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CONDITIONS OF THE EXISTENCE AND UNIQUENESS OF SOLUTIONS OF THE MULTIPOINT BOUNDARY VALUE PROBLEM FOR A SYSTEM OF GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. Effective sufficient conditions are established for the solvability and unique solvability of the boundary value problem

$$dx(t) = dA(t) \cdot f(t, x(t)),$$
$$x_i(t_i) = \varphi_i(x) \quad (i = 1, \dots, n),$$

where $x = (x_i)_{i=1}^n$, $A : [a, b] \to R^{n\times n}$ is a matrix-function with bounded variation components, $f : [a, b] \times R^n \to R^n$ is a vector-function belonging to the Carathéodory class corresponding to A; $t_1, \dots, t_n \in [a, b]$ and $\varphi_1, \dots, \varphi_n$ are the continuous functionals (in general nonlinear) defined on the set of all vector-functions of bounded variation.

1. Statement of the Problem and Formulation of the Main Results

Let $t_1, \dots, t_n \in [a, b]$; $a_{mik} : [a, b] \to R$ be a nondecreasing function on the intervals $[a, t_i[$ and $]t_i, b]$ for $m \in \{1, 2\}$ and $i, k \in \{1, \dots, n\}$; $a_{ik}(t) \equiv a_{1ik}(t) - a_{2ik}(t)$, $A = (a_{ik})_{i,k=1}^n$; $f = (f_k)_{k=1}^n : [a, b] \times R^n \to R^n$ be a vector-function belonging to the Carathéodory class corresponding to the matrix-function A, and $\varphi_i : BV_s([a, b], R^n) \to R$ $(i = 1, \dots, n)$ be continuous functionals, which are nonlinear in general.

For the system of generalized ordinary differential equations

$$dx(t) = dA(t) \cdot f(t, x(t)), \tag{1.1}$$

where $x = (x_i)_{i=1}^n$, consider the multipoint boundary value problem

$$x_i(t_i) = \varphi_i(x) \quad (i = 1, \dots, n). \tag{1.2}$$

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In this paper sufficient conditions are given for the existence and uniqueness of solutions of the boundary value problem (1.1), (1.2). Analogous results are contained in [1–4] for the multipoint boundary value problems for systems of ordinary differential equations.

The theory of generalized ordinary differential equations enables one to investigate ordinary differential and difference equations from the commonly accepted standpoint. Moreover, the convergence conditions for difference schemes corresponding to boundary value problems for systems of ordinary differential equations can be deduced from the correctness results for appropriate boundary value problems for systems of generalized ordinary differential equations [5–15].

Throughout the paper the following notation and definitions will be used.

$R =]-\infty, +\infty[$, $R_+ = [0, +\infty[$; $[a,b]$ $(a, b \in R)$ is a closed segment.

$R^{n\times m}$ is the space of all real $n\times m$-matrices $X = (x_{ij})_{i,j=1}^{n,m}$ with the norm $\|X\| = \max_{j=1,\dots,m} \sum_{i=1}^{n} |x_{ij}|$;

$$R_+^{n\times m} = \Big\{(x_{ij})_{i,j=1}^{n,m} :\ x_{ij} \geq 0\ (i = 1,\dots,n;\ j = 1,\dots,m)\Big\}.$$

If $X = (x_{ij})_{i,j=1}^{n,m}$, then $|X| = (|x_{ij}|)_{i,j=1}^{n,m}$, $[X]_\pm = (|X| \pm X)/2$.

$R^n = R^{n\times 1}$ is the space of all real column n-vectors $x = (x_i)_{i=1}^n$; $R_+^n = R_+^{n\times 1}$.

If $X \in R^{n\times n}$, then X^{-1} and $\det(X)$ are, respectively, the matrix inverse to X and the determinant of X; I_n is the identity $n\times n$-matrix; δ_{ij} is Kronecker symbol, i.e., $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$ $(i, j = 1,\dots,n)$.

$\overset{b}{\underset{a}{\vee}}(X)$ is the total variation of the matrix-function $X : [a,b] \to R^{n\times m}$, i.e., the sum of total variations of the latter's components.

$X(t-)$ and $X(t+)$ are the left and the right limit of the matrix-function $X : [a,b] \to R^{n\times m}$ at the point t[1];

$$d_1X(t) = X(t) - X(t-), \quad d_2X(t) = X(t+) - X(t);$$
$$\|X\|_s = \sup\big\{\|X(t)\| :\ t \in [a,b]\big\}.$$

$BV([a,b], R^{n\times m})$ is the set of all matrix-functions of bounded variation $X : [a,b] \to R^{n\times m}$ (i.e., such that $\overset{b}{\underset{a}{\vee}}(X) < +\infty$);

$BV_v([a,b], R^n)$ is the Banach space $(BV([a,b], R^n), \|\cdot\|_v)$ with the norm

$$\|x\|_v = \|x(a)\| + \overset{b}{\underset{a}{\vee}}(x);$$

$BV_s([a,b], R^n)$ is the normed space $(BV([a,b], R^n), \|\cdot\|_s)$;

$$BV_s([a,b], R_+^n) = \Big\{x \in BV_s([a,b], R^n) :\ x(t) \in R_+^n \text{ for } t \in [a,b]\Big\}.$$

[1]We shall assume $X(t) = X(a)$ for $t \leq a$ and $X(t) = X(b)$ for $t \geq b$, if necessary.

If $D \subset R$, then $C(D, R^n)$ is the set of all continuous vector-functions $x : D \to R^n$; $C(D, R^n_+) = \{x \in C(D, R^n) : \ x(t) \in R^n_+ \text{ for } t \in [a, b]\}$.

If $\alpha \in BV([a, b], R)$ has not more than a finite number of discontinuity points and $m \in \{1, 2\}$, then $D_{\alpha m} = \{t_{\alpha m1}, \dots, t_{\alpha m n_{\alpha m}}\}$ ($t_{\alpha m1} < \dots < t_{\alpha m n_{\alpha m}}$) is the set of all points $t \in [a, b]$ for which $d_m\alpha(t) \neq 0$;

$$\mu_{\alpha m} = \max\big\{d_m\alpha(t) : \ t \in D_{\alpha m}\big\} \quad (m = 1, 2);$$
$$\nu_{\alpha m\beta j} = \max\Bigg\{d_j\beta(t_{\alpha ml}) + \sum_{t_{\alpha m,l+1-m}<\tau<t_{\alpha m,l+2-m}} d_j\beta(\tau) : \ l = 1, \dots, n_{\alpha m}\Bigg\}$$

for $\beta \in BV([a, b], R)$ $(j, m = 1, 2)$, here $t_{\alpha 1 n_{\alpha 1}+1} = b + 1$, $t_{\alpha 20} = a - 1$.

If $\beta \in BV([a, b], R)$, then

$$\mu_{\beta ji}(t) \equiv (-1)^j[\beta(t) - \beta(t_i)] - d_j\beta(t_i) \quad (j = 1, 2; \ \ i = 1, \dots, n).$$

If $g : [a, b] \to R$ is a nondecreasing function, $x : [a, b] \to R$ and $a \le s < t \le b$, then

$$\int_s^t x(\tau)g(\tau) = \int_{]s,t[} x(\tau)dg(\tau) + x(t)d_1g(t) + x(s)d_2g(s),$$

where $\int_{]s,t[} x(\tau)dg(\tau)$ is the Lebesgue–Stieltjes integral over the open interval $]s, t[$ with respect to the measure μ_g corresponding of the function g (if $s = t$, then $\int_s^t x(\tau)dg(\tau) = 0$);

$L^p([a, b], R; g)$ $(1 \le p < +\infty)$ is the space of all μ_g-measurable functions $x : [a, b] \to R$ such that $\int_a^b |x(t)|^p dg(t) < +\infty$ with the norm

$$\|x\|_{p,g} = \Bigg(\int_a^b |x(t)|^p dg(t)\Bigg)^{\frac{1}{p}};$$

$L^{+\infty}([a, b], R; g)$ is the space of all μ_g-measurable essentially bounded functions $x : [a, b] \to R$ with the norm

$$\|x\|_{+\infty,g} = \operatorname{ess\,sup}\big\{|x(t)| : \ t \in [a, b]\big\}.$$

$s_k : BV([a, b], R) \to BV([a, b], R)$ $(k = 0, 1, 2)$ are the operators defined by

$$s_1(x)(a) = s_2(x)(a) = 0,$$
$$s_1(x)(t) = \sum_{a<\tau\le t} d_1x(\tau), \quad s_2(x)(t) = \sum_{a\le\tau<t} d_2x(\tau) \quad \text{for } \ t \in (a, b],$$
$$s_0(x)(t) \equiv x(t) - s_1(x)(t) - s_2(x)(t).$$

A matrix-function is said to be nondecreasing if each of its components is such.

