The Electronic Journal of Linear Algebra.
A publication of the International Linear Algebra Society.
Volume 6, pp. 72-94, August 2000.
ISSN 1081-3810. http://math.technion.ac.il/iic/ela

ELA

A BI-CG TYPE ITERATIVE METHOD FOR DRAZIN-INVERSE SOLUTION OF SINGULAR INCONSISTENT NONSYMMETRIC LINEAR SYSTEMS OF ARBITRARY INDEX*

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Abstract. Consider the linear system $Ax = b$, where b is a vector in $\mathbb{C}^N$, $A \in \mathbb{C}^{N\times N}$ is a singular matrix, and $\mathrm{ind}\,(A) = a$ is arbitrary. Here $\mathrm{ind}\,(\cdot)$ denotes the index of a matrix. The Drazin-inverse solution of this system is defined to be the vector $A^{\mathrm{D}}b$, where the matrix A^{D} is the Drazin inverse of A. The Drazin-inverse solution of singular linear systems has been considered recently by the first author within the context of extrapolation methods, when $\mathrm{ind}\,(A)$ is arbitrary. It has also been considered within the context of Krylov subspace methods, when A is real symmetric (hence $\mathrm{ind}\,(A) = 1$ necessarily). In addition, semi-iterative methods have been developed for the cases in which $\mathrm{ind}\,(A) = 1$ and $\mathrm{ind}\,(A) > 1$, assuming that the spectrum of A is real nonnegative. The purpose of the present work is to develop a Bi-CG type Krylov subspace method suitable for the general case in which A is not necessarily real symmetric, its index is arbitrary, and its spectrum is not necessarily real. The method that is developed can be implemented via a 4-term recursion relation independently of the size of $\mathrm{ind}\,(A)$ and produces $A^{\mathrm{D}}b$ in at most $N - a$ steps. A detailed error analysis for this method is provided and the results are illustrated with suitable numerical examples.

Key words. Singular linear systems, Drazin-inverse solution, Krylov subspace methods, Lanczos method, Bi-Conjugate Gradient algorithm.

AMS subject classifications. 15A06, 15A09, 65F10, 65F50

*Received by the editors on 8 August 1999. Accepted for publication on 12 July 2000. Handling Editor: Daniel Hershkowitz.

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