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MAXIMAL NESTS OF SUBSPACES, THE MATRIX BRUHAT DECOMPOSITION, AND THE MARRIAGE THEOREM – WITH AN APPLICATION TO GRAPH COLORING*

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Abstract. Using the celebrated Marriage Theorem of P. Hall, we give an elementary combinatorial proof of the theorem that asserts that given two maximal nests $\mathcal{N}_1$ and $\mathcal{N}_2$ in a finite dimensional vector space V, there is an ordered basis of V that generates $\mathcal{N}_1$ and a permutation of that ordered basis that generates $\mathcal{N}_2$. From this theorem one easily obtains the Matrix Bruhat Decomposition. A generalization to matroids is discussed, and an application to graph coloring is given.

Key words. Nests of subspaces, matrix Bruhat decomposition, marriage theorem, graph colorings, matroids.

AMS subject classifications. 15A03, 05C15, 05B35

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