If $G = (g_{ik})_{i,k=1}^{l,n} : [a,b] \to R^{l\times n}$ is a nondecreasing matrix-function and $D \subset R^{n\times m}$, then $L([a,b],D;G)$ is the set of all matrix-functions $X = (x_{kj})_{k,j=1}^{n,m} : [a,b] \to D$ such that $x_{kj} \in L^1([a,b],R;g_{ik})$ $(i = 1,\dots,l;\ k = 1,\dots,n;\ j = 1,\dots,m)$;

$$\int_s^t dG(\tau)\cdot X(\tau) = \left(\sum_{k=1}^{n}\int_s^t x_{kj}(\tau)dg_{ik}(\tau)\right)_{i,j=1}^{l,m} \quad \text{for} \quad a \le s \le t \le b,$$

$$S_j(G)(t) \equiv \big(s_j(g_{ik})(t)\big)_{i,k=1}^{l,n} \quad (j = 0,1,2).$$

If $D_1 \subset R^n$ and $D_2 \subset R^{n\times m}$, then $K([a,b]\times D_1, D_2; G)$ is the Carathéodory class, i.e., the set of all mappings $F = (f_{kj})_{k,j=1}^{n,m} : [a,b]\times D_1 \to D_2$ such that for each $i \in \{1,\dots,l\}$, $j \in \{1,\dots,m\}$ and $k \in \{1,\dots,n\}$: a) the function $f_{kj}(\cdot,x) : [a,b] \to D_2$ is $\mu_{g_{ik}}$-measurable for every $x \in D_1$; b) the function $f_{kj}(t,\cdot) : D_1 \to D_2$ is continuous for $\mu_{g_{ik}}$-almost every $t \in [a,b]$, and $\sup\big\{|f_{kj}(\cdot,x)| :\ x \in D_0\big\} \in L([a,b],R;g_{ik})$ for every compact $D_0 \subset D_1$.

If $G_j : [a,b] \to R^{l\times n}$ $(j = 1,2)$ are nondecreasing matrix-functions, $G = G_1 - G_2$ and $X : [a,b] \to R^{n\times m}$, then

$$\int_s^t dG(\tau)\cdot X(\tau) = \int_s^t dG_1(\tau)\cdot X(\tau) - \int_s^t dG_2(\tau)\cdot X(\tau) \quad \text{for} \quad s \le t,$$

$$S_k(G) = S_k(G_1) - S_k(G_2) \quad (k = 0,1,2),$$

$$K([a,b]\times D_1, D_2; G) = \bigcap_{j=1}^{2} K([a,b]\times D_1, D_2; G_j).$$

The inequalities between the vectors and between the matrices are understood componentwise.

If B_1 and B_2 are the normed spaces, then an operator $\varphi : B_1 \to B_2$ is called positive homogeneous if $\varphi(\lambda x) = \lambda\varphi(x)$ for $\lambda \in R_+$ and $x \in B_1$.

An operator $\varphi : BV_s([a,b],R^n) \to R^n$ is called nondecreasing if for every $x, y \in BV_s([a,b],R^n)$ such that $x(t) \le y(t)$ for $t \in [a,b]$, the inequality $\varphi(x)(t) \le \varphi(y)(t)$ is fulfilled for $t \in [a,b]$.

A vector-function $x \in BV([a,b],R^n)$ is said to be a solution of system (1.1) if

$$x(t) = x(s) + \int_s^t dA(\tau)\cdot f(\tau,x(\tau)) \quad \text{for} \quad a \le s \le t \le b.$$

By a solution of the system of generalized differential inequalities

$$dx(t) - dA(t) \cdot f(t, x(t)) \le 0 \ \ (\ge 0)$$

we understand a vector-function $x \in BV([a,b], R^n)$ such that

$$x(t) - x(s) - \int_s^t dA(\tau) \cdot f(\tau, x(\tau)) \le 0 \ \ (\ge 0) \quad \text{for} \quad a \le s \le t \le b.$$

Definition 1.1. We shall say the pair $((c_{il})_{i,l=1}^n; (\varphi_{0i})_{i=1}^n)$, consisting of a matrix-function $(c_{il})_{i,l=1}^n \in BV([a,b], R^{n\times n})$ and a positive homogeneous nondecreasing operator $(\varphi_{0i})_{i=1}^n : BV_s([a,b], R_+^n) \to R_+^n$, belongs to the set $U(t_1, \dots, t_n)$ if the functions c_{il} $(i \ne l;\ i,l = 1,\dots,n)$ are nondecreasing on $[a,b]$ and continuous at the point t_i,

$$d_j c_{ii}(t) \ge 0 \quad \text{for} \quad t \in [a,b] \quad (j=1,2;\ \ i=1,\dots,n) \tag{1.3}$$

and the problem

$$\Big[dx_i(t) - \operatorname{sign}(t-t_i) \sum_{l=1}^n x_l(t) dc_{il}(t)\Big] \operatorname{sign}(t-t_i) \le 0 \ \ (i=1,\dots,n),$$
$$(-1)^j d_j x_i(t_i) \le x_i(t_i) d_j c_{ii}(t_i) \quad (j=1,2;\ \ i=1,\dots,n); \tag{1.4}$$

$$x_i(t_i) \le \varphi_{0i}\big(|x_1|, \dots, |x_n|\big) \quad (i=1,\dots,n) \tag{1.5}$$

has no nontrivial non-negative solution.

Let the nondecreasing matrix-functions $A_m = (a_{ik}^{(m)})_{i,k=1}^n : [a,b] \to R^{n\times n}$ $(m=1,2)$ be defined by

$$a_{ik}^{(m)}(t_i) = a_{mik}(t_i), \quad a_{ik}^{(m)}(t) = a_{mik}(t) +$$
$$+(-1)^j\big([d_j a_{1ik}(t_i)]_- + [d_j a_{2ik}(t_i)]_-\big) \quad \text{for} \quad (-1)^j(t-t_i) > 0 \ \ (j=1,2).$$

Theorem 1.1. *Let the conditions*

$$(-1)^{m+1} f_k(t, x_1, \dots, x_n) \operatorname{sign}[(t-t_i)x_i] \le$$
$$\le \sum_{l=1}^n p_{mikl}(t)|x_l| + q_k\Big(t, \sum_{l=1}^n |x_l|\Big)$$
$$\textit{for} \ \ \mu_{a_{mik}}\text{- almost every} \ \ t \in [a,b] \setminus \{t_i\} \quad (i,k=1,\dots,n) \tag{1.6}$$

and

$$\Big[(-1)^{m+j+1} f_k(t_i, x_1, \dots, x_n) \operatorname{sign} x_i - \sum_{l=1}^n \alpha_{mikjl}|x_l| -$$

$$-q_k\Big(t_i,\sum_{l=1}^n|x_l|\Big)\Big]d_ja_{mik}(t_i)\leq 0 \quad (j=1,2;\;\; i,k=1,\dots,n) \tag{1.7}$$

be fulfilled on R^n *for* $m\in 1,2$*, and let the inequalities*

$$|\varphi_i(x_1,\dots,x_n)|\leq$$
$$\leq \varphi_{0i}\big(|x_1|,\dots,|x_n|\big)+\gamma\Big(\sum_{l=1}^n\|x_l\|_s\Big) \quad (i=1,\dots,n) \tag{1.8}$$

be fulfilled on $BV([a,b],R^n)$*, where* $\alpha_{mikjl}\in R$ $(j,m=1,2;\ i,k,l=1,\dots,n)$*,* $(p_{mikl})_{k,l=1}^n\in L([a,b],R^{n\times n};A_m)$ $(m=1,2;\ i=1,\dots,n)$*,* $q=(q_k)_{k=1}^n\in\cap_{m=1}^2K([a,b]\times R_+,R_+^n;A_m)$ *is a vector-function nondecreasing with respect to the second variable,* $\gamma\in C(R_+,R_+)$ *and*

$$\lim_{\rho\to+\infty}\frac{1}{\rho}\int_a^b d(A_1(t)+A_2(t))\cdot q(t,\rho)=0, \quad \lim_{\rho\to+\infty}\frac{\gamma(\rho)}{\rho}=0\,. \tag{1.9}$$

Let, moreover, there exists a matrix-function $(c_{il})_{i,l=1}^n\in BV([a,b],R^{n\times n})$ *such that*

$$\big((c_{il})_{i,l=1}^n;(\varphi_{0i})_{i=1}^n\big)\in U(t_1,\dots,t_n), \tag{1.10}$$

$$\sum_{m=1}^2\sum_{k=1}^n\int_s^t p_{mikl}(\tau)da_{mik}(\tau)\leq c_{il}(t)-c_{il}(s)$$
$$\text{for}\quad a\leq s<t<t_i \quad\text{and}\quad t_i<s<t\leq b \quad (i,l=1,\dots,n) \tag{1.11}$$

and

$$\sum_{m=1}^2\sum_{k=1}^n\alpha_{mikjl}d_ja_{mik}(t_i)\leq\delta_{il}d_jc_{ii}(t_i) \quad (j=1,2;\;\; i,l=1,\dots,n). \tag{1.12}$$

