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Intuitionistic Fuzzy Sets in Ordered Γ-Semigroups

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Abstract

We consider the intuitionistic fuzzification of the concept of several ideal in an ordered Γ*-semigroup, and investigate some properties of such ideals.*

Keywords: *Ordered* Γ*-semigroup, inituitionistic fuzzy* Γ*-subsemi group, inituitionistic left (rest. right) ideal, inituitionistic fuzzy interior ideal, inituitionistic fuzzy left (resp. right) simple.*

1 Introduction

The concept of a fuzzy set given by L.A. Zadeh in his clasis paper of 1965 [11] has been used by many authors to generalize some of the basic notions of algebra. Fuzzy semigroups have been first considered by N. Kuroki [5], and fuzzy ordered groupoids and ordered semigrous, by Kehayopulu and Tsingelis [7]. The notion of a Γ-semigroup was introduced by Sen [9]. Many classical notions of semigroups have been extended to Γ-semigroups. The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [10]. In [4], N. Kuroki gave some properties of fuzzy ideals and fuzzy semiprime ideals in semigroups [6]. In [1], K. H. Kim gave some properties of several ideals in an ordered semigroup. In this paper, we consider the intuitionistic fuzzification of the concept of several ideals in an ordered Γ-semigroup, and investigate some properties of such ideal.

2 Preliminaries

We include some elementary aspects of ordered Γ-semigroups that are necessary for this paper.

Definition 2.1 *Let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if it satisfies*

(i) $x\gamma y \in S$,

(ii) $(x\beta y)\gamma z = x\beta(y\gamma z)$,

for all $x, y, z \in S$ *and* $\beta, \gamma \in \Gamma$.

Definition 2.2 *Let S be a Γ-semigroup and $(S, \leq)$ a partially ordered set. Then S is called an ordered Γ-semigroup if $x \leq y$ implies $a\gamma z \leq b\gamma z$ and $z\gamma a \leq z\gamma b$, for all $x, y, z \in S$ and $\gamma \in \Gamma$.*

Definition 2.3 *Let S be an ordered Γ-semigroup. A non-empty subset A of an ordered Γ-semigroup S is said to be a Γ-subsemigroup of S if $A\Gamma A \subseteq A$.*

Let S be an ordered Γ-semigroup. For $A \subseteq S$, we denote

$$(A] := \{t \in S \mid t \leq h \text{ for some } h \in A\}.$$

For $A, B \subseteq S$, we denote

$$A\Gamma B := \{a\gamma b \mid a \in A, b \in B, \gamma \in \Gamma\}.$$

Definition 2.4 *Let S be an ordered Γ-semigroup. A non-empty subset A of S is called a left ideal of S if it satisfies*

(i) $S\Gamma A \subseteq A$.

(ii) For any $b \in S$ and $a \in A$ such that $b \leq a$ implies $b \in A$.

Definition 2.5 *Let S be an ordered Γ-semigroup. A non-empty subset A of S is called a right ideal of S if it satisfies*

(i) $A\Gamma S \subseteq A$.

(ii) For any $b \in S$ and $a \in A$ such that $b \leq a$ implies $b \in A$.

Definition 2.6 *Let S be an ordered Γ-semigroup. A non-empty subset A of S is called an ideal of S if it satisfies*

(i) $S\Gamma A \subseteq A$.

(ii) $A\Gamma S \subseteq A$.

(iii) *For any* $b \in S$ *and* $a \in A$ *such that* $b \leq a$ *implies* $b \in A$.

Definition 2.7 *Let* S *be an ordered* Γ*-semigroup. A non-empty subset* A *of* S *is called a bi-ideal of* S *if it satisfies*

(i) $A\Gamma S\Gamma A \subseteq A$.

(ii) *For any* $b \in S$ *and* $a \in A$ *such that* $b \leq a$ *implies* $b \in A$.

Definition 2.8 *Let* S *be an ordered* Γ*-semigroup. A* Γ*-subsemigroup* A *of* S *is called an interior ideal of* S *if it satisfies*

(i) $S\Gamma A\Gamma S \subseteq A$.

(ii) *For any* $b \in S$ *and* $a \in A$ *such that* $b \leq a$ *implies* $b \in A$.

