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On Fuzzy P-Spaces, Weak Fuzzy P-Spaces and Fuzzy Almost P-Spaces

G. Thangaraj[1], C. Anbazhagan[2] and P. Vivakanandan[3]

[1]Department of Mathematics
Thiruvalluvar University, Vellore – 632115
Tamilnadu, India
E-mail: g.thangaraj@rediffmail.com

[2, 3] Department of Mathematics
Jawahar Science College, Neyveli – 607803
Tamilnadu, India
[2] E-mail: emailofca@gmail.com
[3] E-mail: vivekjsc@gmail.com

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Abstract

In this paper we introduce the concepts of weak fuzzy P- spaces, fuzzy almost P-spaces. Also we discuss several characterizations of fuzzy P-spaces and weak fuzzy P-spaces, fuzzy almost P- spaces and study the inter-relations between the spaces introduced. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: *Fuzzy P-space, weak fuzzy P-space, fuzzy almost P-space, fuzzy almost Lindelof, fuzzy weakly Lindelof, fuzzy Baire.*

1 Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A. Zadeh in his classical paper [16] in the year 1965. Thereafter the paper of C.L. Chang [4] in1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting andthus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. Tang [9] has used a slightly changed version of Chang's fuzzy topological spaces to model spatial objects for GIS databases and Structured Query Language (SQL) for GIS.In this paper we introduce the concepts ofweak fuzzy P-spaces, fuzzy almost P-spaces.Also we discuss severalcharacterizations of fuzzy P-spaces,weak fuzzy P-spacesandfuzzyalmost P-spaces, and study the inter-relations between thespaces introduced. Several examples are given to illustrate the concepts introduced in this paper.

2 Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological spacedue to Chang (1968).

Definition 2.1: *Let λ and μ be any two fuzzy sets in (X,T). Then we define $\lambda \vee \mu : X \to [0,1]$ as follows : $(\lambda \vee \mu)(x) = Max\{ \lambda(x), \mu(x) \}$. Also we define $\lambda \wedge \mu : X \to [0,1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{ \lambda(x), \mu(x) \}$.*

Definition 2.2: *Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define $int(\lambda) = \vee\{ \mu / \mu \leq \lambda, \mu \in T \}$ and $cl(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T\}$.*
For any fuzzy set in a fuzzy topological space (X,T), it is easy to see that $1-cl(\lambda) = int(1-\lambda)$ and $1-int(\lambda) = cl(1-\lambda)$ [1].

Definition 2.3[13]: *A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$.*

Definition 2.4[13]: *A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda) = 0$.*

Definition 2.5[3]: *A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy F_σ-set in (X,T) if $\boldsymbol{\lambda} = \bigvee_{i=1}^{\infty} \boldsymbol{\lambda}_i$* where $1-\lambda_i \in T$ for $i \in I$.

Definition 2.6[3]: *A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy G_δ-set in*(X,T) *if $\boldsymbol{\lambda} = \bigwedge_{i=1}^{\infty} \boldsymbol{\lambda}_i$, where $\lambda_i \in T$ for $i \in I$.*

Definition 2.7[1]: *A fuzzy set λ in a fuzzy topological space (X,T) is called*

(i) *a fuzzy regular openset in (X,T) if int cl (λ) = λ and*

(ii) *a fuzzy regular closed set in (X,T) ifcl int (λ) = λ.*

Lemma 2.1[1]: *For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X,T), $\vee$ cl (λ_α) $\leq$ cl ($\vee\lambda_\alpha$). In case $\mathcal{A}$ is a finiteset, $\vee$cl (λ_α)=cl ($\vee\lambda_\alpha$). Also $\vee$int(λ_α)$\leq$int($\vee\lambda_\alpha$).*

3 Fuzzy P- Spaces

Based on the classical notion of P-spaces in [2] and [6],its fuzzy version is definedin[11] as follows:

Definition 3.1[11]: *A fuzzy topological space (X,T) is called a fuzzy P-spaceif countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in(X,T), is fuzzy open in(X,T).*

Example 3.1: Let X = {a, b, c}. The fuzzy sets λ, μ and υ are defined on X as follows:

$\lambda : X \to [0,1]$ is defined as$\lambda(a) = 0.1$; $\lambda(b) = 0.3$; $\lambda(c) = 0.1$.

$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.2$; $\mu(b) = 0.2$; $\mu(c) = 0.3$.

$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = 0.3$; $\upsilon(b) = 0.1$;$\upsilon(c) = 0.2$.

Then, T= {0, λ, μ, υ, (λ∨μ) , (λ∨υ), (μ∨υ), (λ∧μ), (λ∧υ), (μ∧υ),[λ∨(μ∧υ)], [υ∨(λ∧μ)], [μ∧(λ∨υ)],[λ∨μ∨υ], 1} is a fuzzy topology on X. Now the fuzzy sets λ∧υ ={ λ∧υ∧ [μ∨υ] ∧ [λ∨μ] } and μ∧(λ∨υ) = { [λ∨ (μ∧υ)]∧[υ∨(λ∧μ)} are fuzzy G_δ-sets in (X,T) and λ∧υ ,[μ∧(λ∨υ)] are fuzzy openin(X,T). Hence (X,T) is a fuzzy P-space.

