Large quadratic programming problems generated by rigid body simulation

Bogdan Gavrea, Cosmin Petra

Abstract

In the present paper we briefly present how to obtain large quadratic programming (QP) problems from rigid body simulation. The QPs are obtained based on the convex relaxation introduced by Mihai Anitescu in 2006 and its dual formulation.

2000 Mathematical Subject Classification: 65K10, 90C20

1 Introduction

Time-stepping schemes for rigid-body simulation are concerned with the numerical integration of the motion of several rigid bodies experiencing contacts and Coulomb friction. One of the standard approaches is to formulate the integration step as a linear complementarity problem (LCP). In the presence of Coulomb friction the underlying LCP matrix is a copositive

matrix. LCPs with copositive matrices, [3], are generally solved by means of Lemke-type algorithms, and solvers such as PATH, [2], have proved to be robust. However for large systems the PATH solver or any other pivotal algorithms become impractical from a computational point of view.

The convex relaxation introduced in [1] formulates the integration step as a quadratic programming problem (QP), for which polynomial time algorithms and state-of-the-art solvers can be used. Therefore, from a computational point of view such an approach becomes a feasible alternative for the simulation of large rigid-body systems. On the other hand, Anitescu's relaxation in the context of rigid body simulation leads to large QPs that can be used as test problems for dedicated optimization solvers. In the present work we briefly present how such QPs can be generated via rigid body simulation. Computational results, showing the performance of some state-of-the-art solvers on a large rigid body system have been presented in [5] and we refer the interested reader to this paper for more details.

2   LCP based time-stepping schemes and Anitescu's convex relaxation

In [4] we have analyzed convergence properties of a class of semi-implicit LCP-based time-stepping schemes. The integration step can be formulated as follows:

At integration time t_{l+1} find the new system generalized velocity by solving

the following mixed linear complementarity problem:

$$(1) \quad \widetilde{M}v^{l+1} - \sum_{i=1}^{m}\nu^{(i)}c_\nu^{(i)} - \sum_{j\in\mathcal{A}}(n^{(j)}c_n^{(j)} + D^{(j)}\beta^{(j)}) \;=\; \widetilde{M}v^l + \widetilde{k}$$

$$(2) \quad \nu^{(i)T}\left(\alpha v^{(l+1)} + (1-\alpha)v^{(l)}\right) \;=\; 0,\; i = 1,2,\ldots m$$

$$(3) \quad 0 \le \rho^{(j)} := n^{(j)T}\left(\alpha v^{(l+1)} + (1-\alpha)v^{(l)}\right) \;\perp\; c_n^{(j)} \ge 0, \quad j \in \mathcal{A}$$

$$(4)\; 0 \le \sigma^{(j)} := \lambda^{(j)}e^{(j)} + D^{(j)T}\left(\alpha v^{(l+1)} + (1-\alpha)v^{(l)}\right) \;\perp\; \beta^{(j)} \ge 0, \quad j \in \mathcal{A}$$

$$(5) \quad 0 \le \zeta^{(j)} := \mu^{(j)}c_n^{(j)} - e^{(j)^T}\beta^{(j)} \;\perp\; \lambda^{(j)} \ge 0, \quad j \in \mathcal{A}.$$

Equation (1) is a discrete version of Newton's second law at the velocity-impulse level. Here the matrix $\widetilde{M}$ is dominated by the generalized mass matrix and for sufficiently small integration step h, $h > 0$, this matrix is positive definite. The positive definite property of the matrix $\widetilde{M}$ is one of the key elements in reducing the above mixed linear complementarity problem to an LCP with a copositive matrix. In equation (1), $z_j^{l+1} := \sum_{i=1}^{m}\nu^{(i)}c_\nu^{(i)}$ represent the joint constraint impulses, while $\sum_{j\in\mathcal{A}}(n^{(j)}c_n^{(j)} + D^{(j)}\beta^{(j)})$ are the contact (normal and tangential) impulses. The set $\mathcal{A}$ is an index set of all active contacts, while the term $\widetilde{k}$ on the right-hand side of (1) is connected to the applied (external and inertial) impulses. The equations comprised in (2) impose the joint constraints at the velocity level, [4]. The next three equations are complementarity constraints. Here, for two column vectors $u, v \in \mathbb{R}^p$, we write $0 \le u \perp v \ge 0$ if all components of both u and v are non-negative and $u^Tv = 0$. Equation (3) represents the contact and non-penetration constraints, while equations (4) and (5) model Coulomb frictional constraints for a polyhedral friction cone. In all the above equations $\alpha \in [\frac{1}{2}, 1]$ is a parameter used to span an entire family

of time-stepping schemes.