An ordered Γ-semigroup S is called *left-zero* (resp. *right-zero*) if $x \leq x\alpha y$ (resp. $y \leq x\alpha y$) for all $x, y \in S$ and $\alpha \in \Gamma$. An ordered Γ-semigroup S is said to be *left* (resp. *right*) simple if for every left (resp. right) ideal A of S, we have $A = S$. An ordered Γ-semigroup S is said to be *regular* if for every $a \in S$ there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$. $L[x]$ denote the principal left ideal of a Γ-semigroup S generated by x in S, that is, $L[x] = (x \cup S\Gamma x]$. By a *fuzzy set* μ in a non-empty set X, we mean a function $\mu : X \to [0,1]$ and the *complement* of μ, denoted by $\mu^{'}$, is the fuzzy set in X given by $\mu^{'}(x) := 1 - \mu(x)$ for all $x \in X$. For any fuzzy subset μ in S and $t \in [0,1]$, we define

$$U(\mu; f) := \{x \in S \mid \mu(x) \geq t\},$$

which is called an *upper t-level cut* of μ and can be used to the characterization of μ.

An intuitionistic fuzzy set (briefly, IFS) A in a non-empty set X is an object having the form

$$A := \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$$

where the function $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$$

for all $x \in X$. For the sake of simplicity, we shall use the symbol $A := (\mu_A, \gamma_A)$ for the IFS $A := \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$.

Let χ_U denote the characteristic function of a non-empty subset U of an ordered Γ-semigroup.

Definition 2.9 *Let S be an ordered Γ-semigroup. A fuzzy set μ is called a fuzzy Γ-subsemigroup of S if*

$$\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$$

for all $x, y \in S$ and $\gamma \in \Gamma$.

Definition 2.10 *Let S be an ordered Γ-semigroup. A fuzzy Γ-subsemigroup μ of S is called a fuzzy bi-ideal of S, if the following axioms are satisfied:*

(1) If $x \leq y$, then $\mu(x) \geq \mu(y)$, for all $x, y \in S$,

(2) $\mu(x\alpha a\beta y) \geq \min\{\mu(x), \mu(y)\}$, for all $a, x, y \in S$ and $\alpha, \beta \in \Gamma$.

3 Main Results

In what follows, we use S to denote an ordered Γ-semigroup unless otherwise specified.

Definition 3.1 *For an IFS $A = (\mu_A, \gamma_A)$ in S, consider the following axioms:*

(ΓIS_1) $\mu_A(x\alpha y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ΓIS_2) $\gamma_A(x\alpha y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$, for all $x, y \in S$ and $\alpha \in \Gamma$.

*Then $A = (\mu_A, \gamma_A)$ is called a first (*resp. *second) intuitionistic fuzzy Γ-subsemigroup (briefly, $IF\Gamma SS_1$ (resp. $IF\Gamma SS_2$)) of S if satisfies (ΓIS_1) (resp. ($\Gamma\Gamma IS_2$)). Also, $A = (\mu_A, \gamma_A)$ is said to be an intuitionistic fuzzy Γ-semigroup (briefly, $IF\Gamma SS$) of S if it is both a first and a second intuitionistic fuzzy Γ-semigroup.*

Theorem 3.2 *If U is a Γ-subsemigroup of ordered Γ-semigroup S, then $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma SS$ of S.*

Let $x, y \in S$ and $\alpha \in \Gamma$. From the hypothesis, $x\alpha y \in U$ if $x, y \in U$. In this case,

$$\chi_U(x\alpha y) = 1 \geq \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\begin{aligned} \chi'_U(x\alpha y) &= 1 - \chi_U(x\alpha y) \\ &\leq 1 - \min\{\chi_U(x), \chi_U(y)\} \\ &= \max\{1 - \mu_U(x), 1 - \mu_U(y)\} \\ &= \max\{\mu'_U(x), \mu'_U(y)\}. \end{aligned}$$

If $x \notin U$ or $y \notin U$, then $\chi_U(x) = 0$ or $\chi_U(y) = 0$. Thus $\min\{\chi_U(x), \chi_U(y)\} = 0$, which is implies that

$$\chi_U(x\alpha y) \geq 0 = \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\begin{array}{rcl}\chi'_U(x\alpha y) & \leq & 1 \\ & = & 1-\min\{\chi_U(x),\chi_U(y)\} \\ & = & \max\{1-\chi_U(x),1-\chi_U(y)\} \\ & = & \max\{\chi'_U(x),\chi'_U(y)\}.\end{array}$$

This completes the proof.

