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ON SYMMETRIC MATRICES WITH EXACTLY ONE POSITIVE EIGENVALUE*

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Abstract. We present a class of nonsingular matrices, the MC'-matrices, and prove that the class of symmetric MC-matrices introduced by Shen, Huang and Jing [On inclusion and exclusion intervals for the real eigenvalues of real matrices. *SIAM J. Matrix Anal. Appl.*, 31:816-830, 2009] and the class of symmetric MC'-matrices are both subsets of the class of symmetric matrices with exactly one positive eigenvalue. Some other sufficient conditions for a symmetric matrix to have exactly one positive eigenvalue are derived.

Key words. Eigenvalue, Symmetric matrix, MC-matrix, MC'-matrix.

AMS subject classifications. 15A18, 15A48, 15A57.

1. Introduction. The class of symmetric real matrices having exactly one positive eigenvalue will be denoted by $\mathscr{B}$. The class of positive matrices belonging to $\mathscr{B}$ will be denoted by $\mathscr{A}$; see [1]. Clearly, $\mathscr{A} \subseteq \mathscr{B}$. These classes of matrices play important roles in many areas such as mathematical programming, matrix theory, numerical analysis, interpolation of scattered data and statistics; see, e.g., [1, 5, 7].

It was shown in [1] that a symmetric positive matrix $A \in \mathscr{A}$ if and only if the (unique) doubly stochastic matrix of the form $D^T AD$ is conditionally negative definite, where D is a positive diagonal matrix. In [7], Peña presented several properties of a symmetric positive matrix with exactly one positive eigenvalue. In particular, the author first obtained an equivalent condition for $A \in \mathscr{A}$, i.e., a symmetric positive matrix $A \in \mathscr{A}$ if and only if A has the LDL^T decomposition:

$$A = LDL^T,$$

where L is a unit lower triangular matrix, and $D = \mathrm{diag}(d_{11}, d_{22}, \cdots, d_{nn})$ with $d_{11} > 0$ and $d_{ii} < 0, i = 2, \cdots, n$. Secondly, the class of symmetric positive stochastic

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C-matrices was proved to be a subset of $\mathscr{A}$. Finally, the growth factor of Gaussian elimination with a given pivoting strategy applied to $A \in \mathscr{A}$ was analyzed, and a stable test to check whether a matrix belongs to $\mathscr{A}$ was established.

The class of C-matrices mentioned above was first defined by Peña in [6]. A matrix $A \in \mathbb{R}^{n\times n}$ with positive row sums is said to be a C-matrix if all its off-diagonal elements are bounded below by the corresponding row means; see [6]. Recently, Shen, Huang and Jing [8] presented a class of nonsingular matrices-MC-matrices, which is a generalization of the class of C-matrices.

In this paper, we show that every symmetric MC-matrix has exactly one positive eigenvalue. A new class of nonsingular matrices, the MC'-matrices, is introduced. The class of symmetric MC'-matrices is proved to be the subset of $\mathscr{B}$. Moreover, some other sufficient conditions for $A \in \mathscr{B}$ are derived.

The remainder of the paper is organized as follows. After introducing some notation and definitions in Section 2, we shall present some subclasses of $\mathscr{B}$ and some sufficient conditions for a matrix belonging to $\mathscr{B}$ in Section 3.

2. Notation and definitions. Let $A = (a_{ij}), B = (b_{ij}) \in \mathbb{R}^{n\times n}$, and let k be a positive integer. Then we denote by I the identity matrix; A^T the transpose of A; $\rho(A)$ the spectral radius of A; $\lambda_1(A) \le \lambda_2(A) \le \cdots \le \lambda_n(A)$ the eigenvalues of A if A is symmetric; ν_k the row vector $(1, 2, \cdots, k)$;

$$A[\nu_k] = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ \vdots & \vdots & \vdots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kk} \end{pmatrix}$$

the $k \times k$ leading principle submatrix of A. We write $A \ge B$ (respectively, $A > B$) if $a_{ij} \ge b_{ij}$ (respectively, $a_{ij} > b_{ij}$) for $i, j = 1, 2, \cdots, n$.

Definition 2.1. Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$. Then

1. ([2]) A is said to be a *positive* (respectively, *nonnegative*) matrix if $a_{ij} > 0$ (respectively, $a_{ij} \ge 0$) for all $i, j = 1, 2, \cdots, n$.
2. ([9]) The nonsingular matrix A is said to be an *M-matrix* if all its off-diagonal entries are nonpositive, and A^{-1} is nonnegative.

