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GENERALIZED NUMERICAL RANGES OF PERMANENTAL COMPOUNDS ARISING FROM QUANTUM SYSTEMS OF BOSONS*

NATÁLIA BEBIANO†, CHI-KWONG LI‡, AND JOÃO DA PROVIDÊNCIA§

Abstract. Let $\alpha = (\alpha_1, \ldots, \alpha_m)$ be an m-tuple of integers such that $1 \leq \alpha_1 \leq \cdots \leq \alpha_m \leq n$. Denote by per (X) the permanent of a square matrix. The permanental range $W_m^\alpha(A)$ of an $n\times n$ complex matrix A associated with α is the set of complex numbers of the form $\mathrm{per}(X^*AX)/\mathrm{per}(X^*X)$, where X is an $n \times m$ matrix with columns x_{α_j} for $j = 1, \ldots, m$ so that $(x_k, x_l) = \delta_{kl}$, the Kronecker delta. The set $W_m^\alpha(A)$ is related to quantum system of m bosons lying in single-particle states specified by α. Geometrical properties of $W_m^\alpha(A)$ are studied and their physical interpretations are given. Linear operators L on $n \times n$ matrices satisfying $W_m^\alpha(L(A)) = W_m^\alpha(A)$ are characterized. The permanental ranges associated with derivations are also discussed.

Key words. Generalized numerical range, permanent, quantum systems of Bosons

AMS subject classifications. 15A60, 15A90

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†Departamento de Matemática, Universidade de Coimbra, 3000 Coimbra, Portugal (bebiano@ci.uc.pt)

‡Department of Mathematics, College of William and Mary, P.O. Box 8795, Williamsburg, VA 23187-8795, USA (ckli@math.wm.edu)

§Departamento de Física, Universidade de Coimbra, 3000 Coimbra, Portugal (providencia@fteor5.fis.uc.pt)