Then problem (1.1), (1.2) *is solvable.*

Corollary 1.1. *Let conditions* (1.3), (1.9), (1.11), (1.12) *and*

$$|c_{il}(t)-c_{il}(s)|\leq$$
$$\leq\int_s^t h_{il}(\tau)d\alpha_l(\tau) \quad \text{for}\;\; a\leq s<t\leq b \;\;(i,l=1,\dots,n) \tag{1.13}$$

hold and let conditions (1.6) *and* (1.7) *be fulfilled on* R^n *for every* $m\in\{1,2\}$*, where* $\alpha_{mikjl}\in R$ $(j,m=1,2;\ i,k,l=1,\dots,n)$*,* $(p_{mikl})_{k,l=1}^n\in L([a,b],R^{n\times n};A_m)$ $(m=1,2;\ i=1,\dots,n)$*;* $q=(q_k)_{k=1}^n\in\cap_{m=1}^2K([a,b]\times$

$R_+, R_+^n; A_m)$ is a vector-function, nondecreasing with respect to the second variable, $\gamma \in C(R_+, R_+)$; c_{il} $(i \neq l;\ i,l = 1,\dots,n)$ are the functions nondecreasing on $[a,b]$ and continuous at the point t_i, $c_{ii} \in BV([a,b],R)$ $(i = 1,\dots,n)$; α_l $(l = 1,\dots,n)$ are the functions nondecreasing on $[a,b]$ and having not more than a finite number of discontinuity points; $h_{il} \in L^\mu([a,b],R_+;\alpha_l)$ $(i,l = 1,\dots,n)$, $1 \le \mu \le \infty$. Let, moreover, the inequalities

$$|\varphi_i(x_1,\dots,x_n)| \le$$
$$\le \sum_{m=0}^{2}\sum_{k=1}^{n} l_{mik}\|x\|_{\nu,s_m(\alpha_k)} + \gamma\Big(\sum_{l=1}^{n}\|x_l\|_s\Big) \quad (i = 1,\dots,n)$$

be fulfilled for $x = (x_l)_{l=1}^n \in BV([a,b],R^n)$ and the module of every characteristic value of the $3n \times 3n$-matrix $\mathcal{H} = (\mathcal{H}_{j+1,m+1})_{j,m=0}^2$ be less than 1, where $l_{mik} \in R_+$ $(m = 0,1,2;\ i,k = 1,\dots,n)$, $\frac{1}{\mu} + \frac{2}{\nu} = 1$;

$$\mathcal{H}_{j+1,m+1} = \big(\xi_{ij} l_{mik} + \lambda_{kmij}\|h_{ik}\|_{\mu,s_m(\alpha_i)}\big)_{i,k=1}^n \quad (j,m = 0,1,2);$$
$$\xi_{ij} = \big[s_j(\alpha_i)(b) - s_j(\alpha_i)(a)\big]^{\frac{1}{\nu}} \quad (j = 0,1,2;\ \ i = 1,\dots,n);$$
$$\lambda_{k0k0} = \big[(2\pi^{-1})^{\frac{1}{\nu}}\xi_{k0}\big]^2 \quad (k = 1,\dots,n),$$
$$\lambda_{kmij} = \xi_{km}\xi_{ij} \quad \text{for} \quad m^2 + j^2 + (i-k)^2 > 0, \quad mj = 0$$
$$(j,m = 0,1,2;\ \ i,k = 1,\dots,n);$$
$$\lambda_{kmij} = \Big(\frac{1}{4}\,\mu_{\alpha_k m}\nu_{\alpha_k m \alpha_i j}\sin^{-2}\frac{\pi}{4n_{\alpha_k m} + 2}\Big)^{\frac{1}{\nu}}$$
$$(j,m = 1,2;\ \ i,k = 1,\dots,n).$$

Then problem (1.1), (1.2) is solvable.

Corollary 1.2. *Let inequalities (1.12) hold for $i \neq l$ $(i,l = 1,\dots,n)$, let there exist $m, m_1 \in \{1,2\}$ such that $m + m_1 = 3$ and conditions (1.6), (1.7),*

$$(-1)^{m_1+1} f_k(t,x_1,\dots,x_n)\operatorname{sign}[(t-t_i)x_i] \le \sum_{l=1}^{n}\eta_{il}|x_l| + q_k\Big(t,\sum_{l=1}^{n}|x_l|\Big)$$
$$\text{for} \quad \mu_{a_{m_1ik}}\text{- almost every} \quad t \in [a,b]\setminus\{t_i\} \quad (i,k = 1,\dots,n),$$
$$\Big[(-1)^{m_1+j+1} f_k(t_i,x_1,\dots,x_n)\operatorname{sign} x_i - \sum_{l=1}^{n}\alpha_{m_1ikjl}|x_l| -$$
$$-q_k\Big(t_i,\sum_{l=1}^{n}|x_l|\Big)\Big] d_j a_{m_1ik}(t_i) \le 0 \quad (j = 1,2;\ \ i,k = 1,\dots,n)$$

be fulfilled on R^n, and let the inequalities

$$|\varphi_i(x_1,\dots,x_n)| \le c_i|x_i(\tau_i)| + \gamma\Big(\sum_{l=1}^n \|x_l\|_s\Big) \quad (i=1,\dots,n)$$

be fulfilled on $BV([a,b],R^n)$, where $\alpha_{mikjl}\in R$ and $\alpha_{m_1ikjl}\in R$ $(j=1,2;$ $i,k=1,\dots,n)$, $(p_{mikl})_{k,l=1}^n \in L([a,b],R_+^{n\times n};A_m)$ $(i=1,\dots,n)$, $\eta_{il}\in R_+$ $(i\neq l;\ i,l=1,\dots,n)$, $\eta_{ii}<0$ $(i=1,\dots,n)$, a_{m_1ik} $(i,k=1,\dots,n)$ are the functions, nondecreasing and continuous on every interval $[a,t_i[$ and $]t_i,b]$, satisfying the condition

$$\sum_{k=1}^n d_j a_{m_1ik}(t_i) \le 0 \quad (j=1,2;\ \ i=1,\dots,n); \tag{1.14}$$

$q=(q_k)_{k=1}^n \in \cap_{m=1}^2 K([a,b]\times R_+,R_+^n;A_m)$ is the vector-function nondecreasing with respect to the second variable, $\gamma\in C(R_+,R_+)$ and conditions (1.9) *hold; $c_i\in R_+$ and $\tau_i\in[a,b]$, $\tau_i\neq t_i$ $(i=1,\dots,n)$ are such that*

$$c_i\gamma_{ij}\zeta_{ij}<1 \quad \text{if} \quad (-1)^j(\tau_i-t_i)>0 \quad (j=1,2;\ \ i=1,\dots,n); \tag{1.15}$$

$$\gamma_{ij} = 1+\eta_{ii}d_j\alpha_i(t_i)+\sum_{k=1}^n p_{miki}(t_i)d_j a_{mik}(t_i),$$

$$\zeta_{ij} = \exp\big(\eta_{ii}\mu_{\alpha_i ji}(\tau_i)\big), \quad \alpha_i(t)\equiv\sum_{k=1}^n a_{m_1ik}(t) \ \ (i,j=1,\dots,n).$$

Let, moreover,

$$g_{iij}<1 \quad \text{if} \quad (-1)^j(\tau_i-t_i)>0 \quad (j=1,2;\ \ i=1,\dots,n) \tag{1.16}$$

and the real part of every characteristic value of the matrix $(\xi_{il})_{i,l=1}^n$ be negative, where

$$\xi_{il} = \eta_{il}\big[\delta_{il}+(1-\delta_{il})h_{ij}\big]-\eta_{ii}g_{ilj} \quad \text{for} \quad (-1)^j(\tau_i-t_i)>0$$
$$(j=1,2;\ \ i,l=1,\dots,n),$$

$$g_{ilj} = c_i\gamma_{ij}(1-c_i\gamma_{ij}\zeta_{ij})^{-1}\mu_{\beta_{il}ji}(\tau_i)+\max\big\{\mu_{\beta_{il}1i}(a),\mu_{\beta_{il}2i}(b)\big\},$$

$$\beta_{il}(t) = \sum_{k=1}^n \int_a^t p_{mikl}(\tau)da_{mik}(\tau),$$

$h_{ij}=1$ for $c_i\gamma_{ij}\le 1$ and $h_{ij}=1+(c_i\gamma_{ij}-1)(1-c_i\gamma_{ij}\zeta_{ij})^{-1}$ for $c_i\gamma_{ij}>1$. Then problem (1.1), (1.2) *is solvable.*