Given a matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, we define

$$s_i^+(A) := \max\{0, \min\{a_{ij} | j \ne i\}\}, i = 1, 2, \cdots, n.$$

The matrix A can be decomposed into

$$A = C^+(A) + E^+(A),$$

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where

$$C^+(A) = \begin{pmatrix} a_{11} - s_1^+(A) & a_{12} - s_1^+(A) & \cdots & a_{1n} - s_1^+(A) \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} - s_n^+(A) & a_{n2} - s_n^+(A) & \cdots & a_{nn} - s_n^+(A) \end{pmatrix},$$

$$E^+(A) = \begin{pmatrix} s_1^+(A) & s_1^+(A) & \cdots & s_1^+(A) \\ \vdots & \vdots & \vdots & \vdots \\ s_n^+(A) & s_n^+(A) & \cdots & s_n^+(A) \end{pmatrix}.$$

From Proposition 2.3 in [6], A is a C-matrix if and only if all its row sums are positive, and $-C^+(A)$ is a strictly diagonally dominant M-matrix (see [2]).

Definition 2.2 ([8]). A matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ with positive row sums is called an *MC-matrix* if all its off-diagonal elements are positive, and $-C^+(A)$ is an M-matrix.

Obviously, any C-matrix must be an MC-matrix. For a given MC-matrix $A \in \mathbb{R}^{n\times n}$, from Theorem 2.1 in [8] we have $(-1)^{n-1}\det(A) > 0$.

3. MC-matrices, MC'-matrices, and $\mathscr{B}$.

This section is devoted to giving some subclasses of $\mathscr{B}$. The following lemmas are needed.

Lemma 3.1. *Let $A \in \mathbb{R}^{n\times n}$ be an MC-matrix, and let $D \in \mathbb{R}^{n\times n}$ be a positive diagonal matrix. Then DA is an MC-matrix.*

Proof. By $A = C^+(A) + E^+(A)$, it is easy to get

$$DA = DC^+(A) + DE^+(A) = C^+(DA) + E^+(DA). \tag{3.1}$$

Clearly, $-C^+(DA) = -DC^+(A)$ is an M-matrix. Since all row sums and all off-diagonal entries of DA are still positive, from (3.1) and Definition 2.2, DA is an MC-matrix. ◻

The following result is a direct consequence of applying the Geršgorin disc theorem (see, e.g., [3]).

Lemma 3.2. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ with row sums*

$$\sum_{k=1}^{n} a_{ik} = r > 0, i = 1, 2, \cdots, n,$$

let all its off-diagonal elements be nonnegative, and let λ be any positive eigenvalue of A. Then r is an eigenvalue of A, and $\lambda \le r$.

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THEOREM 3.3. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be a symmetric MC-matrix. Then $A \in \mathscr{B}$.*

Proof. Setting

$$r_i = \sum_{k=1}^{n} a_{ik}, i = 1, 2, \cdots, n \text{ and } D = \text{diag}(r_1, r_2, \cdots, r_n),$$

from Definition 2.2, D is a positive diagonal matrix. By Lemma 3.1, $D^{-1}A$ is also an MC-matrix, and all its row sums are 1. Since $D^{-1}A$ is similar to $D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$ which is congruence to A, by Sylvester's law of inertia (Theorem 4.5.8 in [3]), we need only to prove that $D^{-1}A$ has exactly one positive eigenvalue. In fact, 1 is an eigenvalue of $D^{-1}A$ with algebraic multiplicity 1. Assume that λ different from 1 is a positive eigenvalue of A. Then by Lemma 3.2 we have $0 < \lambda < 1$. Obviously, all row sums of $D^{-1}A - \lambda I$ are $1 - \lambda > 0$, and, taking into account Lemma 3.1 and [2, Lemma 6.4.1], $-C^+(D^{-1}A - \lambda I)$ is an M-matrix. Hence, $D^{-1}A - \lambda I$ is an MC-matrix, and then nonsingular. This contradicts that λ is an eigenvalue of $D^{-1}A$. Thus, $D^{-1}A$ has exactly one positive eigenvalue 1. The proof is completed. □

We now define a new class of nonsingular matrices.

