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ON THE RELATION BETWEEN THE NUMERICAL RANGE AND THE JOINT NUMERICAL RANGE OF MATRIX POLYNOMIALS*

P. J. PSARRAKOS† AND M. J. TSATSOMEROS‡

Abstract. It is shown that the numerical range, $\mathrm{NR}[P(\lambda)]$, of a matrix polynomial $P(\lambda) = A_m\lambda^m + \ldots + A_1\lambda + A_0$ consists of the roots of all scalar polynomials whose coefficients correspond to the elements of the convex hull of the joint numerical range of the $(m+1)$-tuple $(A_0, A_1, \ldots, A_m)$. Moreover, the elements of the joint numerical range that give rise to scalar polynomials with a common root belonging to $\mathrm{NR}[P(\lambda)]$ form a connected set. The latter fact is used to examine the multiplicity of roots belonging to the intersection of the root zones of $\mathrm{NR}[P(\lambda)]$. Also an approximation scheme for $\mathrm{NR}[P(\lambda)]$ is proposed, in terms of numerical ranges of diagonal matrix polynomials.

Key words. joint numerical range, matrix polynomial, root zone

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†Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, Canada S4S 0A2 (panos@math.uregina.ca).

‡Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, Canada S4S 0A2 (tsat@math.uregina.ca). Research supported by NSERC research grant.