Theorem 1.2. *Let the conditions*

$$(-1)^{m+1}\big[f_k(t,x_1,\dots,x_n)-f_k(t,y_1,\dots,y_n)\big]\times$$

$$\times \operatorname{sign}\big[(t-t_i)(x_i-y_i)\big] \le \sum_{l=1}^{n} p_{mikl}(t)|x_l-y_l|$$

$$\text{for } \ \mu_{a_{mik}}\text{- almost every } \ t\in[a,b]\backslash\{t_i\} \quad (i,k=1,\dots,n), \tag{1.17}$$

$$\Big\{(-1)^{m+j+1}\big[f_k(t_i,x_1,\dots,x_n)-f_k(t_i,y_1,\dots,y_n)\big]\operatorname{sign}(x_i-y_i)-$$
$$-\sum_{l=1}^{n}\alpha_{mikjl}|x_l-y_l|\Big\}d_j a_{mik}(t_i)\le 0 \ \ (j=1,2;\ \ i,k=1,\dots,n) \tag{1.18}$$

be fulfilled on R^n for every $m\in\{1,2\}$, and let the inequalities

$$\big|\varphi_i(x_1,\dots,x_n)-\varphi_i(y_1,\dots,y_n)\big|\le$$
$$\le \varphi_{0i}\big(|x_1-y_1|,\dots,|x_n-y_n|\big) \quad (i=1,\dots,n) \tag{1.19}$$

be fulfilled on $BV([a,b],R^n)$, where $\alpha_{mikjl}\in R$ $(j,m=1,2;\ i,k,l=1,\dots,n)$, $(p_{mikl})_{k,l=1}^n\in L([a,b],R^{n\times n};A_m)$ $(m=1,2;\ i=1,\dots,n)$. Let, moreover, there exist a matrix-function $(c_{il})_{i,l=1}^n\in BV([a,b],R^{n\times n})$ such that conditios (1.10)–(1.12) *hold. Then problem* (1.1), (1.2) *has only one solution.*

Corollary 1.3. *Let conditions* (1.3), (1.11)–(1.13) *hold and let conditions* (1.17) *and* (1.18) *be fulfilled on R^n for every $m\in\{1,2\}$, where $\alpha_{mikjl}\in R$ $(j,m=1,2;i,k,l=1,\dots,n)$, $(p_{mikl})_{k,l=1}^n\in L([a,b],R^{n\times n};A_m)$ $(m=1,2;\ i=1,\dots,n)$, c_{il} $(i\ne l;\ i,l=1,\dots,n)$ are the functions nondecreasing on $[a,b]$ and continuous at the point t_i, $c_{ii}\in BV([a,b],R)$ $(i=1,\dots,n)$; α_l $(l=1,\dots,n)$ are the functions nondecreasing on $[a,b]$ and having not more than a finite number of discontinuity points; $h_{il}\in L^\mu([a,b],R_+;\alpha_l)$ $(i,l=1,\dots,n)$, $1\le\mu\le+\infty$. Let, moreover, the inequalities*

$$\big|\varphi_i(x_1,\dots,x_n)-\varphi_i(y_1,\dots,y_n)\big|\le\sum_{m=0}^{2}\sum_{k=1}^{n}l_{mik}\|x_k-y_k\|_{\nu,s_m(\alpha_k)}$$
$$(i,k=1,\dots,n)$$

be fulfilled on $BV([a,b],R^n)$, where $l_{mik}\in R_+$ $(m=0,1,2;i,k=1,\dots,n)$; $\frac{1}{\mu}+\frac{2}{\nu}=1$ and the module of every characteristic value of the $3n\times 3n$-matrix $\mathcal{H}=(\mathcal{H}_{j+1,m+1})_{j,m=0}^2$ appearing in Corollary 1.1 *is less than* 1. *Then problem* (1.1), (1.2) *has only one solution.*

Corollary 1.4. *Let inequalities* (1.12) *hold for $i\ne l$ $(i,l=1,\dots,n)$, let there exist $m,m_1\in\{1,2\}$ such that $m+m_1=3$ and conditions* (1.17), (1.18),

$$(-1)^{m_1+1}\big[f_k(t,x_1,\dots,x_n)-f_k(t,y_1,\dots,y_n)\big]\operatorname{sign}\big[(t-t_i)(x_i-y_i)\big]\le$$

$$\leq \sum_{l=1}^{n} \eta_{il}|x_l - y_l| \quad \textit{for } \mu_{a_{m_1ik}}\textit{- almost every } t \in [a,b]\backslash\{t_i\}$$

$$(i,k = 1,\dots,n),$$

$$\Big\{(-1)^{m_1+j+1}\big[f_k(t_i,x_1,\dots,x_n) - f_k(t_i,y_1,\dots,y_n)\big]\operatorname{sign}(x_i - y_i) -$$

$$-\sum_{l=1}^{n} \alpha_{m_1ikjl}|x_l - y_l|\Big\} d_j a_{m_1ik}(t_i) \leq 0 \quad (j=1,2;\ i,k=1,\dots,n)$$

be fulfilled on R^n*, where* $\alpha_{mikjl} \in R$ *and* $\alpha_{m_1ikjl} \in R$ $(j = 1,2;\ i,k,l = 1,\dots,n)$, $(p_{mikl})_{k,l=1}^n \in L([a,b], R_+^{n\times n}; A_m)$ $(i = 1,\dots,n)$, $\eta_{il} \in R_+$ $(i \neq l;\ i,l = 1,\dots,n)$, $\eta_{ii} < 0$ $(i = 1,\dots,n)$; a_{m_1ik} $(i,k = 1,\dots,n)$ *are the functions, nondecreasing and continuous on the intervals* $[a,t_i[$ *and* $]t_i,b]$*, satisfying condition* (1.14)*. Let, moreover,* $c_i \in R_+$ *and* $\tau_i \in [a,b]$, $\tau_i \neq t_i$ $(i = 1,\dots,n)$ *be such that conditions* (1.15) *and* (1.16) *hold and the real part of every characteristic value of the matrix* $(\xi_{il})_{i,l=1}^n$ *be negative, where* γ_{ij}, ζ_{ij}, ξ_{il}, h_{ij}, g_{ilj} $(j = 1,2;\ i,l = 1,\dots,n)$ *and* $\alpha_i(t)$ $(i = 1,\dots,n)$ *are the numbers and the functions, respectively, appearing in Corollary* 1.2*. Then for every* $\lambda_i \in [-c_i,c_i]$ *and* $\gamma_i \in R_+$ $(i = 1,\dots,n)$ *system* (1.1) *has only one solution satisfying the boundary conditions*

$$x_i(t_i) = \lambda_i x_i(\tau_i) + \gamma_i \quad (i = 1,\dots,n). \tag{1.20}$$

Remark 1.1. The $3n\times 3n$-matrix $\mathcal{H}$ appearing in Corollaries 1.1 and 1.3 can be replaced by the $n\times n$-matrix

$$\Big(\max\Big\{\sum_{j=0}^{2}\big(\xi_{ij} l_{mik} + \lambda_{kmij}\|h_{ik}\|_{\mu,s_m(\alpha_k)}\big):\ m = 0,1,2\Big\}\Big)_{i,k=1}^{n}.$$

Theorem 1.3. *Let* $\varphi_{0i} : BV_s([a,b],R_+^n) \to R_+$ $(i=1,\dots,n)$ *be the linear continuous functionals, a matrix-function* $C = (c_{il})_{i,l=1}^n \in BV([a,b],R^{n\times n})$ *be such that the functions* c_{il} $(i \neq l;\ i,l = 1,\dots,n)$ *are nondecreasing on* $[a,b]$ *and continuous at the point* t_i, $c_{ii} \in BV([a,b],R)$ $(i = 1,\dots,n)$*, and condition* (1.3) *holds, but condition* (1.10) *is violated. Then there exist nondecreasing matrix-functions* $A_m = (a_{mkl})_{k,l=1}^n : [a,b] \to R^{n\times n}$ $(m = 1,2)$*, matrix-functions* $(p_{mikl})_{k,l=1}^n \in L([a,b],R^{n\times n};A_m)$ $(m = 1,2;\ i = 1,\dots,n)$*, numbers* $\alpha_{mikjl} \in R$ $(j,m = 1,2;\ i,k,l = 1,\dots,n)$*, a vector-function* $f = (f_k)_{k=1}^n \in \cap_{m=1}^2 K([a,b]\times R^n;A_m)$ *and functionals* $\varphi_i : BV_s([a,b],R^n) \to R$ $(i = 1,\dots,n)$ *for which* (1.11)*,* (1.12) *hold and conditions* (1.17)*,* (1.18) *and* (1.19) *are fulfilled respectively on* R^n *for* $m \in \{1,2\}$ *and on* $BV([a,b],R^n)$*, but problem* (1.1)*,* (1.2) *has no solution.*

2. Auxiliary Propositions

Lemma 2.1. *Let $g \in BV([a,b],R)$. Then*

$$\int_a^b \operatorname{sign} g(t)dg(t) = |g(b)| - |g(a)| + \sum_{a<t\le b} \big[|g(t-)| - g(t-)\operatorname{sign} g(t)\big] - \\ - \sum_{a\le t<b} \big[|g(t+)| - g(t+)\operatorname{sign} g(t)\big]. \tag{2.1}$$