DEFINITION 3.4. A matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ with at least one positive diagonal element is called an MC'-matrix if all its off-diagonal elements are positive, and $-C^+(A)$ is an M-matrix.

For an MC'-matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, similar to the proof of Theorem 2.1 in [8], we can deduce $(-1)^{n-1}\det(A) > 0$. For the reader's convenience, we provide the following simple proof: From the proof of Theorem 2.1 in [8], it follows that

$$\det(-A) = \det(-C^+(A))(1 - x^T(-C^+(A))^{-1}y), \tag{3.2}$$

where

$$x = (1, 1, \cdots, 1)^T, \;\; y = (s_1^+(A), s_2^+(A), \cdots, s_n^+(A))^T.$$

Let $(-C^+(A))^{-1}y := z = (z_i)$. Then $y = -C^+(A)z$, and then

$$s_i^+(A)(z_1 + z_2 + \cdots + z_n) = s_i^+(A) + \sum_{k=1}^{n} a_{ik}z_k, i = 1, 2, \cdots, n. \tag{3.3}$$

Since z is a positive vector and, by the definition of an MC'-matrix, there must exist $1 \le j \le n$ such that $a_{jj} > 0$, we derive $\sum_{k=1}^{n} a_{jk}z_k > 0$, which by (3.3) implies

$$x^T(-C^+(A))^{-1}y = z_1 + z_2 + \cdots + z_n = \frac{s_j^+(A) + \sum_{k=1}^{n} a_{jk}z_k}{s_j^+(A)} > 1.$$

From (3.2) we can get that $(-1)^{n-1}\det(A) > 0$.

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REMARK 3.5. We remark that a positive matrix is an MC'-matrix if and only if it is an MC-matrix. But the classes of symmetric MC'-matrices and symmetric MC-matrices do not contain each other. For example, let

$$A_1 = \begin{pmatrix} -8 & 4 \\ 2 & 1 \end{pmatrix} \text{ and } A_2 = \begin{pmatrix} -2 & 4 \\ 2 & -1 \end{pmatrix}.$$

Then, by simple computations, A_1 is an MC'-matrix, but not an MC-matrix. A_2 is an MC-matrix, but not an MC'-matrix.

LEMMA 3.6. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, and let $P \in \mathbb{R}^{n\times n}$ be a permutation matrix. Then P^TAP is an MC'-matrix if and only if A is an MC'-matrix.*

Proof. The matrix P^TAP can be decomposed into

$$P^TAP = P^TC^+(A)P + P^TE^+(A)P = C^+(P^TAP) + E^+(P^TAP). \qquad (3.4)$$

It is clear that

$$-C^+(P^TAP) = -P^TC^+(A)P$$

is an M-matrix if and only if -$C^+(A)$ is an M-matrix. By (3.4) and Definition 3.4 the conclusion of the lemma holds. □

LEMMA 3.7. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be an MC'-matrix with $a_{11} > 0$. Then any $k \times k$ leading principle submatrix $A[\nu_k]$ of A is an MC'-matrix.*

Proof. The matrices A and $A[\nu_k]$ can be decomposed into

$$A = C^+(A) + E^+(A) \text{ and } A[\nu_k] = C^+(A[\nu_k]) + E^+(A[\nu_k]).$$

We have

$$s_i^+(A) \le s_i^+(A[\nu_k]), i = 1, 2, \cdots, k,$$

which implies

$$-\left(C^+(A)\right)[\nu_k] \le -C^+(A[\nu_k]).$$

Since $-C^+(A)$ is an M-matrix, it is easy to get that $-\left(C^+(A)\right)[\nu_k]$ is an M-matrix. So, from Exercise 6.5.1 in [2], $-C^+(A[\nu_k])$ is also an M-matrix, which, together with all off-diagonal elements of $A[\nu_k]$ being positive and $a_{11} > 0$, implies that $A[\nu_k]$ is an MC'-matrix. □

LEMMA 3.8. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be symmetric, and let*

$$(-1)^{k-1}\det(A[\nu_k]) > 0 \textit{ for all } k = 1, 2, \cdots, n.$$

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Then $A \in \mathscr{B}$.