Example 3.2: Let X = {a, b, c}. The fuzzy sets λ, μ and υ are defined on X as follows:

$\lambda : X \to [0,1]$ is defined as $\lambda(a) =0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 07$.

$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.8$.

$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = .7$; $\upsilon(b) = 0.5$; $\upsilon(c) = 0.9$.

Then, T = {0, λ, μ,υ, λ∨ μ, λ∨υ, μ∨υ, λ∧ μ, λ∧υ, μ∧υ,λ∧(μ∨υ), λ∨(μ∧υ),μ∧ (λ ∨υ), μ∨(λ∧υ), υ∧(λ∨ μ), υ∨(λ∧ μ), [λ∨μ∨υ], 1} is a fuzzy topology on X. Now the fuzzy set ∝ = {[λ∨ (μ∧υ)]∧[μ∨(λ ∧υ)] ∧[υ∨ (λ∧ μ)] }is a fuzzyG_δ-set in (X,T).But ∝ is not a fuzzy open set in (X,T). Hence the fuzzy topological space (X,T) is not a fuzzy P-space.

Proposition 3.1: *If λ is a non-zero fuzzy F_σ–set in a fuzzy P- space (X,T), then λ is a fuzzy closed set in (X,T).*

Proof: Since λ is a non-zero fuzzy F_σ-set in (X,T), $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where the fuzzy sets λ_i' s are fuzzy closedin(X,T). Then $1-\lambda = 1-(\vee_{i=1}^{\infty}(\lambda_i)) = \wedge_{i=1}^{\infty} (1-\lambda_i)$. Now λ_i's are fuzzy closed in (X,T), implies that $(1-\lambda_i)$'s are fuzzy open in (X,T).Hence we have $1-\lambda = \wedge_{i=1}^{\infty} (1-\lambda_i)$, where $1-\lambda_i \in T$. Then $1-\lambda$is a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy P- space, $1-\lambda$is fuzzy open in (X,T). Therefore λ is a fuzzy closedset in (X,T).

Proposition 3.2: *If the fuzzy topological space (X,T) is a fuzzy P-space, then* $cl\, (\vee_{i=1}^{\infty} (\lambda_i)) = \vee_{i=1}^{\infty} cl(\lambda_i)$, *where λ_i' s are non-zero fuzzy closed sets in (X,T).*

Proof: Let λ_i's be non-zero fuzzy closed sets in a fuzzy P-space (X,T).Then$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, is a non–zero fuzzy F_σ–set in(X, T). By proposition 3.1, λ is a fuzzy closed set in (X,T). Hence $Cl(\lambda) = \lambda$, hich implies that $Cl (\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty}(\lambda_i) = \vee_{i=1}^{\infty} cl(\lambda_i)$ [since λ_i's are fuzzy closed, $cl(\lambda_i) = (\lambda_i)$]. Therefore $cl (\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty} cl(\lambda_i)$,where λ_i's are fuzzy closed sets in (X,T).

Proposition 3.3: *If λ_i's are fuzzy regular closed sets in a fuzzy P-space (X,T), then* $cl (\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty} (\lambda_i)$.

Proof: Let λ_i's be fuzzy regular closed sets in a fuzzy P-space (X,T). Then λ_i's are fuzzy closed sets in (X,T), which implies that $(1-\lambda_i)$'s are fuzzy open sets in (X,T). Let $\mu = \wedge_{i=1}^{\infty}[1-(\lambda_i)]$. Then μ is a non-zero fuzzy G_δ-set in (X,T). Since the fuzzy topological space (X,T) is a fuzzy P-space, int (μ) = μ, which implies that $int\ (\wedge_{i=1}^{\infty}[1-(\lambda_i)]) = \wedge_{i=1}^{\infty}[1-(\lambda_i)]$. Then $1 - cl\big(\vee_{i=1}^{\infty} (\lambda_i)\big) = 1 - \vee_{i=1}^{\infty}(\lambda_i)$. Hence we have $cl\ (\vee_{i=1}^{\infty} (\lambda_i)) = \vee_{i=1}^{\infty} (\lambda_i)$.