Theorem 3.3 *Let U be a non-empty subset of ordered Γ-semigroup S. If $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma SS_1$ or $IF\Gamma SS_2$ of S, then U is a Γ-subsemigroup of S.*

Suppose that $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma SS_1$ of S and $x \in U\Gamma U$. In this case $x = u\alpha v$ for some $u, v \in U$ and $\alpha \in \Gamma$. It follows from (ΓIS_1) that

$$\chi_U(x) = \chi_U(u\alpha v) \geq \min\{\chi_U(u), \chi_U(v)\} = 1.$$

Hence $\chi_U(x) = 1$, that is, $x \in U$. Thus U is a Γ-subsemigroup of S. Now, assume that $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma SS_2$ of S and $x' \in U\Gamma U$. Then $x' = u'\alpha' v'$ for some $u', v' \in U$ and $\alpha' \in \Gamma$. Using (ΓIS_2), we get that

$$\begin{array}{rcl}\chi'_U(x') & = & \chi'_U(u'\alpha' v') \\ & \leq & \max\{\chi'_U(u'),\chi'_U(v')\} \\ & = & \max\{1-\chi_U(u'),1-\chi_U(v')\} \\ & = & 0,\end{array}$$

and so $1-\chi_U(x') = \chi'_U(x') = 0$, which implies that $\chi'_U(x') = 1$, i.e. $x' \in U$. Thus U is a Γ-subsemigroup of S. This completes the proof.

Definition 3.4 *For an IFS $A = (\mu_A, \gamma_A)$ in S, consider the following axioms:*

(ΓIL_1) *$x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\mu_A(x\alpha y) \geq \mu_A(y)$,*

(ΓIL_2) *$x \leq y$ implies $\gamma_A(x) \leq \gamma_A(y)$ and $\gamma_A(x\alpha y) \leq \gamma_A(y)$, for all $x, y \in S$ and $\alpha \in \Gamma$.*

Then $A = (\mu_A, \gamma_A)$ is called a first (resp.second) intuitionistic fuzzy left ideal (briefly, $IF\Gamma LI_1$ (resp. $IF\Gamma LI_2$)) of S if it satisfies (ΓIL_1) (resp. (ΓIL_2)). Also, $A = (\mu_A, \gamma_A)$ is said to be an intuitionistic fuzzy left ideal (briefly, $IF\Gamma LI$) of S if it is both a first and a second intuitionistic fuzzy left ideal.

Definition 3.5 *For an IFS $A = (\mu_A, \gamma_A)$ in S, consider the following axioms:*

(ΓIR_1) *$x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\mu_A(x\alpha y) \geq \mu_A(x)$,*

(ΓIR_2) *$x \leq y$ implies $\gamma_A(x) \leq \gamma_A(y)$ and $\gamma_A(x\alpha y) \leq \gamma_A(x)$, for all $x, y \in S$ and $\alpha \in \Gamma$.*

Then $A = (\mu_A, \gamma_A)$ is called a first (resp.second) intuitionistic fuzzy right ideal (briefly, $IF\Gamma RI_1$ (resp. $IF\Gamma RI_2$)) of S if it satisfies (ΓIR_1) (resp. (ΓIR_2)). Also, $A = (\mu_A, \gamma_A)$ is said to be an intuitionistic fuzzy right ideal (briefly, $IF\Gamma RI$) of S if it is both a first and a second intuitionistic fuzzy right ideal.

Definition 3.6 *Let $A = (\mu_A, \gamma_A)$ be an IFS in S. Then A is called an intuitionistic fuzzy ideal of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal.*

Let U be a left-zero Γ-subsemigroup of S. If $A = (\mu_A, \gamma_A)$ is an $IF\Gamma LI$ of S. Then the restriction of A to U is constant, that is, $A(x) = A(y)$ for all $x, y \in S$.

Let $x, y \in S$ and $\alpha \in \Gamma$. Since U is a left-zero of Γ-subsemigroup of S, we have $x \leq x\alpha y$ and $y \leq y\alpha x$. In this case, from the hypothesis, we have

$$\mu_A(x) \geq \mu_A(x\alpha y) \geq \mu_A(y), \quad \mu_A(y) \geq \mu_A(y\alpha x) \geq \mu_A(x)$$

and

$$\gamma_A(x) \leq \gamma_A(x\alpha y) \leq \gamma_A(y), \quad \gamma_A(y) \leq \gamma_A(y\alpha x) \leq \gamma_A(x).$$

Thus we obtain $\mu_A(x) = \mu_A(y)$ and $\gamma_A(x) = \gamma_A(y)$ for all $x, y \in U$. Hence $A(x) = A(y)$.