Proof. Let $(\alpha_k,\beta_k) \subset [a,b]$ $(k=1,2,\dots)$ be a system of all maximal nonoverlapping intervals on which the function $\operatorname{sign} x(t)$ is constant. By $\mathcal{G} = \{\gamma_1,\gamma_2,\dots\}$ we denote a closure of the set $\mathcal{A}\cup\mathcal{B}$, where $\mathcal{A} = \{\alpha_1,\alpha_2,\dots\}$ and $\mathcal{B} = \{\beta_1,\beta_2,\dots\}$. Taking into account the relations $[a,b] = \cup_{k=1}^{+\infty}(\alpha_k,\beta_k)\cup\mathcal{G}$, $g(\gamma_k-) = 0$ for $\gamma_k \notin \{a\}\cup\mathcal{B}$ and $g(\gamma_k+) = 0$ for $\gamma_k \in \{b\}\cup\mathcal{A}$, we have

$$\int_a^b \operatorname{sign} g(t)dg(t) =$$

$$= \sum_{k=1}^{+\infty}\bigg(\lim_{\varepsilon\to0+}\int_{\alpha_k+\varepsilon}^{\beta_k-\varepsilon} \operatorname{sign} g(t)dg(t) + [g(\gamma_k+) - g(\gamma_k-)]\operatorname{sign} g(\gamma_k)\bigg) =$$

$$= |g(b)| + |g(b-)| - g(b-)\operatorname{sign} g(b) - |g(a)| - |g(a+)| + g(a+)\operatorname{sign} g(a) + \\ + \sum_{\gamma_k\in\mathcal{A}\cup\mathcal{B}\setminus\{a,b\}} \Big\{|g(\gamma_k-)| - |g(\gamma_k+)| + \big[g(\gamma_k+) - g(\gamma_k-)\big]\operatorname{sign} g(\gamma_k)\Big\}.$$

From this immediately follows equality (2.1), since

$$|x(t-)| = x(t-)\operatorname{sign} x(t) \quad \text{and} \quad |x(t+)| = x(t+)\operatorname{sign} x(t)$$

for $t \in (\alpha_k,\beta_k)$ $(k=1,2,\dots)$. □

Lemma 2.2. *Let $q = (q_i)_{i=1}^n \in BV([a,b],R^n)$ and $(c_{il})_{i,l=1}^n \in BV([a,b], R^{n\times n})$ be such that the functions c_{il} $(i\ne l;\ i,l=1,\dots,n)$ are nondecreasing and*

$$d_jc_{ii}(t_i) \ge 0, \quad d_jc_{ii}(t) \ge -1 \\ \text{for} \quad (-1)^j(t-t_i) > 0 \quad (j=1,2;\ \ i=1,\dots,n). \tag{2.2}$$

Let, moreover, $B = (b_{il})_{i,l=1}^n \in BV([a,b],R^{n\times n})$ be a matrix-function satisfying the conditions

$$s_0(b_{ii})(t) - s_0(b_{ii})(s) \le \big[s_0(c_{ii})(t) - s_0(c_{ii})(s)\big]\operatorname{sign}(t-s) \\ \text{for} \quad (t-s)(s-t_i) > 0 \quad (i=1,\dots,n), \tag{2.3} \\ (-1)^{j+m}\big(|1+(-1)^m d_mb_{ii}(t)| - 1\big) \le d_mc_{ii}(t)$$

$$\textit{for}\quad (-1)^j(t-t_i)\geq 0 \quad (j,m=1,2;\ \ i=1,\dots,n), \tag{2.4}$$

$$\big|s_0(b_{il})(t)-s_0(b_{il})(s)\big|\leq s_0(c_{il})(t)-s_0(c_{il})(s)$$
$$\textit{for}\quad a\leq s<t\leq b \quad (i\neq l;\ \ i,l=1,\dots,n) \tag{2.5}$$

and

$$|d_jb_{il}(t)|\leq d_jc_{il}(t)\quad \textit{for}\quad t\in[a,b]\quad (i\neq l;\ \ i,l=1,\dots,n). \tag{2.6}$$

Then every solution $x=(x_i)_{i=1}^n$ of the system

$$dx(t)=dB(t)\cdot x(t)+dq(t) \tag{2.7}$$

will be a solution of the system

$$\begin{gathered}\bigg[d|x_i(t)|-\operatorname{sign}(t-t_i)\sum_{l=1}^n|x_l(t)|dc_{il}(t)-\\ -\operatorname{sign}x_i(t)dq_i(t)\bigg]\operatorname{sign}(t-t_i)\leq 0\quad (i=1,\dots,n),\\ (-1)^jd_j|x_i(t_i)|\leq\sum_{l=1}^n|x_l(t_i)|d_jc_{il}(t_i)+(-1)^j\operatorname{sign}x_i(t_i)d_jq_i(t_i)\\ (j=1,2;\ \ i=1,\dots,n).\end{gathered} \tag{2.8}$$

Proof. Taking into account (2.1) and the definition of a solution of system (2.7), it can be easily shown that

$$\begin{aligned}|x_i(t)|-|x_i(s)|=&\int_s^t|x_i(\tau)|ds_0(b_{ii})(\tau)+\\ &+\sum_{l\neq i,\,l=1}^n\int_s^t x_l(\tau)\operatorname{sign}x_i(\tau)ds_0(b_{il})(\tau)+\\ &+\sum_{s<\tau\leq t}\big[|x_i(\tau)|-|x_i(\tau-)|\big]+\sum_{s\leq\tau<t}\big[|x_i(\tau+)|-|x_i(\tau)|\big]+\\ &+\int_s^t\operatorname{sign}x_i(\tau)dq_i(\tau)\quad\text{for}\quad a\leq s\leq t\leq b\quad (i=1,\dots,n).\end{aligned}$$

By (2.2)–(2.6) from this we have

$$|x_i(t)|-|x_i(s)|\leq\sum_{l=1}^n\int_s^t|x_l(\tau)|ds_0(c_{il})(\tau)+\sum_{s<\tau\leq t}\big[|x_i(\tau)|-|x_i(\tau-)|\big]+$$

$$
\begin{aligned}
&+\sum_{s\le\tau<t}\Big[|x_i(\tau+)|-|x_i(\tau)|\Big]+\int_s^t \operatorname{sign} x_i(\tau)dq_i(\tau)\le\\
&\le\sum_{l=1}^{n}\int_s^t |x_l(\tau)|dc_{il}(\tau)+\sum_{s<\tau\le t}\bigg\{|x_i(\tau)|(1-d_1c_{ii}(\tau))+\\
&+\Big|\sum_{l\ne i,\,l=1}^{n} x_l(\tau)d_1b_{il}(\tau)\Big|-|x_i(\tau)|\,|1-d_1b_{ii}(\tau)|\bigg\}-\\
&-\sum_{s\le\tau<t}\bigg\{|x_i(\tau)|(1+d_2c_{ii}(\tau))-\\
&-\Big|\sum_{l\ne i,\,l=1}^{n} x_l(\tau)d_2b_{il}(\tau)\Big|-|x_i(\tau)|\,|1+d_2b_{ii}(\tau)|\bigg\}-\\
&-\sum_{s<\tau\le t}\sum_{l\ne i,\,l=1}^{n}|x_l(\tau)|d_1c_{il}(\tau)-\\
&-\sum_{s\le\tau<t}\sum_{l\ne i,\,l=1}^{n}|x_l(\tau)|d_2c_{il}(\tau)+\int_s^t \operatorname{sign} x_i(\tau)dq_i(\tau)\le\\
&\le\sum_{l=1}^{n}\int_s^t |x_l(\tau)|dc_{il}(\tau)+\int_s^t \operatorname{sign} x_i(\tau)dq_i(\tau)+\\
&+\sum_{s<\tau\le t}\bigg\{|x_i(\tau)|\Big[1-d_1c_{ii}(\tau)-|1-d_1b_{ii}(\tau)|\Big]+\\
&+\sum_{l\ne i,\,l=1}^{n}|x_l(\tau)|\Big[|d_1b_{il}(\tau)|-d_1c_{il}(\tau)\Big]\bigg\}-\\
&-\sum_{s\le\tau<t}\bigg\{|x_i(\tau)|\Big[1+d_2c_{ii}(\tau)-|1+d_2b_{ii}(\tau)|\Big]-\\
&-\sum_{l\ne i,\,l=1}^{n}|x_l(\tau)|\Big[|d_2b_{il}(\tau)|-d_2c_{il}(\tau)\Big]\bigg\}\le\\
&\le\sum_{l=1}^{n}\int_s^t |x_l(\tau)|dc_{il}(\tau)+\int_s^t \operatorname{sign} x_i(\tau)dq_i(\tau)\\
&\qquad\text{for}\quad t_i<s\le t\le b\quad(i=1,\dots,n).
\end{aligned}
$$

Therefore inequalities (2.8) are fulfilled for $t>t_i$ and $j=2$.