Proposition 3.4: *If the fuzzy topological space (X,T) is a fuzzy P- space and if λ is a fuzzy first category set in (X, T), then λ is not a fuzzy dense set in (X,T).*

Proof: Assume the contrary. Suppose that λ is a fuzzy first category set in (X,T) such that $cl (\lambda) = 1$. Then, $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now $1- cl (\lambda_i)$ is a fuzzy open set in (X,T). Let $\mu = \wedge_{i=1}^{\infty}[1 - cl\ (\lambda_i)]$. Then μ is a non-zero fuzzy G_δ-set in (X,T). Now we have $\wedge_{i=1}^{\infty}\big[1 - cl\ (\lambda_i)\big] = 1 - \vee_{i=1}^{\infty}(cl\, \lambda_i) \leq 1 - \vee_{i=1}^{\infty}(\lambda_i) = 1 - \lambda$.Hence $\mu \leq 1 - \lambda$. Then $int (\mu) \leq int (1- \lambda) = 1 - cl (\lambda) = 1-1= 0$. That is, $int (\mu) = 0$. Since(X,T) is a fuzzy P-space, $\mu = int(\mu)$ which implies that $\mu = 0$, a contradiction to μ being a non-zero fuzzy G_δ-set in (X,T). Hence $cl(\lambda) \neq 1$.

Remarks: In classical topology, every meager (first category) set in a P-space X is a nowhere dense set in X. But in the case of fuzzy topology, a fuzzy first category set λ in a fuzzy P-space (X,T) is not a fuzzy nowhere dense set in (X,T). For, considerthe following proposition.

Proposition 3.5: *If the fuzzy topological space (X,T) is a fuzzy P-space and if λ is a fuzzy first category set in (X,T), then λ is not a fuzzy nowhere dense set in (X,T).*

Proof: Let λ be a fuzzy first category set in a fuzzy P-space (X,T). Then, we have $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now intcl $(\lambda) =$ int cl $(\bigvee_{i=1}^{\infty}(\lambda_i)) \geq$ int $[\bigvee_{i=1}^{\infty}$ cl $(\lambda_i)]$ and $[\bigvee_{i=1}^{\infty}$ cl $(\lambda_i)]$ is a fuzzy F_σ-set in (X,T). Since (X,T) is a fuzzy P-space, by proposition 3.1, $[\bigvee_{i=1}^{\infty}$ cl $(\lambda_i)]$ is a non- zero fuzzy closed set in (X,T). Also interior of a fuzzy closed is a fuzzy regular open set, int$[\bigvee_{i=1}^{\infty}$ cl $(\lambda_i)]$ is a non-zero fuzzy regular open set in (X,T). Hence we have $0 \neq$ int$[\bigvee_{i=1}^{\infty}$ cl $(\lambda_i)] \leq$ int cl (λ) implies that intcl $(\lambda) \neq 0$. Therefore λ is not a fuzzy nowhere dense set in (X,T).

Proposition 3.6: *If λ is a fuzzy first category set in a fuzzy P- space (X,T) such that $\mu \leq 1 - \lambda$, where μ is a non-zero dense fuzzy G_δ-set in (X,T), then $\lambda$is a fuzzy nowhere dense set in (X,T).*

Proof: Let λ be a fuzzy first category set in (X,T). Then, $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now $1-$ cl (λ_i) is a fuzzy open set in (X,T). Let $\mu = \bigwedge_{i=1}^{\infty}[1 - \text{cl}\,(\lambda_i)]$. Then μ is a non- zero fuzzy G_δ-set in (X,T). Now we have $\bigwedge_{i=1}^{\infty}[1 - \text{cl}\,(\lambda_i)] = 1 - \bigvee_{i=1}^{\infty}(\text{cl}\,\lambda_i) \leq 1 - \bigvee_{i=1}^{\infty}(\lambda_i) = 1 - \lambda$. Hence $\mu \leq (1 - \lambda)$. Then we have $\lambda \leq (1-\mu)$. Now intcl $(\lambda) \leq$ intcl $(1-\mu)$, which implies that int cl $(\lambda) \leq 1-$ cl int(μ). Since (X,T) is a fuzzy P-space, the fuzzy G_δ-set μ is fuzzy open in (X,T) and int$(\mu) = \mu$. Therefore int cl $(\lambda) \leq 1-$ cl $(\mu) = 1-1 = 0$ (sinceμ is fuzzy dense). Then int cl $(\lambda) = 0$ and hence λ is a fuzzy nowhere dense set in (X,T).

Theorem 3.1[10]: *Let (X,T) be a fuzzy topological space. Then the following areequivalent:*

(1) (X,T) is a fuzzy Bairespace.
(2) Int $(\lambda) = 0$ forevery fuzzy first category set λ in (X,T).
(3) Cl $(\mu) = 1$ for every fuzzy residual set μ in (X,T).

Proposition 3.7: *If λ is a fuzzy first category set in a fuzzy P- space (X,T) such that $\mu \leq 1 - \lambda$, where μ is a non-zero dense fuzzy G_δ-set in (X,T), then (X,T) is a fuzzy Bairespace.*

Proof: Let λ be a fuzzy first category set in (X,T). As in proof 3.6, we have int cl $(\lambda) = 0$. Then int $(\lambda) \leq$ int cl (λ) implies that int $(\lambda) = 0$ and hence by theorem 3.1, (X,T) is a fuzzy Baire space.

