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THE STRUCTURE OF LINEAR OPERATORS STRONGLY PRESERVING MAJORIZATIONS OF MATRICES*

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Abstract. A matrix majorization relation $A \prec_r B$ (resp., $A \prec_\ell B$) on the collection M_n of all $n \times n$ real matrices is a relation $A = BR$ (resp., $A = RB$) for some $n \times n$ row stochastic matrix R (depending on A and B). These right and left matrix majorizations have been considered by some authors under the names "matrix majorization" and "weak matrix majorization," respectively. Also, a multivariate majorization $A \prec_{rmul} B$ (resp., $A \prec_{\ell mul} B$) is a relation $A = BD$ (resp., $A = DB$) for some $n \times n$ doubly stochastic matrix D (depending on A and B). The linear operators $T : M_n \to M_n$ which strongly preserve each of the above mentioned majorizations are characterized. Recall that an operator $T : M_n \to M_n$ strongly preserves a relation $\mathcal{R}$ on M_n when $\mathcal{R}(T(X), T(Y))$ if and only if $\mathcal{R}(X, Y)$. The results are the sharpening of well-known representations $TX = CXD$ or $TX = CX^tD$ for linear operators preserving invertible matrices.

Key words. Row stochastic matrix, Doubly stochastic matrix, Matrix majorization, Weak matrix majorization, Left(right) multivariate majorization, Linear preserver.

AMS subject classifications. 15A04, 15A21, 15A51.

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