**Electronic Journal of Linear Algebra ISSN 1081-3810**
**A publication of the International Linear Algebra Society**
**Volume 10, pp. 291-319, December 2003**

**ELA**
http://math.technion.ac.il/iic/ela

# MULTIPLICATIVE MAPS ON INVERTIBLE MATRICES THAT PRESERVE MATRICIAL PROPERTIES*

ROBERT M. GURALNICK†, CHI-KWONG LI‡, AND LEIBA RODMAN‡

**Abstract.** Descriptions are given of multiplicative maps on complex and real matrices that leave invariant a certain function, property, or set of matrices: norms, spectrum, spectral radius, elementary symmetric functions of eigenvalues, certain functions of singular values, $(p, q)$ numerical ranges and radii, sets of unitary, normal, or Hermitian matrices, as well as sets of Hermitian matrices with fixed inertia. The treatment of all these cases is unified, and is based on general group theoretic results concerning multiplicative maps of general and special linear groups, which in turn are based on classical results by Borel - Tits. Multiplicative maps that leave invariant elementary symmetric functions of eigenvalues and spectra are described also for matrices over a general commutative field.

**Key words.** Linear group, Multiplicative Preserver, Norm, Eigenvalue, Spectral radius, Generalized numerical range, Generalized numerical radius.

**AMS subject classifications.** 15A99, 20G99.

*Received by the editors on 16 April 2003. Accepted for publication on 30 November 2003. Handling Editor: Shmuel Friedland.

†Department of Mathematics, University of Southern California, Los Angeles, CA 90089-1113, USA (guralnic@math.usc.edu). The authors gratefully acknowledge the support of the NSF grants DMS-0140578, DMS-0071944, DMS-9988579, respectively.

‡Department of Mathematics, College of William & Mary, Williamsburg, VA 23187-8795, USA (ckli@math.wm.edu, lxrodm@math.wm.edu).