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COMMUTING PAIRS OF PATTERNS AND SYMMETRIC REALIZATIONS*

CHARLES R. JOHNSON† AND OLIVIA WALCH†

Abstract. The patterns that commute with tridiagonal and with other tree patterns are studied. New "ratio" equations are given for the entries of a matrix that commutes with an irreducible tridiagonal one, and these equations imply that a pattern commuting with an irreducible tridiagonal one must be combinatorially symmetric. For an irreducible tridiagonal pattern and another pattern that commutes with it, it is shown that there is always a complex symmetric example of commutativity. However, there need not be a real symmetric commuting instance. An 8-by-8 example is given that settles a natural and long-standing question (whether real, commuting, combinatorially symmetric patterns may be realized by real symmetric matrices). Finally, similar results are given for other patterns, in place of irreducible tridiagonal, under additional hypotheses.

Key words. Commuting matrices, pattern, zero-nonzero pattern, symmetric realization, ratio equations, tree, tridiagonal pattern

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†Department of Mathematics, College of William and Mary, Williamsburg, VA, 23187 USA (crjohnso@math.wm.edu and ojwalch@email.wm.edu).