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On the Solution of Fractional Kinetic Equation

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Abstract

In this paper, the solution of a class of fractional Kinetic equation involving generalized I-function has been discussed. Special cases involving the I-function, H-function, generalized M-series, generalized Mittag-Leffler functions are also discussed. Results obtained are related to recent investigations of possible astrophysical solutions of the solar neutrino problem.

Keywords: *Generalized I-function, Fractional Kinetic Equation, Mittag-Leffler function, Laplace transform, Riemann-Liouville operator.*

1 Introduction

Developments in the field of nuclear physics in the last few years makes it possible to decide rather definitely which processes can and which cannot occur in the interior of stars. The five different fusion paths can be divided into two sets of processes for the evolution of energy in an ordinary star are pp (proton-proton) chain and CNO (carbon, nitrogen and oxygen) cycle. The pp chain is more important in the low-mass star like sun or less, where the hydrogen converted to helium. CNO cycle depends on the amount of carbon, nitrogen, and oxygen in addition to the amount of hydrogen and helium in the star for the greater gravitational force. Theoretical models shows that the CNO or Catalytic cycle is the dominant source of energy in the star which is more massive, about 1.3 times the mass of the sun. The CNO process was independently proposed by Weizsäcker and Bethe in the year 1938 and 1939 respectively. For more details we refer to [1, 2].

A spherically symmetric, non-rotating, non-magnetic, self gravitating model of a star like sun assumed to be in thermal and hydrostatic equilibrium, with a non-uniform chemical composition throughout. For thermal and hydrostatic equilibrium there is no time dependence in the equation describing the internal structure of star, the rate of change of chemical composition of a star species in terms of reaction rates for destruction and production (Kourganoff 1973, Perdang 1976, Clayton 1983). For details we refer to [3].

Due to the importance of kinetic equation in mathematical physics many authors have generalized the standard kinetic equation time to time. In the recent paper of Haubold and Mathai [3] have derived the fractional kinetic equation and thermonuclear function in terms of well known Mittag-Leffler function. As an extension of the work of Haubold and Mathai, Saxena et al. [10] have generalized the standard kinetic equation with generalized Mittag-Leffler functions. Further, Chaurasia and Kumar [15] generalized and studied the kinetic equation with generalized M-series of Sharma [7].

Haubold and Mathai [3] have established a functional differential equation between rate of change of reaction, the destruction rate and the production rate as follows

$$\frac{dN}{dt} = -d(N_t) + p(N_t), \tag{1}$$

where $N = N(t)$ the rate of reaction, $d = d(N)$ the rate of destruction, $p = p(N)$ the rate of production and N_t denotes the function define by $N_t(t^*) = N(t - t^*)$, $t^* > 0$.

They have studied a the special case of (1), for spatial fluctuations or inhomogeneities in the quantity $N(t)$ are neglected, namely the equation

$$\frac{dN_i}{dt} = -c_i N_i(t), \tag{2}$$

together with the initial condition that $N_i(t = 0) = N_0$ is the number of density of species i at time $t = 0$, $c_i > 0$. Dropping the index i and integrate the standard kinetic equation (2) we obtain

$$N(t) - N_0 = -c_0 D_t^{-1} N(t), \tag{3}$$

Replacing the Riemann integral operator D_t^{-1} by the fractional Riemann-Liouville operator $D_t^{-\nu}$ [14] in equation (3), we obtain

$$N(t) - N_0 = -c_0 D_t^{-\nu} N(t), \tag{4}$$

Haubold and Mathai in [3] found the solution of (4) as follows

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\nu k + 1)} (ct)^{\nu k}. \tag{5}$$

2 Preliminaries

In this section, we first recall some definitions and fundamental facts of special function and fractional integral operator.

2.1 Aleph function

In the study of generalized fractional driftless Fokker-Planck equation with power law diffusion constant, there arises a special function, which is a particular case of the Aleph function [9, 12]. The idea of Aleph function was first introduced by Südland et al. [8]. In the recent paper of Saxena and Pogány [13] studied the Mathieu-type Series for the Aleph-function.