Lemma 3.7 *If U is a left ideal of S, then $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma LI$ of S.*

Let $x, y \in S$ and $\alpha \in \Gamma$ be such that $x \leq y$. Since U is a left ideal of S, we have $x \in U$ and $x\alpha y \in U$ if $y \in U$. It follows that $x \leq y$ implies $\chi_U(x) = 1 = \chi_U(y)$ and

$$\begin{aligned} \chi'_U(x) &= 1 - \chi_U(x) \\ &= 0 \\ &= 1 - \chi_U(y) \\ &= \chi'_U(y). \end{aligned}$$

Also, we have $\chi_U(x\alpha y) = 1 = \chi_U(y)$ and

$$\begin{aligned} \chi'_U(x\alpha y) &= 1 - \chi_U(x\alpha y) \\ &= 0 \\ &= 1 - \chi_U(y) \\ &= \chi'_U(y). \end{aligned}$$

If $y \notin U$, then $\chi_U(y) = 0$. In this case, $x \leq y$ implies $\chi_U(x) \geq 0 = \chi_U(y)$ and $\chi'_U(x) \leq \chi'_U(y) = 1 - \chi_U(y) = 1$. Also, we obtain $\chi_U(x\alpha y) \geq 0 = \chi_U(y)$ and $\chi'_U(y) = 1 - \chi_U(y) = 1 \geq \chi'_U(x\alpha y)$. Consequently, $U' = (\chi, \chi'_U)$ is an $IF\Gamma LI$ of S.

An element e in an ordered Γ-semigroup S is called an *idempotent* it $e\alpha e \geq e$, for all $\alpha \in \Gamma$. Let E_S denote the set of all idempotents in an ordered Γ-semigroup S.

Theorem 3.8 *Let $A = (\mu_A, \gamma_A)$ be an $IF\Gamma LI$ of S. If E_S is a left-zero Γ-subsemigroup of S, then $A(e) = A(e')$ for all $e, e' \in E_S$.*

Let $e, e^{'} \in E_S$. From the hypothesis, $e\alpha e^{'} \geq e$ and $e^{'}\beta e \geq e^{'}$ for all $\alpha, \beta \in \Gamma$. Thus, since $A = (\mu_A, \gamma_A)$ is an $IF\Gamma LI$ of S, we get that

$$\mu_A(e) \geq \mu_A(e\alpha e^{'}) \geq \mu_A(e^{'}),\ \mu_A(e^{'}) \geq \mu_A(e^{'}\beta e) \geq \mu_A(e)$$

and

$$\gamma_A(e) \leq \gamma_A(e\alpha e^{'}) \leq \gamma_A(e^{'}),\ \gamma_A(e^{'}) \leq \gamma_A(e^{'}\beta e) \leq \gamma_A(e).$$

Hence we have $\mu_A(e) = \mu_A(e^{'})$ and $\gamma_A(e) = \gamma_A(e^{'})$ for all $e, e^{'} \in E_S$. This completes the proof.

Definition 3.9 *Let S be an ordered Γ-semigroup. A fuzzy Γ-subsemigroup μ of S is called a fuzzy interior ideal of S, if the following axioms are satisfied:*
(1) $\mu(x\alpha a\beta y) \geq \mu(a)$,
(2) If $x \leq y$, then $\mu(x) \geq \mu(y)$ for all $a, x, y \in S$ and $\alpha, \beta \in \Gamma$.

Definition 3.10 *For an IFS $A = (\mu_A, \gamma_A)$ in S, consider the following axioms:*
(ΓII_1) *$x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\mu_A(x\alpha s\beta y) \geq \mu_A(s)$,*
(ΓII_2) *$x \leq y$ implies $\gamma_A(x) \leq \gamma_A(y)$ and $\gamma_A(x\alpha s\beta y) \leq \gamma_A(s)$ for all $s, x, y \in S$*
and $\alpha, \beta \in \Gamma$.
Then $A = (\mu_A, \gamma_A)$ is called a first (resp. second) intuitionistic fuzzy interior ideal (briefly, $IF\Gamma II_1$ (resp. $IF\Gamma II_2$)) of S if it is an $IF\Gamma S_1$ (resp. $IF\Gamma S_2$) satisfying (ΓII_1) *(resp.* (ΓII_2)*). Also, $A = (\mu_A, \gamma_A)$ is said to be an intuitionistic fuzzy interior ideal (briefly, $IF\Gamma II$) of S if it is both a first and a second intuitionistic fuzzy interior ideal of S.*