Proof. Clearly, $a_{11} > 0$. By the interlacing theorem for eigenvalues of Hermitian matrices (see Theorem 4.3.8 in [3]), we have

$$\lambda_1(A[\nu_2]) \leq a_{11} \leq \lambda_2(A[\nu_2]),$$

which together with $\det(A[\nu_2]) < 0$ implies

$$\lambda_1(A[\nu_2]) < 0 \text{ and } \lambda_2(A[\nu_2]) > 0. \tag{3.5}$$

Similarly, it can be seen that

$$\lambda_1(A[\nu_3]) \leq \lambda_1(A[\nu_2]) \leq \lambda_2(A[\nu_3]) \leq \lambda_2(A[\nu_2]) \leq \lambda_3(A[\nu_3]),$$

which together with $\det(A[\nu_3]) > 0$ and (3.5) lead to

$$\lambda_1(A[\nu_3]) < 0, \lambda_2(A[\nu_3]) < 0 \text{ and } \lambda_3(A[\nu_3]) > 0.$$

Thus, a similar induction argument completes the proof of the lemma. □

Remark 3.9. It was shown in [1, Theorem 4.4.6] that a symmetric positive matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ belongs to $\mathscr{A}$ if and only if, for any $k \times k$ principle submatrix B of A, $(-1)^{k-1}\det(B) > 0$ for all $k = 1, 2, \cdots, n$. Thus, Lemma 3.8 provides a weaker condition such that a symmetric matrix has exactly one positive eigenvalue.

The following theorem shows that the class of symmetric MC'-matrices is a subset of $\mathscr{B}$.

Theorem 3.10. *Let* $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ *be a symmetric* MC'*-matrix. Then* $A \in \mathscr{B}$.

Proof. There exists a permutation matrix P such that the first diagonal element of P^TAP is positive. From Lemma 3.6, P^TAP is also an MC'-matrix. By Lemma 3.7, any $k \times k$ leading principle submatrix of P^TAP is an MC'-matrix, and hence

$$(-1)^{k-1}\det\left((P^TAP)[\nu_k]\right) > 0, k = 1, 2, \cdots, n.$$

Thus, due to Lemma 3.8, we have $P^TAP \in \mathscr{B}$, and then $A \in \mathscr{B}$. □

Remark 3.11. Compared with Proposition 4.3 and Corollary 4.4 in [7], Theorems 3.3 and 3.10 establish two wider classes of matrices with exactly one positive eigenvalue.

Remark 3.12. Assume that $A \in \mathscr{A}$. Then Theorem 4.4.6 in [1] implies that any principle submatrix of A also belongs to $\mathscr{A}$. From Lemma 3.7, we can see that any principle submatrix with at least one positive diagonal element of an MC'-matrix is

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also an MC'-matrix. However, we have not similar conclusions for MC-matrices. For example, let

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ 1 & 2 & -2 \end{pmatrix}.$$

Then A is an MC-matrix. But all 1×1 and 2×2 principle submatrices of A are not MC-matrices.

The following theorems establish some sufficient conditions for a matrix in $\mathscr{B}$.

THEOREM 3.13. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be symmetric, let all row sums of A be positive, and let A be decomposed into*

$$A = D^+(A) + F^+(A),$$

where

$$D^+(A) = \begin{pmatrix} a_{11} - s_{11} & a_{12} - s_{12} & \cdots & a_{1n} - s_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} - s_{n1} & a_{n2} - s_{n2} & \cdots & a_{nn} - s_{nn} \end{pmatrix},$$

$$F^+(A) = \begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{pmatrix}.$$

If $F^+(A) \in \mathscr{A}$, and $-D^+(A)$ is an M-matrix, then $A \in \mathscr{B}$.

Proof. From Exercise 6.2.6 in [2], $-D^+(A)$ is positive definite. By Sylvester's law of inertia we need only prove

$$(-D^+(A))^{-\frac{1}{2}} A (-D^+(A))^{-\frac{1}{2}} = -I + (-D^+(A))^{-\frac{1}{2}} F^+(A) (-D^+(A))^{-\frac{1}{2}} \in \mathscr{B}. \quad (3.6)$$

