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THE MAXIMAL SPECTRAL RADIUS OF A DIGRAPH WITH $(M+1)^2 - S$ EDGES*

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Abstract. It is known that the spectral radius of a digraph with k edges is at most $\sqrt{k}$, and that this inequality is strict except when k is a perfect square. For $k = m^2 + \ell$, ℓ fixed, m large, Friedland showed that the optimal digraph is obtained from the complete digraph on m vertices by adding one extra vertex, a corresponding loop, and then connecting it to the first $\lfloor \ell/2 \rfloor$ vertices by pairs of directed edges (for even ℓ an extra edge is added to the new vertex).

Using a combinatorial reciprocity theorem, and a classification by Backelin on the digraphs on s edges having a maximal number of walks of length two, the following result is obtained: for fixed $0 < s \neq 4$, $k = (m+1)^2 - s$, m large, the maximal spectral radius of a digraph with k edges is obtained by the digraph which is constructed from the complete digraph on $m+1$ vertices by removing the loop at the last vertex together with $\lfloor s/2 \rfloor$ pairs of directed edges that connect to the last vertex (if s is even, remove an extra edge connecting the last vertex).

Key words. Spectral radius, Digraphs, $(0,1)$-matrices, Perron-Frobenius theorem, Number of walks.

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