Proposition 3.8: *If the fuzzy topological space (X,T) is a fuzzy P- space and if λ is a fuzzy dense and fuzzy first category set in (X,T), then there is no non -zero fuzzy G_δ-set μ in (X,T) such that $\mu \leq 1 - \lambda$.*

Proof: Let λ be a fuzzy first category set in (X,T). As in proof 3.6, we have a fuzzy G_δ-set μ in (X,T) such that $\mu \leq 1 - \lambda$. Then int (μ) ≤int (1− λ) implies that int (μ) ≤1− cl (λ) = 1−1 = 0 [since λ is fuzzy dense,cl (λ) = 1]. That is, int (μ) = 0. Since (X,T) is a fuzzy P- space, int (μ) = μ and hence we have μ = 0. Hence, if λ is a fuzzy dense and fuzzy first category set in (X,T), then there is no non-zero fuzzy G_δ-set μ in (X,T) such that $\mu \leq 1 - \lambda$.

Proposition 3.9: *If A ⊂ X is such that χ_A (the characteristic function of A⊂ X) is fuzzy open ina fuzzy P- space (X,T), then the fuzzy subspace (A, T_A)is a fuzzy P-space.*

Proof: Let λ be a fuzzy G_δ-set in (A,T_A). Then λ= $\wedge_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy open sets in (A, T_A). Now $\lambda_i$is a fuzzy open set in (A, T_A) implies that there exists a fuzzy open set μ_i in (X,T) such that μ_i/A $= \lambda_i$. That is, $\mu_i \wedge \chi_A = \lambda_i$. Since μ_i and χ_A are fuzzy open sets in (X,T), λ_i is a fuzzy open set in (X,T).Now$\wedge_{i=1}^{\infty}(\mu_i \wedge \chi_A) = \wedge_{i=1}^{\infty}(\lambda_i)$, implies that $[\wedge_{i=1}^{\infty}(\mu_i)] \wedge (\chi_A) = \wedge_{i=1}^{\infty}(\lambda_i)$and $[\wedge_{i=1}^{\infty}(\mu_i)]$ is a fuzzy G_δ-set in (X,T).Since (X,T) is a fuzzy P- space, $[\wedge_{i=1}^{\infty}(\mu_i)]$ is fuzzy open in (X,T). Now $[\wedge_{i=1}^{\infty}(\mu_i)] \wedge (\chi_A) = \wedge_{i=1}^{\infty}(\lambda_i)$ implies that $[\wedge_{i=1}^{\infty}(\mu_i)]$ A is fuzzy open in (A, T_A).Then$\wedge_{i=1}^{\infty}(\lambda_i)$ is fuzzy open in (A, T_A). Hence the fuzzy G_δ-set λ is fuzzy open in (A, T_A). Thereforethe fuzzy subspace (A, T_A) is a fuzzy P-space.

4 Weak Fuzzy P-Spaces

Motivated by the classical concept introduced in [7] we shall now define:

Definition 4.1: *A fuzzy topological space (X,T) is called a weak fuzzy P-space if the countable intersection fuzzy regular open sets in (X,T) is a fuzzy regular open set in (X,T). That is, $\wedge_{i=1}^{\infty}(\lambda_i)$is fuzzyregular open in (X,T), where λ_i's are fuzzy regular open sets in (X,T).*

Example 4.1: Let X = {a, b, c}. The fuzzy sets λ, μ andυ are defined on X as follows:

λ : X → [0,1] is defined as λ(a)= 0.4; λ(b) = 0.5; λ(c) = 0.6.
μ :X → [0,1] is defined as μ(a) = 0.6; μ (b) = 0.4;μ(c) =0.5.
υ : X → [0,1] is definedas υ(a) = 0.7; υ (b) = 0.6; υ(c) =0.4.

Then, T={0, λ, μ,υ, λ∨μ, λ∨υ, μ∨υ, λ∧μ, λ∧υ, μ ∧υ,λ∧(μ∨υ), μ∨(λ∧υ), υ∧(λ∨ μ), [λ∧ μ∧υ], [λ∨μ∨υ], 1}is a fuzzy topology on X. Now λ , λ∨ μ , λ∧υ, λ∧(μ ∨υ), μ∨(λ∧υ), υ∧(λ∨μ) are fuzzy regular open sets in (X,T) and{λ∧[λ∨ μ]∧[λ∧υ]∧[λ∧(μ∨υ)]∧[μ∨(λ∧υ)]∧[υ∧(λ∨μ)]} = λ∧υ, is a fuzzy regular open in (X,T) and hence (X,T) is a weak fuzzy P-space.