Analogously, we shall show that

$$|x_i(t)| - |x_i(s)| \geq -\sum_{l=1}^{n} \int_s^t |x_l(\tau)| dc_{il}(\tau) + \int_s^t \operatorname{sign} x_i(\tau) dq_i(\tau)$$
$$\text{for} \quad a \leq s \leq t < t_i \quad (i = 1, \dots, n).$$

The above inequality implies (2.8) for $t < t_i$ and $j = 1$. □

Lemma 2.3. *Let a matrix-function* $(c_{il})_{i,l=1}^n \in BV([a,b], R^{n\times n})$ *be such that the functions* c_{il} $(i \neq l;\ i,l = 1,\dots,n)$ *are nondecreasing, the condition* (2.2) *holds and system* (1.4) *has no nontrivial non-negative solution satisfying the condition*

$$x_i(t_i) = 0 \quad (i = 1, \dots, n). \tag{2.9}$$

Let, moreover, a matrix-function $B = (b_{il})_{i,l=1}^n \in BV([a,b], R^n)$ *be such that the functions* b_{il} $(i \neq l;\ i,l = 1,\dots,n)$ *are continuous at the point* t_i,

$$1 + (-1)^j d_j b_{ii}(t_i) \neq 0 \quad (j = 1,2;\ \ i = 1,\dots,n) \tag{2.10}$$

and conditions (2.3)–(2.6) *and*

$$\left|1 + (-1)^j d_j b_{ii}(t)\right| \geq 1 - d_{3-j} c_{ii}(t)$$
$$\textit{for} \quad (-1)^j (t - t_i) > 0 \quad (j = 1,2;\ \ i = 1,\dots,n) \tag{2.11}$$

hold. Then

$$\det\big(I_n + (-1)^j d_j B(t)\big) \neq 0 \quad \textit{for} \quad t \in [a,b] \quad (j = 1,2). \tag{2.12}$$

Proof. Assume the contrary. Then there exist $j \in \{1,2\}$, $\tau_0 \in [a,b]$ and $\xi = (\xi_l)_{l=1}^n \in R^n$ such that

$$\det\big(I_n + (-1)^j d_j B(\tau_0)\big) = 0$$

and the following system of linear algebraic equations

$$\big(I_n + (-1)^j d_j B(\tau_0)\big) y = \xi \tag{2.13}$$

is not solvable.

Let $\mathcal{N}_0 = \{i_1, \dots, i_{n_0}\}$ be a set of all indices $i \in \{1,\dots,n\}$ for which $t_i = \tau_0$. Put $\eta = \xi$ if $\mathcal{N}_0 = \varnothing$ and $\eta = (\eta_l)_{l=1}^n$, $\eta = \xi - \sum_{k=1}^{n_0} \xi^{(k)}$ if $\mathcal{N}_0 \neq \varnothing$, where $\xi^{(k)} = (\delta_{l i_k} \xi_{i_k})_{l=1}^n$ $(k = 1,\dots,n_0)$. Then the system

$$\big(I_n + (-1)^j d_j B(\tau_0)\big) y = \eta \tag{2.14}$$

is not solvable, either, since in view of (2.1) the system

$$\big(I_n + (-1)^j d_j B(\tau_0)\big) y = \xi^{(k)}$$

has a solution $(\delta_{l i_k} \xi_{i_k} (1 + (-1)^j d_j b_{i_k i_k}(t_{i_k}))^{-1})_{l=1}^n$ for every $k \in \{1,\dots,n_0\}$.

Let $B^*(t) = (b^*_{il}(t))^n_{i,l=1}$ be a matrix-function whose elements are defined by the equalities $b^*_{il}(\tau_0) = 0$, $b^*_{il}(t) = (-1)^j d_j b_{il}(\tau_0)$ for $t \neq \tau_0$ $(i \neq l)$; $b^*_{ii}(t) \equiv b_{ii}(t)$ if $t_i = \tau_0$; $b^*_{ii}(t) = s_0(b_{ii})(t)$ for $(\tau_0 - t)(\tau_0 - t_i) \leq 0$ $(t_i \neq \tau_0)$ and $b^*_{ii}(t) = s_0(b_{ii})(t) + (-1)^j d_j b_{ii}(\tau_0)$ for $(\tau_0 - t)(\tau_0 - t_i) > 0$ $(t_i \neq \tau_0)$.

By (2.3)–(2.6) and (2.11) it is easy to show that the matrix-function B^* satisfies conditions (2.3)–(2.6). Consequently, according to Lemma 2.2 it is not difficult to ascertain that the system

$$dx(t) = dB^*(t) \cdot x(t)$$

has only a trivial solution satisfying condition (2.9). This, in turn, is equivalent to the condition

$$\mathrm{Ker}(I_n - F) = \varnothing,$$

where $F : BV_v([a,b], R^n) \to BV_v([a,b], R^n)$ is a completely continuous operator given by

$$(Fx)(t) \equiv \left(\sum_{l=1}^{n} \int_{t_i}^{t} x_l(\tau) db^*_{il}(\tau) \right)_{i=1}^{n} \quad \text{for} \quad x = (x_l)^n_{l=1} \in BV_v([a,b], R^n).$$

Therefore the operator equation

$$(I_n - F)x = \varphi$$

has a unique solution $y = (y_i)^n_{i=1}$, where $\varphi(t) = 0$ for $t \neq \tau_0$ and $\varphi(\tau_0) = \eta$ (see [7], p. 28). It is clear that

$$y_i(t) = \sum_{l=1}^{n} \int_{t_i}^{t} y_l(\tau) db^*_{il}(\tau) \quad \text{for} \quad t \neq \tau_0 \quad (i = 1, \dots, n) \tag{2.15}$$

and

$$y_i(\tau_0) = \sum_{l=1}^{n} \int_{t_i}^{\tau_0} y_l(\tau) db^*_{il}(\tau) + \eta_i \quad (i = 1, \dots, n). \tag{2.16}$$

Assume that an index $i \in \{1, \dots, n\}$ is such that $\tau_0 \neq t_i$. Then by (2.15) and the definition of B^* the function y_i is a solution of the problem

$$dy_i(t) = y_i(t) ds_0(b_{ii})(t), \quad y_i(t_i) = 0$$

on $[\tau_0 + \varepsilon, t_i]$ for $t_i > \tau_0$ and on $[t_i, \tau_0 - \varepsilon]$ for $t_i < \tau_0$, where $\varepsilon > 0$ is an arbitrary sufficiently small number. But the latter problem has only a trivial solution ([7], p. 106). Therefore

$$y_i(\tau_0) + (-1)^m d_m y_i(\tau_0) = 0 \quad \text{for} \quad (-1)^m (t_i - \tau_0) > 0 \quad (m = 1, 2).$$

On the other hand, it follows from (2.15) and (2.16) that

$$d_m y_i(\tau_0) = \sum_{l=1}^{n} y_l(\tau_0) d_m b^*_{il}(\tau_0) - (-1)^m \eta_i \quad (m = 1, 2).$$

Hence, with regard for the equalities

$$d_m b^*_{il}(\tau_0) = (-1)^{m+j} d_j b_{il}(\tau_0)$$
$$\text{for} \quad (-1)^m (t_i - \tau_0) > 0 \quad (m = 1, 2; \;\; l = 1, \dots, n),$$

we have

$$y_i(\tau_0) + (-1)^j \sum_{l=1}^{n} y_l(\tau_0) d_j b_{il}(\tau_0) = \eta_i \quad \text{for} \quad t_i \neq \tau_0.$$

The latter equality is obviously true for $t_i = \tau_0$ as well because $\eta_i = 0$ and functions b_{il} $(i \neq l;\, l = 1, \dots, n)$ are continuous at the point t_i.

According to what has been said, it is clear that

$$\big(I_n + (-1)^j d_j B(\tau_0)\big) y(\tau_0) = \eta,$$

i.e., system (2.14) is solvable. The contradiction abtained proves the lemma. □

Lemma 2.4. *Let condition* (1.10) *hold. Then there exists a positive number ρ_* such that every solution of the problem*

$$\bigg[d|x_i(t)| - \operatorname{sign}(t - t_i)\Big(\sum_{l=1}^{n} |x_l(t)| dc_{il}(t) + $$
$$+ du_i(t)\Big)\bigg] \operatorname{sign}(t - t_i) \leq 0 \quad (i = 1, \dots, n), \tag{2.17}$$

$$(-1)^j d_j |x_i(t_i)| \leq |x_i(t_i)| d_j c_{ii}(t_i) + d_j u_i(t_i) \;\; (j = 1, 2; \;\; i = 1, \dots, n);$$
$$|x_i(t_i)| \leq \varphi_{0i}\big(|x_1|, \dots, |x_n|\big) + \gamma \quad (i = 1, \dots, n) \tag{2.18}$$

admits an estimate

$$\sum_{i=1}^{n} \|x_i\|_s \leq \rho_* \Big[\gamma + \frac{1}{n} \|u(\cdot) - u(a)\|_s\Big], \tag{2.19}$$

where $\gamma \in R_+$ and $u = (u_i)_{i=1}^n \in BV([a, b], R^n_+)$ are an arbitrary number and vector-function, respectively.