In fact, by $F^+(A) \in \mathscr{A}$ we get that

$$(-D^+(A))^{-\frac{1}{2}} F^+(A) (-D^+(A))^{-\frac{1}{2}} \in \mathscr{B}.$$

Since $(-D^+(A))^{-\frac{1}{2}} F^+(A) (-D^+(A))^{-\frac{1}{2}}$ is similar to $-(D^+(A))^{-1} F^+(A)$, it follows that $-(D^+(A))^{-1} F^+(A)$ has exactly one positive eigenvalue

$$\lambda_+ := \rho(-(D^+(A))^{-1} F^+(A)).$$

Assume that $x = (x_i)$ is a positive eigenvector corresponding to λ_+ . Then we have

$$-(D^+(A))^{-1} F^+(A) x = \lambda_+ x,$$

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and then

$$F^+(A)x = -\lambda_+ D^+(A)x,$$

which is equivalent to

$$\sum_{j=1}^{n} s_{ij}x_j = \lambda_+(\sum_{j=1}^{n} s_{ij}x_j - \sum_{j=1}^{n} a_{ij}x_j), i = 1, 2, \cdots, n,$$

i.e.,

$$\lambda_+ = \frac{\sum\limits_{j=1}^{n} s_{ij}x_j}{\sum\limits_{j=1}^{n} s_{ij}x_j - \sum\limits_{j=1}^{n} a_{ij}x_j}, i = 1, 2, \cdots, n. \tag{3.7}$$

Setting

$$x_m = \min_{1\le i\le n}\{x_i\},$$

we get

$$\sum_{j=1}^{n} a_{mj}x_j = a_{mm}x_m + \sum_{j\neq m} a_{mj}x_j \geq x_m \sum_{j=1}^{n} a_{mj} > 0,$$

which, by (3.7), implies

$$\lambda_+ = \frac{\sum\limits_{j=1}^{n} s_{mj}x_j}{\sum\limits_{j=1}^{n} s_{mj}x_j - \sum\limits_{j=1}^{n} a_{mj}x_j} > 1.$$

Since λ_+ is also the only positive eigenvalue of $(-D^+(A))^{-\frac{1}{2}}F^+(A)(-D^+(A))^{-\frac{1}{2}}$, we can get that (3.6) holds. The proof is completed. □

The following theorem shows that the condition on $F^+(A)$ can be weakened if the condition that $-D^+(A)$ is an M-matrix is strengthened.

Theorem 3.14. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be symmetric, let all row sums of A be positive, and let A be decomposed into*

$$A = D^+(A) + F^+(A).$$

If $F^+(A)$ belonging to $\mathscr{B}$ is nonnegative, and $-D^+(A)$ is a strictly diagonally dominant M-matrix, then $A \in \mathscr{B}$.

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Proof. Let λ_+ be defined as in the proof of Theorem 3.13. Then, by the proof of Theorem 3.13, we need only prove $\lambda_+ > 1$. In fact, we have, from Theorem 2 in [4],

$$\lambda_+ \geq \min_{1 \leq i \leq n} \frac{\sum_{j=1}^{n} s_{ij}}{\sum_{j=1}^{n} s_{ij} - \sum_{j=1}^{n} a_{ij}} > 1.$$

The proof is completed. ◻

By an analogous argument as in the proof of Theorems 3.13, we derive immediately the following result.

THEOREM 3.15. *Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be symmetric, let A have at least one positive diagonal element, and let A be decomposed into*

$$A = D^+(A) + F^+(A).$$

If $F^+(A) \in \mathscr{A}$, and $-D^+(A)$ is an M-matrix, then $A \in \mathscr{B}$.

REMARK 3.16. We remark that, similar to the relation between MC-matrices and MC'-matrices, the classes of matrices described by Theorem 3.13 and Theorem 3.15 do not contain each other.

From Theorems 3.3, 3.10, 3.13-3.15, we can obtain the following simple sufficient conditions.

COROLLARY 3.17. *Let $A \in \mathbb{R}^{n\times n}$ be symmetric, and let A be decomposed into*

$$A = D^+(A) + F^+(A).$$

If one of the following conditions holds:

1. *all row sums of A are positive, $F^+(A)$ is a symmetric positive MC-matrix, and $-D^+(A)$ is an M-matrix;*
2. *all row sums of A are positive, $F^+(A)$ is a symmetric nonnegative MC-matrix, and $-D^+(A)$ is a strictly diagonally dominant M-matrix;*
3. *A has at least one positive diagonal element, $F^+(A)$ is a symmetric positive MC-matrix, and $-D^+(A)$ is an M-matrix,*

then $A \in \mathscr{B}$.

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