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POLAR DECOMPOSITION UNDER PERTURBATIONS OF THE SCALAR PRODUCT*

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Abstract. Let $\mathcal{A}$ be a unital C*-algebra with involution $*$ represented in a Hilbert space $\mathcal{H}$, G the group of invertible elements of $\mathcal{A}$, $\mathcal{U}$ the unitary group of $\mathcal{A}$, G^s the set of invertible selfadjoint elements of $\mathcal{A}$, $Q = \{\varepsilon \in G : \varepsilon^2 = 1\}$ the space of reflections and $P = Q \cap \mathcal{U}$. For any positive $a \in G$ consider the a-unitary group $\mathcal{U}_a = \{g \in G : a^{-1}g^*a = g^{-1}\}$, i.e., the elements which are unitary with respect to the scalar product $\langle \xi, \eta \rangle_a = \langle a\xi, \eta \rangle$ for $\xi,\ \eta \in \mathcal{H}$. If π denotes the map that assigns to each invertible element its unitary part in the polar decomposition, it is shown that the restriction $\pi|_{\mathcal{U}_a} : \mathcal{U}_a \to \mathcal{U}$ is a diffeomorphism, that $\pi(\mathcal{U}_a \cap Q) = P$, and that $\pi(\mathcal{U}_a \cap G^s) = \mathcal{U}_a \cap G^s = \{u \in G : u = u^* = u^{-1} \text{ and } au = ua\}$.

Key words. Polar decomposition, C*-algebras, positive operators, projections.

AMS subject classifications. Primary 47A30, 47B15

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