Definition 2.1. *The Aleph is define as Mellin-Barnes type contour integrals:*

$$\begin{aligned}\aleph[z] :=& \aleph^{m,n}_{p_i,q_i,c_i;r}\left[z \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, & [c_i(a_{ji}, A_{ji})]_{n+1,p_i;r} \\ (b_j, B_j)_{1,m}, & [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right] \\ =& \frac{1}{2\pi\omega} \int_{\mathfrak{L}} \Omega^{m,n}_{p_i,q_i,c_i;r}(\zeta) z^{-\zeta}\, d\zeta \end{aligned} \tag{6}$$

for all $z \neq 0$, *where* $\omega = \sqrt{-1}$ *and*

$$\Omega^{m,n}_{p_i,q_i,c_i;r}(\zeta) = \frac{\prod_{j=1}^{m} \Gamma(b_j + B_j\zeta). \prod_{j=1}^{n} \Gamma(1 - a_j - A_j\zeta)}{\sum_{i=1}^{r} c_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji}\zeta). \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji}\zeta)}. \tag{7}$$

The $\mathfrak{L} = \mathfrak{L}_{\omega\gamma\infty}$ *is a suitable contour of the Mellin-Barnes type which runs from* $\gamma - \omega\infty$ *to* $\gamma + \omega\infty$ *with* $\gamma \in \mathbb{R}$, *the integers* m, n, p, q *satisfy the inequality* $0 \leq n \leq p_i$, $1 \leq m \leq q_i$, $c_i > 0$; $i = 1, \cdots, r$. *The parameters* A_j, B_j, A_{ji}, B_{ji} *are positive real numbers and* a_j, b_j, a_{ji}, b_{ji} *are complex numbers, such that the poles of* $\Gamma(b_j + B_j\zeta)$, $j = 1, \cdots, m$ *separating from those of* $\Gamma(1 - a_j - A_j\zeta)$, $j = 1, \cdots, n$. *All the poles of the integrand* (6) *are assumed to be simple, and empty products are interpreted as unity. The existence conditions [12] for the Aleph function* (6) *are given below:*

$$\varphi_k > 0, \quad |\arg(z)| < \frac{\pi}{2}\varphi_k; \quad k = 1, \cdots, r, \tag{8}$$

$$\varphi_k \geq 0, \quad |\arg(z)| < \frac{\pi}{2}\varphi_k \text{ and } \Re\{\Lambda_k\} + 1 < 0, \tag{9}$$

where

$$\varphi_k = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - c_k \left(\sum_{j=n+1}^{p_k} A_{jk} + \sum_{j=m+1}^{q_k} B_{jk} \right), \tag{10}$$

$$\Lambda_k = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + c_k \left(\sum_{j=1}^{q_k} b_{jk} - \sum_{j=n+1}^{p_k} a_{jk} \right) + \frac{1}{2}(p_k - q_k). \tag{11}$$

For $c_i = 1$, $i = 1, \cdots, r$, in (2.1) the Aleph function coincide with the I-function of Saxena [16, p. 30]. Again for $r = 1$ and $c_1 = 1$ in (2.1) the Aleph function reduces to Fox's H-function. For more details we refer to [12, 9].

2.2 Fractional integral operator

Fractional calculus is a generalization of ordinary differentiation and integration to arbitrary (non-integer) order. The Riemann-Liouville fractional integral is the most useful definition of fractional calculus. The expression of Riemann-Liouville definition will generally arrived from n-fold iterated integral.