Proposition 4.1: *A fuzzy topological space (X,T) is a weak fuzzy P-space if and only if* $\vee_{i=1}^{\infty}(\mu_i)$*,where* μ_i*'s are fuzzy regular closed sets in (X,T),is fuzzy regular closed in(X,T).*

Proof: Let (X,T) be a weak fuzzy P-space. Then intcl$(\wedge_{i=1}^{\infty}(\lambda_i))= \wedge_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy regular open sets in (X,T). Now 1–[int cl $(\wedge_{i=1}^{\infty}(\lambda_i))$] $= 1 - \wedge_{i=1}^{\infty}(\lambda_i)$, implies that cl int[($\vee_{i=1}^{\infty}(1-\lambda_i)$)] = $\vee_{i=1}^{\infty}$ $(1-\lambda_i)$. Let $\mu_i= (1-\lambda_i)$. Since λ_i is a fuzzy regular open set in (X,T), μ_i is a fuzzy regular closed set in(X,T). Then we have cl int $(\vee_{i=1}^{\infty}(\mu_i))$ = $\vee_{i=1}^{\infty}(\mu_i)$. Hence$\vee_{i=1}^{\infty}(\mu_i)$ is a fuzzy regular closed in (X,T). Conversely, suppose that cl int$(\vee_{i=1}^{\infty}(\mu_i))$ =$\vee_{i=1}^{\infty}(\mu_i)$, where μ_i's arefuzzy regular closed sets in (X,T). Then 1−cl int $(\vee_{i=1}^{\infty}(\mu_i))$ = 1− $\vee_{i=1}^{\infty}(\mu_i)$, which implies that int cl $(\wedge_{i=1}^{\infty}(1-\mu_i))$ = $\wedge_{i=1}^{\infty}(1-\mu_i)$,where$(1-\mu_i)$'s are fuzzy regular open sets in (X,T). Therefore (X,T) is a weak fuzzy P-space.

Theorem 4.1[1]:

(a) *The closure of a fuzzy open set is a fuzzy regular closed set, and*
(b) *The interior of a fuzzy closed set is a fuzzy regular open set.*

Proposition 4.2: *If a fuzzy topological space (X,T) is a weak fuzzy P-space, then cl* $(\vee_{i=1}^{\infty}\ (\lambda_i\,))$ = $\vee_{i=1}^{\infty} cl(\lambda_i\,)$*, where* λ_i*' s are non-zero fuzzy open sets in (X,T).*

Proof: Let λ_i's be fuzzy open setsin (X,T). Then by lemma 4.1, cl (λ_i)'s are fuzzy regular closed in (X,T). Since (X,T) is a weak fuzzy P-space, by proposition 4.1, $\vee_{i=1}^{\infty}$ [cl(λ_i)], where cl(λ)'s are non-zero fuzzyregular closed setsin(X,T), isfuzzy regular closed in (X,T). That is, cl int[$\vee_{i=1}^{\infty}$ cl(λ_i)] = $\vee_{i=1}^{\infty}$cl(λ_i). Then we have cl int [$\vee_{i=1}^{\infty}(\lambda_i)$] ≤ cl int [$\vee_{i=1}^{\infty}$ cl(λ_i)] = $\vee_{i=1}^{\infty}$ cl(λ_i). That is, cl int [$\vee_{i=1}^{\infty}(\lambda_i)$] ≤ $\vee_{i=1}^{\infty}$ cl(λ_i).Since $\lambda_i \in$ T, $\vee_{i=1}^{\infty}(\lambda_i) \in$ T and int[$\vee_{i=1}^{\infty}(\lambda_i)$] = [$\vee_{i=1}^{\infty}$ (λ_i)]. Hence cl [$\vee_{i=1}^{\infty}$ (λ_i)] ≤ $\vee_{i=1}^{\infty}$ cl(λ_i)……(1).But $\vee_{i=1}^{\infty}$ cl(λ_i) ≤ cl [$\vee_{i=1}^{\infty}$ (λ_i)]………(2). From (1) and (2), we have cl($\vee_{i=1}^{\infty}(\lambda_i)$) =$\vee_{i=1}^{\infty}$ cl(λ_i), where λ_i's are non-zero fuzzy open sets in (X,T).

Definition 4.2[12]: *A fuzzy topological space (X,T) is called a fuzzy almost Lindelof space if every fuzzy open cover* $\{\lambda_\alpha\}_{\alpha\in\Delta}$ *of (X,T) admits a countable subcover* $\{\lambda_n\}_{\,n\in N}$ *such that* $\vee_{n\in N}\, cl(\lambda_n) =$ *1.*

Definition 4.3[12]: *A fuzzy topological pace (X,T) is said to be fuzzy weakly Lindelof space if every fuzzy open cover* $\{\lambda_\alpha\}_{\alpha\in\Delta}$ *of (X,T) there exists a countable sub cover* $\{\lambda_n\}_{n\in N}$ *such that cl [* $\vee_{n\in N}(\lambda_n)$*] =1.*

Remarks: Obviously every fuzzy almost Lindelof space is a fuzzy weakly Lindelof space. For, $\vee_{n\in N}\, cl(\lambda_n)$ ≤ cl[$\vee_{n\in N}(\lambda_n)$] and$\vee_{n\in N}\, cl(\lambda_n)$ =1, implies that cl [$\vee_{n\in N}(\lambda_n)$] =1.