Theorem 3.11 *If S is regular, then every $IF\Gamma II$ of S is an $IF\Gamma I$ of S.*
Let $A = (\mu_A, \gamma_A)$ be an $IF\Gamma II$ of S and $x, y \in S$. In this case, because S is regular, there exist $s, s^{'} \in S$ and $\alpha, \beta, \alpha^{'}, \beta^{'} \in \Gamma$ such that $x \leq x\alpha s\beta x$ and $y \leq y\alpha^{'}s^{'}\beta^{'}y$. Thus

$$\begin{aligned}\mu_A(xy) &\geq \mu_A(x\gamma^{'}y\alpha^{'}s^{'}\beta^{'}y)\\ &= \mu_A(x\gamma^{'}y\alpha^{'}(s^{'}\beta^{'}y))\\ &\geq \mu_A(y).\end{aligned}$$

and

$$\begin{aligned}\gamma_A(xy) &\leq \gamma_A(x\gamma^{'}y\alpha^{'}s^{'}\beta^{'}y)\\ &= \gamma_A(x\gamma^{'}y\alpha^{'}(s^{'}\beta^{'}y))\\ &\leq \gamma_A(y),\end{aligned}$$

for some $\gamma^{'} \in \Gamma$. It follows that $A = (\mu_A, \gamma_A)$ is an $IF\Gamma LI$ of S. Similarly, we can show that $A = (\mu_A, \gamma_A)$ is an $IF\Gamma RI$ of S. This completes the proof.

Theorem 3.12 *If U is an interior ideal of S, then $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma II$ of S.*

Since U is a Γ-subsemigroup of S, we have $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma SS$ of S by Theorem 3.2. Let $x, y \in S$ be such that $x \leq y$. Then we have $x \in U$ if $y \in U$. Thus $x \leq y$ implies $\chi_U(x) = 1 = \chi_U(y)$ and

$$\begin{aligned} \chi_U^{'}(x) &= 1 - \chi_U(x) \\ &= 0 \\ &= 1 - \chi_U(y) \\ &= \chi_U^{'}(y). \end{aligned}$$

If $y \notin U$, then $\chi_U(x) \geq 0 = \chi_U(y)$ and $\chi_U^{'}(x) \leq \chi_U^{'}(y) = 1 - \chi_U(y) = 1$. Now, let $s, x, y \in S$ and $\alpha, \beta \in \Gamma$. From the hypothesis, $x\alpha s\beta y \in U$ if $s \in U$. In this case, $\chi_U(x\alpha s\beta y) = 1 = \chi_U(s)$ and

$$\begin{aligned} \chi_U^{'}(x\alpha s\beta y) &= 1 - \chi_U(x\alpha s\beta y) \\ &= 0 \\ &= 1 - \chi_U(s) \\ &= \chi_U^{'}(s). \end{aligned}$$

If $s \notin U$, then $\chi_U(s) = 0$. Thus $\chi(x\alpha s\beta y) \geq 0 = \chi_U(s)$ and

$$\begin{aligned} \chi_U^{'}(s) &= 1 - \chi_U(s) \\ &= 1 \\ &\geq \chi_U^{'}(x\alpha s\beta y). \end{aligned}$$

Consequently, $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma II$ of S.

Theorem 3.13 *Let S be regular and U a non-empty subset of S. If $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma II_1$ or $IF\Gamma II_2$ of S, then U is an interior ideal of S.*