In [5] we have used Anitescu's convex relaxation, [1], to simulate a large system of rigid bodies. More precisely, in [5], we simulated the loading of identical pebbles into a vat whose geometry is composed of a finite cylinder and a truncated cone. For this specific application, Anitescu's convex relaxation formulates the integration step as a quadratic programming problem (QP) as follows,[5]:

Find the new generalized velocity v^{l+1} as the solution to the following QP:

$$\min \quad \tfrac{1}{2}v^T M v + \left(f^l\right)^T v$$

(6) $$\text{s.t.} \quad \left(n^{(k)}(q^l)\right)^T v + \mu \left(d_s^{(k)}(q^l)\right)^T v \geq -\tfrac{1}{h}\Phi^{(k)}(q^l)$$

$$k \in \mathcal{A}(q^l, \epsilon), \ s = 1, 2, \ldots, p_k$$

In (6), M denotes the generalized mass-matrix (a symmetric positive definite matrix) and $f^l = -v^l - hk_{app}$ uses the (known) velocity v^l and the applied forces k_{app}, which for this application remain constant for the entire simulation. In the inequality constraints of (6), $n^{(k)}(q^l)$ denotes the generalized normal direction at the active contact k, $\mu \in [0, 1]$ is the friction coefficient and $\Phi^{(k)}(\cdot)$ is the k-th contact signed distance function, which for a valid (no penetration) configuration q, must satisfy $\Phi(q) \geq 0$. The tangential directions $d_s(k)$, $s = 1, 2, \ldots, p_k$ corresponding to contact k are used in the polyhedral approximation of the k-th friction cone. The set $\mathcal{A}(q^l, \epsilon)$ is an index set of all active contacts, which is calculated based on the known position q^l and a certain tolerance ϵ, $\epsilon > 0$.

If we write the inequality constraints of (6) in matrix form and if we denote by A^l the matrix appearing on the left-hand side and by b^l the vector on the left-hand side, then the dual program of (6), takes the form

of the bound constrained QP:

$$(7) \qquad \begin{aligned} \min \quad & \tfrac{1}{2}\lambda^T P^l \lambda + \left(\kappa^l\right)^T \lambda, \\ \text{s.t.} \quad & \lambda \geq 0, \end{aligned}$$

where P^l and κ^l are computed by standard duality techniques, i.e., $P^l = A^l M^{-1} \left(A^l\right)^T$ and $\kappa^l = -b^l - A^l f^l$.

The above two formulations are equivalent, i.e., there is no duality gap, if the friction cone is pointed, [5]. The friction cone at configuration q^l is defined based on the set of active contacts $\mathcal{A}(q^l, \epsilon)$ by

$$(8) \qquad \mathcal{FC}(q^l, \epsilon) = \left\{ \sum_{k \in \mathcal{A}(q^l,\epsilon)} c_n^{(k)} n^{(k)} + \widetilde{D}^{(k)} \beta^{(k)} \;\Big|\; c_n^{(k)} \geq 0, \; \beta^{(k)} \in \mathbb{R}^{p_k}, \right.$$

$$\left. \beta^{(k)} \geq 0, ||\beta^{(k)}||_1 \leq \mu c_n^{(k)} \right\},$$

where $\widetilde{D}^{(k)}$ denotes the matrix of generalized tangential directions, namely,

$$(9) \qquad \widetilde{D}^{(k)}(q^l) := \widetilde{D}^{(k)} = \left(d_1^{(k)}, d_2^{(k)}, \ldots, d_{p_k}^{(k)}\right).$$

We say that the friction cone $\mathcal{FC}(q^l, \epsilon)$ is pointed if it doesn't contain any proper linear subspaces. From a physical point of view lack of pointedness corresponds to jamming, a situation that is addressed in a different manner. Therefore when the friction cone is pointed the new velocity v^{l+1} can be obtained by solving either (6) or (7). We note, once again, that (7) is a bound-constrained optimization problem for which dedicated state-of-the-art solvers exist.

3   Conclusions

We have briefly presented how large quadratic programming problems can be obtained via rigid body simulation using Anitescu's convex relaxation. The QPs can be used as test problems by both standard QP solvers as well as bound constrained optimization solvers, in the case when no jamming occurs. For a set of computational results in which several QP solvers have been tested we refer the reader to [5].

References

[1] M. Anitescu, *Optimization-based simulation of nonsmooth dynamics*, Mathematical Programming 105, 2006, pp. 113–143.

[2] S. Dirkse and M. Ferris, *The PATH Solver: A Non-Monotone Stabilization Scheme for Mixed Complementarity Problems*, Optimization Methods and Software 5, 1995, pp. 123–156.

[3] R.W. Cottle, J-S. Pang and R.E. Stone, *The Linear Complementarity Problem*, Academic Press, Boston, 1992.

[4] B. Gavrea, M. Anitescu and F.A. Potra, *Convergence of a Class of Semi-Implicit Time-Stepping Schemes for Nonsmooth Rigid Multibody Dynamics*, SIAM Journal on Optimization, to appear.

[5] C. Petra, B. Gavrea, M. Anitescu and F.A. Potra, *A computational study of the use of an optimization-based method for simulating large multibody systems*, submitted to Optimization Methods and Software.

Bogdan Gavrea

Technical University of Cluj-Napoca

Department of Mathematics,Faculty of Automation and Computer Science

Str. G. Baritiu nr. 26-28, 400027 Cluj-Napoca, Romania

e-mail: Bogdan.Gavrea@math.utcluj.ro

Cosmin Petra

University of Maryland, Baltimore County

Department fo Mathematics and Statistics

1000 Hilltop Circle Baltimore, MD 21250, USA

e-mail: cpetra1@math.umbc.edu