Definition 2.2. *The Riemann-Liouville fractional integral operator of order α define by*

$$ {}_0\mathcal{D}_t^{-\alpha}\{f(t)\} = {}_0I_t^{\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^t (x-t)^{\alpha-1} f(t)dt, \, \Re(\alpha) > 0. \tag{12}$$

Lemma 2.3. *The Laplace transform of Riemann-Liouville fractional derivative (see in [5, p. 105]) is given by*

$$\mathcal{L}\left\{{}_0\mathcal{D}_t^{\alpha} f(t); s\right\} = s^{\alpha}F(s) - \sum_{k=0}^{n-1} s^k \left[{}_0\mathcal{D}_t^{\alpha-k-1} f(t)\right]_{t=0}, \, n-1 \leq \alpha < n. \tag{13}$$

3 Generalized Fractional Kinetic Equation

Lemma 3.1. *The Laplace transform of the $\aleph$-function as follows*

$$\begin{aligned} &\mathcal{L}\left\{x^{\lambda-1} \aleph_{p_i,q_i,c_i;r}^{m,n}\left[\eta x^{\rho} \left| \begin{matrix} (a_j, A_j)_{1,n}, & [c_i(a_{ji}, A_{ji})]_{n+1,p_i;r} \\ (b_j, B_j)_{1,m}, & [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right]\right\} \\ &= s^{-\lambda} \aleph_{p_i+1,q_i,c_i;r}^{m,n+1}\left[\frac{\eta}{s^{\rho}} \left| \begin{matrix} (1-\lambda,\rho), (a_j, A_j)_{1,n}, & [c_i(a_{ji}, A_{ji})]_{n+1,p_i;r} \\ (b_j, B_j)_{1,m}, & [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right], \end{aligned} \tag{14}$$

where $\lambda, s, \eta \in \mathbb{C}; \Re(s) > 0, \rho > 0, c_i > 0, i = 1, \cdots, r$ and

$$\Re(\lambda) + \rho \min_{1 \leq j \leq m} \frac{\Re(b_j)}{B_j} > 0, \, |\arg(\eta)| < \frac{\pi}{2} \min_{1 \leq k \leq r} (\Lambda_k),$$

Λ_k defined in (11).

Proof. For convenience, we denote the left side of (14) by $\mathcal{I}$. Using the Definition 2.1, we have

$$\mathcal{I} := \frac{1}{2\pi\omega} \int_0^{\infty} \exp(-st) t^{\lambda-1} \int_{\mathfrak{L}} \Omega_{p_i,q_i,c_i;r}^{m,n}(-\zeta)(\eta x^{\rho})^{\zeta} \, d\zeta dt$$

Changing the order of integration, which is permissible under the stated conditions and applying the formula of Laplace transform we have

$$\mathcal{I} := \frac{s^{-\lambda}}{2\pi\omega}\int_{\mathfrak{L}} \Omega^{m,n}_{p_i,q_i,c_i;r}(-\zeta)\eta^{\zeta}s^{-\rho\zeta}\Gamma(\lambda+\rho\zeta)d\zeta$$

After simple adjustment we finally arrived at (14).

Lemma 3.2. *From the Lemma 3.1 it is clear that*

$$\begin{aligned}&\mathcal{L}^{-1}\left\{s^{-\lambda}\aleph^{m,n}_{p_i,q_i,c_i;r}\left[\eta s^{\rho}\left|\begin{matrix}(a_j,A_j)_{1,n}, & [c_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\ (b_j,B_j)_{1,m}, & [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]\right\}\\ &= t^{\lambda-1}\aleph^{m,n}_{p_i+1,q_i,c_i;r}\left[\frac{\eta}{t^{\rho}}\left|\begin{matrix}(a_j,A_j)_{1,n}, & [c_i(a_{ji},A_{ji})]_{n+1,p_i;r},(\lambda,\rho)\\ (b_j,B_j)_{1,m}, & [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right], \qquad (15)\end{aligned}$$

where $\Re(s)>0, \rho>0, c_i>0, i=1,\cdots,r$ *and*

$$\Re\left(\lambda+\rho\min_{1\le j\le n}\frac{1-a_j}{A_j}\right)>0,\ |\arg(t)|<\frac{\pi}{2}\Lambda_k.$$