It is clear that U is a Γ-subsemigroup of S be Theorem 3.3. Suppose that $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma II_1$ of S and $x \in S\Gamma U\Gamma S$. In this case, $x = s\alpha u\beta t$ for some $s, t \in S$, $u \in U$ and $\alpha, \beta \in \Gamma$. It follows from (ΓII_1) that

$$\chi_U(x) = \chi_U(s\alpha u\beta t) \geq \chi_U(u) = 1.$$

Hence $\chi_U(x) = 1$, i.e. $x \in U$. Let $x \leq y$ and $y \in U$. Then

$$\chi_U(x) \geq \chi_U(y) = 1.$$

Hence $\chi_U(x) = 1$, i.e. $x \in U$. Thus U is an interior ideal of S. Now, assume that $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma II_2$ of S and $x^{'} = s^{'}\alpha^{'}u^{'}\beta^{'}t^{'}$ for some $s, t^{'} \in S, u^{'} \in U$ and $\alpha^{'}, \beta^{'} \in \Gamma$. Using (ΓII_2), we obtain

$$\begin{aligned} \chi_U^{'}(x^{'}) &= \chi_U^{'}(s^{'}\alpha^{'}u^{'}\beta^{'}t^{'}) \\ &\leq \chi_U^{'}(u^{'}) \\ &= 1 - \chi_U(u^{'}) \\ &= 0, \end{aligned}$$

and so $\chi_U^{'}(x^{'}) = 1 - \chi_U(x^{'}) = 0$. Therefore, $\chi_U(x^{'}) = 1$, i.e. $x^{'} \in U$. Also, let $x, y \in S$ be such that $x \leq y$ and $y \in U$. Then we have $\chi_U^{'}(x) \leq \chi_U^{'}(y)$, i.e. $1 - \chi_U(x) \leq 1 - \chi_U(y)$. Thus $\chi_U(x) \geq \chi_U(y)$, i.e. $\chi_U(x) = 1$, and so $x \in U$. This completes the proof.

Definition 3.14 *S is called first (resp. second) intuitionistic fuzzy left simple if $IF\Gamma LI_1$ (resp. $IF\Gamma LI_2$) of S is constant. Also, S is said to be intuitionistic fuzzy left simple if is both first and second intuitionistic fuzzy left simple, i.e. every $IF\Gamma LI$ of S is constant.*

Lemma 3.15 *An ordered Γ-semigroup S is left (resp. right) simple if and only if $(S\Gamma a] = S$ (resp. $(a\Gamma S] = S$) for every $a \in S$.*

Theorem 3.16 *If S is left simple, then S is intuitionistic fuzzy left simple.*

Let $A = (\mu_A, \gamma_A)$ be an $IF\Gamma LI$ of S and $x, x^{'} \in S$. In this case, because S is left simple, there exist $s, s^{'} \in S$ and $\alpha, \beta \in \Gamma$ such that $x \leq s\alpha x^{'}$ and $x^{'} \leq s^{'}\beta x$. Thus, since $A = (\mu_A, \gamma_A)$ is an $IF\Gamma LI$ of S, we get that

$$\mu_A(x) \geq \mu_A(s\alpha x^{'}) \geq \mu_A(x^{'}),\ \mu_A(x^{'}) \geq \mu_A(s^{'}\beta x) \geq \mu_A(x)$$

and

$$\gamma_A(x) \leq \gamma_A(s\alpha x^{'}) \leq \gamma_A(x^{'}),\ \gamma_A(x^{'}) \leq \gamma_A(s^{'}\beta x) \leq \gamma_A(x).$$

Hence we have $\mu_A(x) = \mu_A(x^{'})$ and $\gamma_A(x) = \gamma_A(x^{'})$ for all $x, x^{'} \in S$, that is, $A(x) = A(x^{'})$ for all $x, x^{'} \in S$. Consequently, S is intuitionistic fuzzy left simple. This completes the proof.

Theorem 3.17 *If S is first or second intuitionistic fuzzy left simple, then S is left simple.*

Let U be a left ideal of S. Suppose that S is first (or second) intuitionistic fuzzy left simple. Because $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma LI$ of S by Lemma 3.8, $U^{'} = (\chi_U, \chi_U^{'})$ is an $IF\Gamma LI_1$ (and $IF\Gamma LI_2$) of S. From the hypothesis, χ_U (and $\chi_U^{'}$) is constant. Since U is non-empty, it follows that $\chi_U = 1$ (or $\chi_U^{'} = 0$), where 1 and 0 are fuzzy sets in S defined by $1(x) = 1$ and $0(x) = 0$ for all $x \in S$, respectively. Thus $x \in U$ for all $x \in S$. This completes the proof.