Proof. Let

$$g(x) = \big(|x_i(t_i)| - \varphi_{0i}(|x_1|, \dots, |x_n|)\big)_{i=1}^n$$

and let S be a set of all matrix-functions $B = (b_{il})_{i,l=1}^n \in BV([a,b], R^{n\times n})$ for which conditions (2.3)–(2.6), (2.10) and (2.11) are fulfilled and the function b_{il} is continuous at the point t_i for every $i \neq l$ $(i, l = 1, \dots, n)$. According to Lemma 2.2 and (1.10) the problem

$$dx(t) = dB(t) \cdot x(t), \quad g(x) \leq 0$$

has no nontrivial solution if only $B \in S$. Moreover, by Lemma 2.3 the condition (2.12) holds. By Lemma 1 from [13] there exists a positive number ρ_0 such that

$$\|y\|_s \leq \rho_0 \Bigg[\big\| [g(y)]_+ \big\| + \\ + \sup \bigg\{ \bigg\| y(t) - y(a) - \int_a^t dB(\tau) \cdot y(\tau) \bigg\| : \ t \in [a,b] \bigg\} \Bigg] \tag{2.20}$$

for every $y \in BV([a,b], R^n)$ and $B \in S$.

Let $(x_i)_{i=1}^n$ be a solution of problem (2.17), (2.18). For every $i \in \{1, \dots, n\}$ we assume

$$b_i(t) \equiv \big[s_0(c_{ii})(t) - s_0(c_{ii})(t_i) \big] \operatorname{sign}(t - t_i), \tag{2.21}$$

$$\psi_i(t) \equiv \Bigg[\sum_{l=1}^n \int_{t_i}^t |x_l(\tau)| dc_{il}(\tau) - \\ - \int_{t_i}^t |x_i(\tau)| ds_0(c_{ii})(\tau) + u_i(t) - u_i(t_i) \Bigg] \operatorname{sign}(t - t_i).$$

Since the function b_i is continuous, the Cauchy problem

$$dy_i(t) = y_i(t) db_i(t) + d\psi_i(t), \quad y_i(t_i) = |x_i(t_i)|$$

has a unique solution y_i for every $i \in \{1, \dots, n\}$ (see [7], p. 106).

Taking (2.17) into account it is not difficult to verify that for every $i \in \{1, \dots, n\}$ the function

$$z_i(t) \equiv |x_i(t)| - y_i(t)$$

satisfies the conditions of Lemma 2.4 from [15], where $t_0 = t_i$, $c_0 = 0$, $\alpha(t) \equiv b_i(t)$ and $\varphi(t, x) \equiv x$. In addition, the problem

$$du(t) = u(t) db_i(t), \quad u(t_i) = 0 \tag{2.22}$$

has only a trivial solution. Hence, by this lemma

$$|x_i(t)| \leq y_i(t) \quad \text{for} \quad t \in [a,b] \quad (i = 1, \dots, n) \tag{2.23}$$

and

$$|x_i(t)| \equiv \eta_i(t) y_i(t) \quad (i = 1, \dots, n),$$

where $\eta_i : [a, b] \to [0, 1]$ $(i = 1, \dots, n)$ are measurable functions. Moreover, it is easy to see that

$$y_i(t) \equiv y_i(t_i) + \sum_{l=1}^{n} \int_{t_i}^{t} y_l(\tau) db_{il}(\tau) + \\ + [u_i(t) - u_i(t_i)] \operatorname{sign}(t - t_i) \quad (i = 1, \dots, n) \tag{2.24}$$

and

$$\big\| [g(y)]_+ \big\| \le n\gamma, \tag{2.25}$$

where

$$b_{ii}(t) = b_i(t) + \operatorname{sign}(t - t_i) \int_{t_i}^{t} \eta_i(\tau) d(c_{ii}(\tau) - s_0(c_{ii})(\tau)), \tag{2.26}$$

$$b_{il}(t) = \operatorname{sign}(t - t_i) \int_{t_i}^{t} \eta_l(\tau) dc_{il}(\tau) \quad (i \ne l;\ \ i, l = 1, \dots, n). \tag{2.27}$$

On the other hand, according to Lemma 2.3 and condition (1.3) it can be easily shown that $B = (b_{il})_{i,l=1}^{n} \in S$. Therefore, inequality (2.20) holds. By (2.23)–(2.25) inequality (2.20) inplies estimate (2.19), where $\rho_* = n^2 \rho_0$. □

3. Proof of the Main Results

Proof of Theorem 1.1. Let ρ_* be a positive number such that the conclusion of Lemma 2.4 is true.

By (1.9) there exists a positive number ρ such that

$$\rho_* \left[\gamma(\rho) + \frac{1}{n} \left\| \int_a^b d(A_1(t) + A_2(t)) \cdot q(t, \rho) \right\| \right] < \rho \quad \text{for} \quad \rho \ge \rho_0. \tag{3.1}$$

Let $b_i(t)$ $(i = 1, \dots, n)$ be defined by (2.21),

$$\chi(s) = \begin{cases} 1 & \text{for} \quad |s| \le \rho_0, \\ 2 - \dfrac{|s|}{\rho_0} & \text{for} \quad \rho_0 < |s| < 2\rho_0, \\ 0 & \text{for} \quad |s| \ge 2\rho_0; \end{cases} \tag{3.2}$$

$$\widetilde{\varphi}_i(x_1, \dots, x_n) = \chi\Big(\sum_{m=1}^{n} \|x_m\|_s \Big) \varphi_i(x_1, \dots, x_n) \quad (i = 1, \dots, n), \tag{3.3}$$

and let $B(t)$ be a diagonal matrix with diagonal elements $b_1(t),\dots,b_n(t)$. Then

$$\sup\left\{\chi\Big(\sum_{m=1}^n |x_m|\Big)\cdot|f(t,x_1,\dots,x_n)|:\ (x_i)_{i=1}^n\in R^n\right\}\in \\ \in L([a,b],R_+^n;A_1+A_2) \tag{3.4}$$

and

$$\sup\left\{\sum_{m=1}^n |\widetilde{\varphi}_m(x_1,\dots,x_n)|:\ (x_i)_{i=1}^n\in BV_s([a,b],R^n)\right\}<+\infty. \tag{3.5}$$

It is clear that the problem

$$dx(t)=dB(t)\cdot x(t),\quad x_i(t_i)=0\quad (i=1,\dots,n)$$

has only a trivial solution. With regard to this, conditions (3.4) and (3.5), by Lemma 2.4 from [14] the following problem

$$dx(t)=dB(t)\cdot x(t)\Big[1-\chi\Big(\sum_{m=1}^n |x_m(t)|\Big)\Big]+ \\ +dA(t)\cdot\chi\Big(\sum_{m=1}^n |x_m(t)|\Big)f(t,x(t))^2, \tag{3.6}$$

$$x_i(t_i)=\widetilde{\varphi}_i(x_1,\dots,x_n)\quad (i=1,\dots,n) \tag{3.7}$$

is solvable. Let $x=(x_i)_{i=1}^n$ be an arbitrary solution of problem (3.6), (3.7). According to Lemma 2.2, (1.6), (1.7), (1.11) and (1.12) we have

$$(-1)^j\big[|x_i(t)|-|x_i(s)|\big]\le \\ \le\int_s^t |x_i(\tau)|\Big[1-(-1)^j\chi\Big(\sum_{m=1}^n |x_m(\tau)|\Big)\Big]db_i(\tau)+ \\ +(-1)^j\sum_{k=1}^n\int_s^t \chi\Big(\sum_{m=1}^n |x_m(\tau)|\Big)\cdot f_k(\tau,x(\tau))\operatorname{sign}x_i(\tau)da_{ik}(\tau)\le \\ \le\int_s^t |x_i(\tau)|ds_0(c_{ii})(\tau)+\int_s^t \chi\Big(\sum_{m=1}^n |x_m(\tau)|\Big)\cdot|x_i(\tau)|d\big(c_{ii}(\tau)-s_0(c_{ii})(\tau)\big)+ \\ +\sum_{l\ne i,\,l=1}^n\int_s^t \chi\Big(\sum_{m=1}^n |x_m(\tau)|\Big)\cdot|x_l(\tau)|dc_{il}(\tau)+$$

[2]A vector-function from $BV([a,b],R^n)$ is said to be a solution of this system if it satisfies the corresponding integral equality.

$$+\sum_{k=1}^{n}\int\limits_{s}^{t} q_k(\tau,\|x(\tau)\|)d(a_{1ik}(\tau)+a_{2ik}(\tau)) \le$$

$$\le \sum_{l=1}^{n}\int\limits_{s}^{t} |x_l(\tau)|dc_{il}(\tau)+\sum_{k=1}^{n}\int\limits_{s}^{t} q_k(\tau,\|x(\tau)\|)d(a_{1ik}(\tau)+a_{2ik}(\tau))$$

for $s\le t,\ \ (-1)^j(t-t_i)\ge 0,\ \ (-1)^j(s-t_i)\ge 0\ \ (j=1,2;\ \ i=1,\dots,n)$,

since by conditions (1.3) and (1.10) the functions $c_{ii}-s_0(c_{ii})$ and c_{il} $(i\ne l)$ are nondecreasing. Consequently, condition (2.17) holds, where

$$u_i(t)\equiv\sum_{k=1}^{n}\int\limits_{a}^{t} q_k(\tau,\|x(\tau)\|)d(a_{1ik}(\tau)+a_{2ik}(\tau))\quad (i=1,\dots,n).$$

On the other hand, by (1.8), (3.2), (3.3) and (3.7) condition (2.18) holds. Therefore, according to the conclusion of Lemma 2.4

$$\sum_{i=1}^{n}\|x_i\|_s\le\rho_*\Bigg[\gamma\Big(\sum_{i=1}^{n}\|x_i\|_s\Big)+ + \frac{1}{n}\Bigg\|\int\limits_{a}^{b} d(A_1(t)+A_2(t))\cdot q\Big(t,\sum_{i=1}^{n}\|x_i\|_s\Big)\Bigg\|\Bigg].$$

From here with regard for (3.1) we conclude that

$$\sum_{i=1}^{n}\|x_i\|_s<\rho_0.$$

According to this inequality from (3.2), (3.3) (3.6) and (3.7) it follows that $x=(x_i)_{i=1}^n$ is a solution of problem (1.1), (1.2), too. □

Proof of Theorem 1.2. (1.17)–(1.19) imply inequalities (1.6)–(1.8), where

$$q_k(t,\|x\|)\equiv|f_k(t,0,\dots,0)|\quad (k=1,\dots,n),\quad \gamma(r)\equiv\sum_{i=1}^{n}|\varphi_i(0,\dots,0)|.$$

Hence all conditions of Theorem 1.1 are fulfilled so that the solvability of the problem (1.1), (1.2) is guaranteed. It remains to prove that problem (1.1), (1.2) has not more than one solution.

