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ON THE CAYLEY TRANSFORM OF POSITIVITY CLASSES OF MATRICES*

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Abstract. The Cayley transform of A, $F = (I+A)^{-1}(I-A)$, is studied when A is a P-matrix, an M-matrix, an inverse M-matrix, a positive definite matrix, or a totally nonnegative matrix. Given a matrix A in each of these positivity classes and using the fact that the Cayley transform is an involution, properties of the ensuing factorization $A = (I+F)^{-1}(I-F)$ are examined. Specifically, it is investigated whether these factors belong to the same positivity class as A and, conversely, under what conditions on these factors does A belong to one of the above positivity classes.

Key words. Cayley transform, P-matrices, M-matrices, Positive definite matrices, Totally nonnegative matrices, Stable matrices, Matrix factorizations.

AMS subject classifications. 15A23, 15A24, 15A48

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