Proposition 4.3: *If the fuzzy topological space (X,T) is a weak fuzzy P-space, then every fuzzy weakly Lindelof space is a fuzzy almost Lindelof space.*

Proof: Let (X,T) be a fuzzy weakly Lindelof space and $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a fuzzy open cover of (X,T). Then there exists a countable subcover $\{\lambda_n\}_{n\in N}$ such that cl $[\vee_{n\in N}(\lambda_n)] = 1$. Since (X,T) is a weak fuzzy P-space, cl $[\vee_{n\in N}(\lambda_n)] = \vee_{n\in N} cl(\lambda_n)$ where λ_i's are non-zero fuzzy open sets in (X,T). Hence for the fuzzy open cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of (X,T), there exists a countable subcover $\{\lambda_n\}_{n\in N}$such that $\vee_{n\in N} cl(\lambda_n) = 1$. Therefore(X,T) is a fuzzy almost Lindelof space.

Proposition 4.4: *If a fuzzy topological space (X,T) is a fuzzy P-space, then (X,T) is a weak fuzzy P-space.*

Proof: Let λ_i's be fuzzy regular closed sets in (X,T). Since (X,T) is a fuzzy P-space by Proposition 3.3, we have cl $(\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty}(\lambda_i)$. Now cl$int$ $(\vee_{i=1}^{\infty}(\lambda_i)) \le$ cl $(\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty}(\lambda_i)$. That is, cl$int$ $(\vee_{i=1}^{\infty}(\lambda_i)) \le \vee_{i=1}^{\infty}(\lambda_i)$ ……..(1). Since λ_i's are fuzzy regular closed sets in (X,T), cl int $(\lambda_i) = \lambda_i$. Then $\vee_{i=1}^{\infty}$ cl int $(\lambda_i) = \vee_{i=1}^{\infty}(\lambda_i)$, which implies that $\vee_{i=1}^{\infty}(\lambda_i) \le$ clint $(\vee_{i=1}^{\infty}(\lambda_i))$……(2). From (1) and (2), we have clint $(\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty}(\lambda_i)$. Hence by proposition 4.1, (X,T) is a weak fuzzy P-space.

Remarks: A weak fuzzy P-space need not be a fuzzy P-space. For, consider the following example:

Example 4.2: Let X = {a, b, c}. The fuzzy sets λ, μ andυ are defined on X as follows:
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.4$; $\lambda(b) = 0.5$; $\lambda(c) = 0.6$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.4$; $\mu(c) = 0.5$.
$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = 0.5$; $\upsilon(b) = 0.6$; $\upsilon(c) = 0.4$.

Then, $T=\{0, \lambda, \mu, \upsilon, \lambda\vee\mu, \lambda\vee\upsilon, \mu\vee\upsilon, \lambda\wedge\mu, \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\wedge(\mu\vee\upsilon), \lambda\vee(\mu\wedge\upsilon), \mu\wedge(\lambda\vee\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\wedge(\lambda\vee\mu), \upsilon\vee(\lambda\wedge\mu), [\lambda\wedge\mu\wedge\upsilon], [\lambda\vee\mu\vee\upsilon], 1\}$ is a fuzzy topology on X. Now the fuzzy sets $\lambda\vee\mu, \lambda\vee\upsilon, \mu\vee\upsilon, \lambda\wedge\mu, \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\wedge(\mu\vee\upsilon), \lambda\vee(\mu\wedge\upsilon), \mu\wedge(\lambda\vee\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\wedge(\lambda\vee\mu), \upsilon\vee(\lambda\wedge\mu), [\lambda\wedge\mu\wedge\upsilon], [\lambda\vee\mu\vee\upsilon]$ are fuzzy regular open sets in (X,T) and $\{[\lambda\wedge\mu] \wedge [\lambda\wedge\upsilon] \wedge [\mu\wedge\upsilon] \wedge [\lambda\wedge(\mu\vee\upsilon)] \wedge [\lambda\vee(\mu\wedge\upsilon)] \wedge [\mu\wedge(\lambda\vee\upsilon)] \wedge [\mu\vee(\lambda\wedge\upsilon)] \wedge [\upsilon\wedge(\lambda\vee\mu)] \wedge [\upsilon\vee(\lambda\wedge\mu)]\} = [\lambda\wedge\mu\wedge\upsilon]$, is a fuzzy regular open set in (X,T) and hence (X,T) is a weak fuzzy P-space. But (X,T) is not a fuzzy P-space, since the fuzzy G_δ-set $\{[\lambda\vee\mu]\wedge[\lambda\vee\upsilon]\wedge[\mu\vee\upsilon]\}$ is not fuzzy open in (X,T).

