Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 10, pp. 223-231, September 2003

ELA
http://math.technion.ac.il/iic/ela

THE MAXIMUM NUMBER OF 2 × 2 ODD SUBMATRICES IN $(0,1)$-MATRICES*

MICHAEL MARKS†, RICK NORWOOD†, AND GEORGE POOLE†‡

Abstract. Let A be an $m \times n$, $(0,1)$-matrix. A submatrix of A is odd if the sum of its entries is an odd integer and even otherwise. The maximum number of 2×2 odd submatrices in a $(0,1)$-matrix is related to the existence of Hadamard matrices and bounds on Turán numbers. Pinelis [On the minimal number of even submatrices of 0-1 matrices, *Designs, Codes and Cryptography*, 9:85–93, 1994] exhibits an asymptotic formula for the minimum possible number of $p \times q$ even submatrices of an $m \times n$ $(0,1)$-matrix. Assuming the Hadamard conjecture, specific techniques are provided on how to assign the 0's and 1's, in order to yield the maximum number of 2×2 odd submatrices in an $m \times n$ $(0,1)$-matrix. Moreover, formulas are determined that yield the exact maximum counts with one exception, in which case upper and lower bounds are given. These results extend and refine those of Pinelis.

Key words. $(0,1)$-matrices, Even and odd matrices, Hadamard matrices.

AMS subject classifications. 15A36, 15A37.

*Received by the editors on 15 October 2002. Accepted for publication on 18 September 2003. Handling Editor: Bryan Shader.

†Department of Mathematics, East Tennessee State University, Johnson City, Tennessee 37614, USA.

‡Corresponding author: George Poole (pooleg@mail.etsu.edu).