Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 10, pp. 16-30, January 2003

ELA
http://math.technion.ac.il/iic/ela

GENERALIZED INVERSES OF BORDERED MATRICES*

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Abstract. Several authors have considered nonsingular borderings $A = \begin{pmatrix} B & C \\ D & X \end{pmatrix}$ of B and investigated the properties of submatrices of A^{-1}. Under specific conditions on the bordering, one can recover any g-inverse of B as a submatrix of A^{-1}. Borderings A of B are considered, where A might be singular, or even rectangular. If A is $m \times n$ and if B is an $r \times s$ submatrix of A, the consequences of the equality $m + n - rank(A) = r + s - rank(B)$ with reference to the g-inverses of A are studied. It is shown that under this condition many properties enjoyed by nonsingular borderings have analogs for singular (or rectangular) borderings as well. We also consider g-inverses of the bordered matrix when certain rank additivity conditions are satisfied. It is shown that any g-inverse of B can be realized as a submatrix of a suitable g-inverse of A, under certain conditions.

Key words. Generalized inverse, Moore-Penrose inverse, Bordered matrix, Rank additivity.

AMS subject classifications. 15A09, 15A03.

*Received by the editors on 15 July 2002. Accepted for publication on 14 January 2003. Handling Editor: Steve J. Kirkland.

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