5 Fuzzy Almost P-Spaces

Almost P-spaces was introduced by A.I. Veksler [14] and was also studied further by R. Levy [15]. Motivated by the classical concept introduced in [5] and [8] we shall now define:

Definition 5.1: *A fuzzy topological space (X,T) is called a fuzzy almost P-space if for every non-zero fuzzy G_δ-set λ in (X,T), $int(\lambda) \neq 0$ in (X,T).*

Example 5.1: Let X = {a, b, c}. The fuzzy sets λ, μ and υ are defined on X as follows:
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.4$.
$\mu : X \to [0,1]$ isdefined as $\mu(a) = 0.4$; $\mu(b) = 0.7$; $\mu(c) = 0.5$.
$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = 0.5$; $\upsilon(b) = 0.8$; $\upsilon(c) = 0.6$.

Then, $T=\{0, \lambda, \mu, \upsilon, \lambda\vee\mu, \lambda\wedge\mu, 1\}$ is a fuzzy topology on X. Now for the fuzzy G_δ-sets $[\lambda\wedge\mu\wedge\upsilon]$ and $\{[\lambda\vee\mu] \wedge [\lambda\wedge\mu]\}$ in (X,T), int $([\lambda\wedge\mu\wedge\upsilon]) = \lambda\wedge\mu \neq 0$ and int $(\{[\lambda\vee\mu] \wedge [\lambda\wedge\mu]\}) = \lambda\wedge\mu \neq 0$. Hence (X,T) is a fuzzy almost P-space.

Remarks:

(1) Clearly every fuzzy P-space is a fuzzy almost fuzzy P-space, since for every non-zero fuzzy G_δ-set δ in (X,T), we have $\mathrm{int}(\delta) = \delta \neq 0$. But the converse need not be true. For, in example 4.2, for every non-zero fuzzy G_δ -set δ in (X,T), we have int $(\delta) \neq 0$ in (X,T). Hence (X,T) is a fuzzy almost P-space, but (X,T) is not a fuzzy P-space, since the fuzzy G_δ-set $\{[\lambda\vee\mu]\wedge[\lambda\vee\upsilon]\wedge[\mu\vee\upsilon]\}$ is not fuzzy open in (X,T).

(2) A fuzzy almost P-space need not be a weak fuzzy P-space. For, consider the following example:

Example 5.2: Let X = {a, b, c}. The fuzzy sets λ, μ and υ are defined on X as follows:
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$.
$\mu : X \to [0,1]$ isdefined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.8$.
$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = 0.7$; $\upsilon(b) = 0.5$; $\upsilon(c) = 0.9$.

Then, $T=\{0, \lambda, \mu, \upsilon, \lambda\vee\mu, \lambda\vee\upsilon, \mu\vee\upsilon, \lambda\wedge\mu, \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\wedge(\mu\vee\upsilon), \lambda\vee(\mu\wedge\upsilon), \mu\wedge(\lambda\vee\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\wedge(\lambda\vee\mu), \upsilon\vee(\lambda\wedge\mu), [\lambda\wedge\mu\wedge\upsilon], [\lambda\vee\mu\vee\upsilon], 1\}$ is a fuzzy topology on X. In (X,T), for every non-zero fuzzy G_δ -set δ we have int $(\delta) \neq 0$. Hence (X,T) is a fuzzy almost P-space, but (X,T) is not a weak fuzzy P-space.

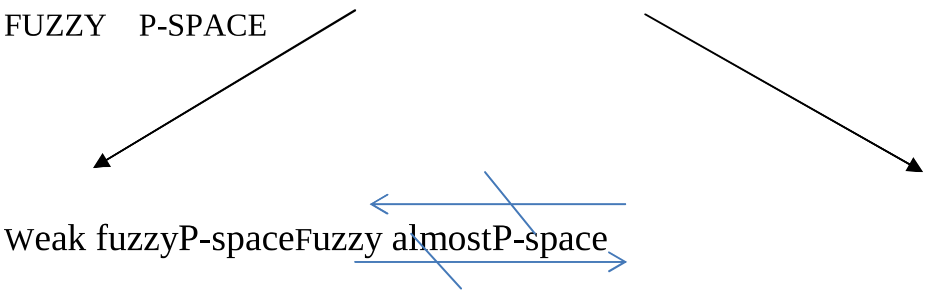
(3) A weak fuzzy P-space need not be a fuzzy almost P-space. For, consider the following example:

Example 5.3: Let X = {a, b, c}. The fuzzy sets λ, μ and υ are defined on X as follows:
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0$; $\lambda(b) = 0.4$; $\lambda(c) = 0.5$.
$\mu : X \to [0,1]$ isdefined as $\mu(a) = 0.6$; $\mu(b) = 0$; $\mu(c) = 0.4$.
$\upsilon : X \to [0,1]$is defined as $\upsilon(a) = 0.5$; $\upsilon(b) = 0.6$; $\upsilon(c) = 0$.