Theorem 3.3. *If* $\nu>0, \varepsilon>0, \tau_i>0, c>0, d>0, \lambda>0, \Re(s)>0, c\neq d$ *then the generalized fractional kinetic equation*

$$\begin{aligned}N(t)-N_0t^{\lambda-1}\aleph^{m,n}_{p_i,q_i,\tau_i;r}&\left[dt^{\nu}\left|\begin{matrix}(a_j,A_j)_{1,n}, & [\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\ (b_j,B_j)_{1,m}, & [\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]\\ &= -c\,{}_0D_t^{-\varepsilon}N(t), \qquad (16)\end{aligned}$$

has a solution of the form

$$\begin{aligned}N(t) =&N_0t^{\lambda-1}\sum_{k=0}^{\infty}(-ct^{\varepsilon})^k\\ &\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left[dt^{\nu}\left|\begin{matrix}(1-\lambda,\nu),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(1-\lambda-\varepsilon k,\nu)\end{matrix}\right.\right] \quad (17)\end{aligned}$$

Proof. Taking Laplace transform on the both side of (16) and using Lemma 2.3 and 3.1 and solving for $\mathcal{N}(s)=\mathcal{L}[N(t);s]$ we get

$$\mathcal{N}(s)=\frac{N_0s^{-\lambda}}{1+cs^{-\varepsilon}}\aleph^{m,n+1}_{p_i+1,q_i,\tau_i;r}\left[\frac{d}{s^{\nu}}\left|\begin{matrix}(1-\lambda,\nu),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right] \qquad (18)$$

Taking inverse Laplace transform on both side of (18) and using Lemma 3.2 we get

$$\begin{aligned}N(t) =& N_0t^{\lambda-1}\sum_{k=0}^{\infty}(-ct^{\varepsilon})^k\\ &\aleph^{m,n+1}_{p_i+1,q_i,\tau_i;r}\left[ds^{-\nu}\left|\begin{matrix}(1-\lambda,\nu),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r},(\lambda+\varepsilon k,-\nu)\\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]\end{aligned}$$

After little arrangement we finally arrived at the desired result (17).

3.1 Special case

When $\tau_i = 1$, $i = 1, \cdots, r$, then we arrive the result

Corollary 3.4. *If $\nu > 0, \varepsilon > 0, d > 0, \lambda > 0, \Re(s) > 0, c \neq d$ then the solution of integral equation*

$$N(t) - N_0 t^{\lambda-1} \mathbf{I}^{m,n}_{p_i,q_i+1;r}\left[dt^\nu \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,m}, (b_{ji}, B_{ji})_{m+1,q_i}, (1-\lambda, \nu) \end{matrix}\right] = -c\, {}_0D_t^{-\varepsilon} N(t), \tag{19}$$

there holds the formula

$$N(t) = N_0 t^{\lambda-1} \sum_{k=0}^{\infty} (-ct^\varepsilon)^k \mathbf{I}^{m,n}_{p_i,q_i+1;r}\left[dt^\nu \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, & (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,m}, & (b_{ji}, B_{ji})_{m+1,q_i}, (1-\lambda-\varepsilon k, \nu) \end{matrix}\right], \tag{20}$$

For the existence conditions of I-function we refer to the book of Saxena [16].

When $r = 1$ and $\tau_1 = 1, p_1 = p, q_1 = q$, then we arrive the result

Corollary 3.5. *If $\nu > 0, \varepsilon > 0, d > 0, \lambda > 0, \Re(s) > 0, c \neq d$ then the solution of integral equation*

$$N(t) - N_0 t^{\lambda-1} \mathbf{H}^{m,n}_{p,q+1}\left[dt^\nu \,\middle|\, \begin{matrix} (a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q}, (1-\lambda, \nu) \end{matrix}\right] = -c\, {}_0D_t^{-\varepsilon} N(t), \tag{21}$$

there holds the formula

$$N(t) = N_0 t^{\lambda-1} \sum_{k=0}^{\infty} (-ct^\varepsilon)^k \mathbf{H}^{m,n}_{p,q+1}\left[dt^\nu \,\middle|\, \begin{matrix} (a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q}, (1-\lambda-\varepsilon k, \nu) \end{matrix}\right], \tag{22}$$

For the existence conditions of H-function we refer to the book of Srivastava et al. [4].