Lemma 3.18 *An ordered Γ-semigroup S is simple if and only if for every $a \in S$, we have $S = (S\Gamma a\Gamma S]$.*

Theorem 3.19 *If S is simple, then every $IF\Gamma II$ of S is constant.*

Let $A = (\mu_A, \gamma_A)$ be an $IF\Gamma II$ of S and $x, x^{'} \in S$. In this case, because S is simple, there exist $s, s^{'}, t, t^{'} \in S$ and $\alpha, \beta, \alpha^{'}, \beta^{'} \in \Gamma$ such that $x \leq s\alpha x^{'}\beta t$ and $x^{'} \leq s^{'}\alpha^{'}x\beta^{'}t^{'}$. Thus, since $A = (\mu_A, \gamma_A)$ is an $IF\Gamma II$ of S, we obtain that

$$\mu_A(x) \geq \mu_A(s\alpha x^{'}\beta t) \geq \mu_A(x^{'}),\ \mu_A(x^{'}) \geq \mu_A(s^{'}\alpha^{'}x\beta^{'}t) \geq \mu_A(x)$$

and

$$\gamma_A(x) \leq \gamma_A(s\alpha x^{'}\beta t) \leq \gamma_A(x^{'}),\ \gamma_A(x^{'}) \leq \gamma_A(s^{'}\alpha^{'}x\beta^{'}t) \leq \gamma_A(x).$$

Hence we get $\mu_A(x) = \mu_A(x^{'})$ and $\gamma_A(x) = \gamma_A(x^{'})$ for all $x, x^{'} \in S$. consequently, $A = (\mu_A, \gamma_A)$ is constant.

Definition 3.20 *For an IFS $A = (\mu_A, \gamma_A)$ in S, consider the following axioms:*

(ΓIB_1) $x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\mu_A(x\alpha s\beta y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ΓIB_2) $x \leq y$ implies $\gamma_A(x) \leq \gamma_A(y)$ and $\gamma_A(x\alpha s\beta y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $s, x, y \in S$ and $\alpha, \beta \in \Gamma$. Then $A = (\mu_A, \gamma_A)$ is called an intuitionistic fuzzy bi-ideal (briefly, $IF\Gamma B$) of S if it satisfies (ΓIB_1) and (ΓIB_2).

Theorem 3.21 *If S is left simple, then every $IF\Gamma B$ of S is an $IF\Gamma RI$ of S.* Let $A = (\mu_A, \gamma_A)$ be an $IF\Gamma B$ of S and $x, y \in S$. In this case, from the hypothesis, there exist $s \in S$ and $\alpha, \beta \in \Gamma$ such that $y \leq s\alpha x$. Thus, because $A = (\mu_A, \gamma_A)$ is an $IF\Gamma B$ of S. we have that

$$\mu_A(x\beta y) \geq \mu_A(x\beta s\alpha x) \geq \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$$

and

$$\gamma_A(x\beta y) \leq \gamma_A(x\beta s\alpha x) \leq \max\{\gamma_A(x), \gamma_A(x)\} = \gamma_A(x).$$

It follows that $A = (\mu_A, \gamma_A)$ is an $IF\Gamma RI$ of S.

Theorem 3.22 *If U is a bi-ideal of S, then $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma B$ of S.*

Since U is a Γ-subsemigroup of S, we obtain that $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma S$ of S by Theorem 3.2. Let $x, y \in S$ be such that $x \leq y$ and $y \in U$. Then $x \in U$, and so $\chi_U(x) = 1 = \chi_U(y)$ and $\chi'_U(x) = 1 - \chi_U(x) = 0 = 1 - \chi_U(y) = \chi'_U(y)$. Let $s, x, y \in S$ and $\alpha, \beta \in \Gamma$. From the hypothesis, $x\alpha s\beta y \in U$ if $x, y \in U$. In this case,

$$\chi_U(x\alpha s\beta y) = 1 = \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\chi'_U(x\alpha s\beta y) = 1 - \chi_U(x\alpha s\beta y) = 0 = \max\{\chi'_U(x), \chi'_U(y)\}.$$

If $x \notin U$ or $y \notin U$, then $\chi_U(x) = 0$ or $\chi_U(y) = 0$. Thus

$$\chi_U(x\alpha s\beta y) \geq 0 = \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\begin{aligned} \max\{\chi'_U(x), \chi'_U(y)\} &= \max\{1 - \chi_U(x), 1 - \chi_U(y)\} \\ &= 1 - \min\{\chi_U(x), \chi_U(y)\} \\ &= 1 \\ &\geq \chi'_U(x\alpha s\beta y). \end{aligned}$$

Consequently, $U' = (\chi_U, \chi'_U)$ is an $IF\Gamma B$ of S.

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