Let $(x_i)_{i=1}^n$ and $(y_i)_{i=1}^n$ be arbitrary solutions of problem (1.1), (1.2) and

$$z_i(t)\equiv x_i(t)-y_i(t)\quad (i=1,\dots,n).$$

According to Lemma 2.2

$$\Big[d|z_i(t)| - \operatorname{sign} z_i(t) \sum_{k=1}^{n} \varphi_k(t) da_{ik}(t)\Big] \operatorname{sign}(t - t_i) \le 0$$
$$\text{for } \ t \in [a,b] \ \ (i = 1, \dots, n)$$

and

$$(-1)^j d_j |z_i(t_i)| \le (-1)^j \operatorname{sign} z_i(t_i) \sum_{k=1}^{n} \varphi_k(t_i) da_{ik}(t_i)$$
$$(j = 1, 2; \ \ i = 1, \dots, n),$$

where

$$\varphi_k(t) \equiv f_k(t, x_1(t), \dots, x_n(t)) - f_k(t, y_1(t), \dots, y_n(t)) \quad (k = 1, \dots, n).$$

By virtue of (1.11), (1.12) and (1.17)–(1.19) this implies that $(|z_i(t)|)_{i=1}^n$ is a non-negative solution of problem (1.4), (1.5). Therefore, in view of (1.10) $z_i(t) \equiv 0$ $(i = 1, \dots, n)$. □

Corollaries 1.1–1.4 are immediately obtained from the proved theorems and Lemmas 2.6 and 2.7 from [15]. In addition, c_{il} $(i, l = 1, \dots, n)$ are defined in just the same way as in [15] (see p. 36) for Corollaries 1.2 and 1.4.

Remark 1.1 follows from the proof of Lemma 2.6 from [15].

Proof of Theorem 1.3. As condition (1.10) is violated, problem (1.4), (1.5) has a nontrivial non-negative solution $(x_i)_{i=1}^n$.

By y_i we shall denote unique solution of the following Cauchy problem

$$dy(t) = y(t) db_i(t) + d\psi_i(t), \quad y(t_i) = x_i(t_i), \tag{3.8}$$

where $b_i(t)$ is defined by (2.21), and

$$\psi_i(t) \equiv \Bigg[\sum_{l=1}^{n} \int_{t_i}^{t} x_l(\tau) dc_{il}(\tau) - \int_{t_i}^{t} x_i(\tau) ds_0(c_{ii})(\tau)\Bigg] \operatorname{sign}(t - t_i)$$

for every $i \in \{1, \dots, n\}$. Moreover, it is easy to verify that the function

$$z_i(t) \equiv x_i(t) - y_i(t)$$

satisfies the conditions of Lemma 2.4 from [15], where $t_0 = t_i$, $c_0 = 0$, $\alpha(t) \equiv b_i(t)$, $\varphi(t, x) \equiv x$ and problem (2.22) has only a trivial solution for every $i \in \{1, \dots, n\}$. Consequently, according to this lemma

$$x_i(t) \le y_i(t) \quad \text{for} \quad t \in [a,b] \quad (i = 1, \dots, n)$$

and

$$x_i(t) = \eta_i(t) y_i(t) \quad \text{for} \quad t \in [a,b] \quad (i = 1, \dots, n),$$

where $\eta_i : [a,b] \to [0,1]$ $(i = 1,\dots,n)$ are some measurable functions. By this, (1.5) and (3.8) the vector-function $y = (y_i)_{i=1}^n$ is a nontrivial non-negative solution of the linear homogeneous problem

$$dy(t) = dA(t) \cdot y(t), \quad y_i(t_i) = \delta_i \varphi_{0i}(y_1,\dots,y_n) \quad (i = 1,\dots,n),$$

where $\delta_i \in [0,1]$ $(i = 1,\dots,n)$, $A(t) = (a_{il}(t))_{i,l=1}^n$ for $t \in [a,b]$, and $a_{ii}(t)$ and $a_{il}(t)$ $(i \neq l)$ are equal to the right-hand side of (2.26) and (2.27), respectively.

Assume that system (1.4) has no nontrivial non-negative solution satisfying condition (2.9). With regard to (1.3) it is easy to verify that the matrix-function $A = (a_{il})_{i,l=1}^n$ satisfies the conditions of Lemma 2.3. Therefore, there exist numbers c_i $(i = 1,\dots,n)$ such that problem (1.1), (1.2) has no solution if for every $j,m \in \{1,2\}$ and $i,k,l \in \{1,\dots,n\}$

$$\begin{gathered} f_k(t,x_1,\dots,x_n) \equiv x_k, \quad \varphi_i(x_1,\dots,x_n) \equiv \delta_i \varphi_{0i}(x_1,\dots,x_n) + c_i; \\ p_{mikl}(t) \equiv 0 \quad \text{and} \quad \alpha_{mikjl} = 0 \quad \text{for} \quad l \neq k, \\ p_{mikk}(t) \equiv 1 \quad \text{and} \quad \alpha_{mikjk} = 1 \quad \text{for} \quad i \neq k, \\ p_{mkkk}(t) = (-1)^{m+1} \operatorname{sign}(t - t_k), \quad \alpha_{mkkjk} = (-1)^{m+j+1}; \\ a_{mil}(t) = 0 \quad \text{for} \quad (-1)^m (t - t_i) \geq 0 \quad (i \neq l), \\ a_{mil}(t) = a_{il}(t) \operatorname{sign}(t - t_i) \quad \text{for} \quad (-1)^m (t - t_i) < 0 \quad (i \neq l), \end{gathered}$$

and a_{1ii} and a_{2ii} are arbitrary nondecreasing functions for which

$$a_{1ii}(t) - a_{2ii}(t) \equiv a_{ii}(t).$$

Moreover, f_k, φ_i, a_{mkl}, p_{mikl} and α_{mikjl} $(j,m = 1,2;\ i,k,l = 1,\dots,n)$ satisfy conditions (1.11), (1.12), (1.17)–(1.19).

Assume now that system (1.4) has a nontrivial non-negative solution $(x_i)_{i=1}^n$ satisfying condition (2.9). Then it is clear that the conditions of Theorem 1.3 are fulfilled for functionals $\varphi_{0i}(x_1,\dots,x_n) \equiv 0$ $(i = 1,\dots,n)$. Reasoning as above, it is not difficult to ascertain that the problem

$$dy(t) = dA(t) \cdot y(t), \quad y_i(t_i) = 0 \quad (i = 1,\dots,n)$$

has a nontrivial solution. Therefore

$$\operatorname{Ker}(I_n - F) \neq \varnothing,$$

where F is a completely continuous operator of the type appearing in the proof of Lemma 2.3. By Fredholm's alternative (see [7], p. 28) there exists a vector-function $q = (q_i)_{i=1}^n \in BV([a,b], R^n)$ such that the problem

$$dy(t) = dA(t) \cdot y(t) + dq(t), \quad y_i(t_i) = 0 \quad (i = 1,\dots,n)$$

is not solvable. Assuming $z(t) = y(t) - q(t)$ we find that the problem

$$dz(t) = dA(t) \cdot (z(t) + q(t)), \quad z_i(t_i) = c_i \quad (i = 1,\dots,n),$$

where $c_i = q_i(t_i)$ $(i = 1, \dots, n)$ is not solvable either.

Let a_{mil}, p_{mikl} and α_{mikjl} $(j, m = 1, 2;\ i, k, l = 1, \dots, n)$ be defined as above and

$$f_k(t, x_1, \dots, x_n) \equiv x_k + q_k(t) \quad \text{and} \quad \varphi_i(x_1, \dots, x_n) \equiv q_i(t_i).$$

Then conditions (1.11), (1.12) and (1.17)–(1.19) are fulfilled and problem (1.1), (1.2) is not solvable either.

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