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AN ALGORITHM THAT CARRIES A SQUARE MATRIX INTO ITS TRANSPOSE BY AN INVOLUTORY CONGRUENCE TRANSFORMATION[*]

D.Ž. ĐOKOVIĆ[†], F. SZECHTMAN[‡], AND K. ZHAO[§]

Abstract. For any matrix X let X' denote its transpose. It is known that if A is an n-by-n matrix over a field F, then A and A' are congruent over F, i.e., $XAX' = A'$ for some $X \in \mathrm{GL}_n(F)$. Moreover, X can be chosen so that $X^2 = I_n$, where I_n is the identity matrix. An algorithm is constructed to compute such an X for a given matrix A. Consequently, a new and completely elementary proof of that result is obtained.

As a by-product another interesting result is also established. Let G be a semisimple complex Lie group with Lie algebra $\mathfrak{g}$. Let $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ be a $\mathbf{Z}_2$-gradation such that $\mathfrak{g}_1$ contains a Cartan subalgebra of $\mathfrak{g}$. Then L.V. Antonyan has shown that every G-orbit in $\mathfrak{g}$ meets $\mathfrak{g}_1$. It is shown that, in the case of the symplectic group, this assertion remains valid over an arbitrary field F of characteristic different from 2. An analog of that result is proved when the characteristic is 2.

Key words. Congruence of matrices, Transpose, Rational solution, Symplectic group.

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[†]Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada (djokovic@uwaterloo.ca). Research was supported in part by the NSERC Grant A-5285.

[‡]Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada. Current Address: Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada (szechtf@math.uregina.ca).

[§]Institute of Mathematics, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing 100080, P.R. China, and Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario, N2L 3C5, Canada (kzhao@mail2.math.ac.cn). Research supported by the NSF of China (Grant 10371120).