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POSITIVE EIGENVALUES AND TWO-LETTER GENERALIZED WORDS*

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Abstract. A generalized word in two letters A and B is an expression of the form $W = A^{\alpha_1}B^{\beta_1}A^{\alpha_2}B^{\beta_2}\cdots A^{\alpha_N}B^{\beta_N}$ in which the exponents are nonzero real numbers. When independent positive definite matrices are substituted for A and B, it is of interest whether W necessarily has positive eigenvalues. This is known to be the case when $N = 1$ and has been studied in case all exponents are positive by two of the authors. When the exponent signs are mixed, however, the situation is quite different (even for 2-by-2 matrices), and this is the focus of the present work.

Key words. Positive definite matrices, projections, generalized word.

AMS subject classifications. 15A18, 15A57

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