Then, $T=\{0, \lambda, \mu, \upsilon, \lambda\vee\mu, \lambda\vee\upsilon, \mu\vee\upsilon, \lambda\wedge\mu, \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\wedge(\mu\vee\upsilon), \lambda\vee(\mu\wedge\upsilon), \mu\wedge(\lambda\vee\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\wedge(\lambda\vee\mu), \upsilon\vee(\lambda\wedge\mu), [\lambda\wedge\mu\wedge\upsilon], [\lambda\vee\mu\vee\upsilon], 1\}$ is a fuzzy topology on X.

Now $\lambda\vee\upsilon$, $\lambda\vee(\mu\wedge\upsilon)$ and $[\lambda\vee\mu\vee\upsilon]$, are fuzzy regular open sets in (X,T) and $\{[\lambda\vee\upsilon]\wedge[\lambda\vee(\mu\wedge\upsilon)]\wedge[\lambda\vee\mu\vee\upsilon]\} = [\lambda\vee(\mu\wedge\upsilon)]$ is a fuzzy regular open in (X,T) and hence (X,T) is a weak fuzzy P-space.But (X,T) is not a fuzzy almost P-space, since for the non-zero fuzzy G_δ -set$\{[\lambda\wedge(\mu\vee\upsilon)]\wedge[\mu\wedge(\lambda\vee\upsilon)]\wedge[\upsilon\wedge(\lambda\vee\mu)]\}$, we have int$\{[\lambda\wedge(\mu\vee\upsilon)]\wedge[\mu\wedge(\lambda\vee\upsilon)]\wedge[\upsilon\wedge(\lambda\vee\mu)]\}= 0$.

Therelationship among the classes of fuzzy P-space, weak fuzzy P-space, fuzzy almost P-space can be summarized as follows:



Proposition 5.1: *If a fuzzy topological space (X,T) is a fuzzy almost P-space, if and only if the only fuzzy F_σ-set λ such that cl(λ) = 1 in (X,T) is 1_x.*

Proof: Let $\lambda(\neq 1)$ be a fuzzy F_σ-set such that cl(λ) = 1 in (X,T). Then, $1-\lambda$ is a non-zero fuzzy G_δ -set in (X,T). Now $1-$cl(λ) = 0, implies that int $(1-\lambda)$ =0, which is a contradiction to (X,T) being a fuzzy almost P-space in which int $(\delta)\neq 0$ for any non- zero fuzzy G_δ-set δin (X,T). Hence our assumption that cl(λ) = 1 does not hold. Therefore there is no non-zero fuzzy F_σ-set (other than 1_x) in (X,T) such that cl(λ) =1.

Conversely, let us assume that the only fuzzy F_σ-set λ in (X,T) such thatcl(λ)=1 is 1_x. Let be a fuzzy G_δ-set in (X,T).Then $\lambda= \bigwedge_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy open sets in (X,T). Now $1-\lambda = \bigvee_{i=1}^{\infty} (1-\lambda_i)$. Then, $1-\lambda$ is a fuzzy F_σ - set in (X,T). By hypothesis, cl$(1-\lambda)\neq 1$. Then, 1 – int $(\lambda)\neq 1$, which implies that int $(\lambda)\neq 0$. Hence for the non- zero fuzzy G_δ-set λ in (X,T), we have int $(\lambda)\neq 0$ and therefore (X,T) is a fuzzy almost P-space.

Remarks: If a fuzzy topological space (X,T) has non- zero fuzzy nowhere dense fuzzy G_δ-sets, then (X,T) is not a fuzzy almost P-space. For, consider the following proposition.

Proposition 5.2: *If λ is a non- zero fuzzy nowhere dense fuzzy G_δ-set in a fuzzy topological space (X,T), then (X,T) is not a fuzzy almost P-space.*

Proof: Let λ be a non- zero fuzzy nowhere dense fuzzy G_δ-set λ in (X,T). Then int$(\lambda)\leq$ int cl(λ) and int cl$(\lambda)= 0$, implies that int(λ) =0. Hence for the non- zero fuzzy G_δ-set λ in (X,T), int (λ) = 0 in (X,T). Therefore (X,T) is not a fuzzy almost P-space.

Theorem 5.1[11]: *For any fuzzy topological space (X,T), then following are equivalent:*

(a) X is fuzzy basically disconnected.

(b) For each fuzzy closed G_δ-set λ, int (λ) is fuzzy closed.

(c) For each fuzzy open F_σ-set λ, $cl\ (\lambda) + cl\big(1 - cl(\lambda)\big) = 1$.

Proposition 5.3: *If λ is a non-zero fuzzy closed fuzzy G_δ-set in a fuzzy basically disconnected space (X,T) and fuzzy almost P-space, then $cl[int(\lambda)] \neq 0$.*

Proof: Let λ be a non-zero fuzzy closed G_δ-set λ in (X,T). Since (X,T) is a fuzzy basically disconnected space, by theorem 5.1, int(λ) is fuzzy closed. That is, cl[int (λ)] $= int(\lambda)$. Since (X,T) is a fuzzy almost P-space, $int(\lambda) \neq 0$. Hence we have cl[int(λ)] $\neq 0$.

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