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ITERATIONS OF CONCAVE MAPS, THE PERRON–FROBENIUS THEORY, AND APPLICATIONS TO CIRCLE PACKINGS*

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Abstract. The theory of pseudo circle packings is developed. It generalizes the theory of circle packings. It allows the realization of almost any graph embedding by a geometric structure of circles. The corresponding Thurston's relaxation mapping is defined and is used to prove the existence and the rigidity of the pseudo circle packing. It is shown that iterates of this mapping, starting from an arbitrary point, converge to its unique positive fixed point. The coordinates of this fixed point give the radii of the packing. A key property of the relaxation mapping is its superadditivity. The proof of that is reduced to show that a certain real polynomial in four variables and of degree 20 is always nonnegative. This in turn is proved by using recently developed algorithms from real algebraic geometry. Another important ingredient in the development of the theory is the use of nonnegative matrices and the corresponding Perron–Frobenius theory.

Key words. Circle packings, Discrete conformal geometry, Nonnegative matrices, Perron–Frobenius, Monotone and positive mappings, Min-max principles, Fixed-point theorems, Rigidity of circle packings, Real algebraic geometry, Nonnegative polynomials, Algorithms for sums of squares of polynomials.

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