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ADDITIONAL RESULTS ON INDEX SPLITTINGS FOR DRAZIN INVERSE SOLUTIONS OF SINGULAR LINEAR SYSTEMS *

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Abstract. Given an $n \times n$ singular matrix A with $\mathrm{Ind}(A) = k$, an index splitting of A is one of the form $A = U - V$, where $R(U) = R(A^k)$ and $N(U) = N(A^k)$. This splitting, introduced by the first author, generalizes the proper splitting proposed by Berman and Plemmons. Regarding singular systems $Au = f$, the first author has shown that the iterations $u^{(i+1)} = U^{\#}Vu^{(i)} + U^{\#}f$ converge to $A^D f$, the Drazin inverse solution to the system, if and only if the spectral radius of $U^{\#}V$ is less than one. The aim of this paper is to further study index splittings in order to extend some previous results by replacing the Moore-Penrose inverse A^+ and A^{-1} with the Drazin inverse A^D. The characteristics of the Drazin inverse solution $A^D f$ are established. Some criteria are given for comparing convergence rates of $U_i^{\#}V_i$, where $A = U_1 - V_1 = U_2 - V_2$. Results of Collatz, Marek and Szyld on monotone-type iterations are extended. A characterization of the iteration matrix of an index splitting is also presented.

Key words. Index, Drazin inverse, group inverse, Moore-Penrose inverse, index splitting, proper splitting, comparison theorem, monotone iteration.

AMS subject classifications. 15A09, 65F15, 65F20

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