}{a_{ij*}}, 1 + \frac{a_{ij}^{+}}{a_{ij*}}\right\} = \frac{11}{5}$. $\theta_1 = 0.18$, $\theta_2 = 0.36$, $\theta_1 \cdot \max\limits_{i,j}\{\frac{1}{a_{ij*}}, 1 + \frac{a_{ij}^{+}}{a_{ij*}}\} = 0.396 \leq \frac{1}{2}$, $\theta = \theta_2 \cdot \max\limits_{i,j}\left\{\frac{1}{a_{ij*}}, 1 + \frac{a_{ij}^{+}}{a_{ij*}}\right\} = 0.792 < 1$. So all the conditions of theorem 2.1 are satisfied, hence by the theorem 2.1, Eq.(1.1) has a unique almost periodic solution.

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