When $r = 1$ and $\tau_1 = 1, p_1 = p, q_1 = q$ and replacing c, d by c^ν, d^ν respectively, then we arrive the result

Corollary 3.6. *If $\nu > 0, \varepsilon > 0, c > 0, d > 0, \lambda > 0, \Re(s) > 0, c \neq d$ then the solution of integral equation*

$$N(t) - N_0 t^{\lambda-1} {}_p\overset{\nu,\lambda}{M}_q(a_1, \cdots, a_p; b_1, \cdots, b_q, -d^\nu t^\nu) = -c^\nu {}_0D_t^{-\nu} N(t), \tag{23}$$

there holds the formula

$$N(t) = N_0 t^{\lambda-1} \sum_{r=0}^{\infty} (-1)^r (ct)^{\nu r} {}_p\overset{\nu,\lambda+r\nu}{M}_q(a_1, \cdots, a_p; b_1, \cdots, b_q, -d^\nu t^\nu) \tag{24}$$

Which is the main result obtained by the Chaurasia and Kumar [15].

When $r = 1$, $p_i = q_i = 0$, $\tau_1 = 1$, $\nu = \varepsilon$, and replacing c, d by c^ν, d^ν respectively, then we arrive the results

Corollary 3.7. *If $\nu > 0, d > 0, c > 0, \lambda > 0, \Re(s) > 0, c \neq d$ then the solution of integral equation*

$$N(t) - N_0 t^{\lambda-1} E_{\nu,\lambda}(-d^\nu t^\nu) = -c^\nu {}_0D_t^{-\nu} N(t), \tag{25}$$

there holds the formula

$$N(t) = N_0 \frac{t^{\lambda-\nu-1}}{c^\nu - d^\nu} \left[E_{\nu,\lambda-\nu}(-d^\nu t^\nu) - E_{\nu,\lambda-\nu}(-c^\nu t^\nu)\right] \tag{26}$$

Which is the result obtained by the Saxena et al. [10].

Corollary 3.8. *If $\nu > 0, c > 0, \lambda > 0, \Re(s) > 0$ then the solution of integral equation*

$$N(t) - N_0 t^{\lambda-1} E_{\nu,\lambda}(-c^\nu t^\nu) = -c^\nu {}_0D_t^{-\nu} N(t), \tag{27}$$

there holds the formula

$$N(t) = N_0 \frac{t^{\lambda-1}}{\nu} \left[E_{\nu,\lambda-1}(-c^\nu t^\nu) + (1 + \nu - \lambda) E_{\nu,\lambda}(-c^\nu t^\nu)\right] \tag{28}$$

Which is the result obtained by the Saxena et al. [10].

When $r = 1$, $p_i = q_i = 0$, $\tau_1 = 1$, $\nu = \varepsilon$, and replacing c, d by c^ν, d^ν respectively, then we arrive the result

Corollary 3.9. *If $\nu > 0, c > 0, \lambda > 0, \Re(s) > 0$ then the solution of integral equation*

$$N(t) - N_0 t^{\lambda-1} E^{\gamma}_{\nu,\lambda}(-c^\nu t^\nu) = -c^\nu {}_0D_t^{-\nu} N(t), \tag{29}$$

there holds the formula

$$N(t) = N_0 t^{\lambda-1} \left[E^{\gamma+1}_{\nu,\lambda}(-c^\nu t^\nu)\right]. \tag{30}$$

Which is the result obtained by the Saxena et al. [11].

4 Conclusion

Aleph function is general in nature and includes a number of known and new results as particular cases. This extended fractional kinetic equation can be used to compute the particle reaction rate and may be utilized in other branch of mathematics. Results obtained in this paper provide an extension of [3, 10, 11, 15].

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