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A Common Fixed Point Theorem for Sub Compatibility and Occasionally Weak Compatibility in Intuitionistic Fuzzy Metric Spaces

Krishnapal Singh Sisodia[1], Deepak Singh[2] and M.S. Rathore[3]

[1]Department of Mathematics, SISTec-E, Ratibad, Bhopal
E-mail: sisodiakps@gmail.com
[2]NITTTR, Bhopal, Ministry of HRD, Govt. of India
E-mail: dk.singh1002@gmail.com
[3]C.S.A. Government P.G College, Sehore
E-mail: dr.rathore11@gmail.com

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Abstract

In this paper, we establish a common fixed point theorem for six maps using concept of subcompatibility and occasionally weak compatibility in Intuitionistic Fuzzy metric space. S. kutukcu [10] obtained a fixed point theorem for Menger spaces; we obtain its Intuitionistic Fuzzy metric space version with more generalized conditions relaxing completeness criteria. We also justify our findings with an example.

Keywords: *Intuitionistic fuzzy metric space, Subcompatible mapping, Occasionally weakly compatible mapping, Common fixed point.*

1 Introduction

The concept of fuzzy sets was introduced initially by Zadeh [17] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Atanassov [4] Introduced and studied the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [2] proved the well known fixed point theorems of Banach [5] in the setting of intuitionisitc fuzzy metric spaces. Later on, Turkoglu et al. [16] Proved Jungcks [8] common fixed point theorem in the setting of intuitionisitc fuzzy metric space. Turkoglu et al. [16] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of pants theorem [12]. Gregori et al. [7], Saadati and Park [13] studied the concept of intuitionistic fuzzy metric space and its applications.

Recently, Saurabh Manro et al. [11] introduced the notion of subcompatibility and subsequential continuity in Intuitionistic Fuzzy metric space and proved some result for four self maps. Inspired by the result of Saurabh Manro et al. [11], in this paper we prove a common fixed point theorem for six self maps which is a generalization of [10].

2 Preliminaries

Definition 2.1[14] *A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm, if * is satisfying the following conditions:*

(i) $*$ is commutative and associative
(ii) $*$ is continuous
(iii) $a * 1 = a$ for all $a \in [0,1]$
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for $a, b, c, d \in [0,1]$.

Definition 2.2[14] *A binary operation* $\diamondsuit: [0,1] \times [0,1] \rightarrow [0,1]$ *is a continuous* $t-conorm$ *if* $\diamondsuit$ *it satisfies the following conditions:*

(i) $\diamondsuit$ is commutative and associative
(ii) $\diamondsuit$ is continuous
(iii) $a \diamondsuit 0 = a$ for all $a \in [0, 1]$
(iv) $a \diamondsuit b \leq c \diamondsuit d$ whenever $a \leq c$ and $b \leq d$ for $a, b, c, d \in [0,1]$.

Definition 2.3[2] *A 5-tuple* $(X, M, N, *, \diamondsuit)$ *is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, * is a continuous t-*

norm, $\diamondsuit$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions:

For all $x, y, z \in X$ and $s, t > 0$,

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$

(IFM-2) $M(x, y, 0) = 0$

(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$

(IFM-4) $M(x, y, t) = M(y, x, t)$

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(IFM-6) $M(x, y, \bullet): [0, \infty) \to [0,1]$ is left continuous

(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$

(IFM-8) $N(x, y, 0) = 1$

(IFM-9) $N(x, y, t) = 0$ if and only if $x = y$

(IFM-10) $N(x, y, t) = N(y, x, t)$

(IFM-11) $N(x, y, t) \diamondsuit N(y, z, s) \geq N(x, z, t + s)$

(IFM-12) $N(x, y, \bullet): [0, \infty) \to [0,1]$ is right continuous.

(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.1([1], [3]) Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space if X is of the form $(X, M, 1 - M, *, \diamondsuit)$ such that t- norm * and t-conorm $\diamondsuit$ are associated, that is, $x \diamondsuit y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in X$. But the converse is not true.

Example 2.1 Let (X, d) be a metric space. Define $a * b = \min\{a, b\}$ and t–conorm $a \diamondsuit b = \max\{a, b\}$ for all $x, y \in X$ and $t > 0$, $M_d(x, y, t) = \frac{t}{t+d(x,y)}$ and $N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$.

Then $(X, M, N, * \diamondsuit)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

Definition 2.4[2] *Let $(X, M, N, *, \diamondsuit)$ be an Intuitionistic Fuzzy metric space, then*

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for all $t > 0$, $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy if for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(c) An Intuitionistic Fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5[15] *Self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ are said to be compatible if for all $t > 0$, $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.*

Definition 2.6[9] *Self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ are said to be weakly compatible if $ABx = BAx$ when $Ax = Bx$ for some $x \in X$.*

Definition 2.7[11] *Self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ are said to be occasionally weakly compatible (owc) iff there is point $x \in X$ which is a coincidence point of A and B at which A and B commute.*

Definition 2.8[11] *Self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ are said to be sub compatible iff there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$ and satisfy $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$ for all $t > 0$.*

Lemma 2.1[1] *Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and for all $x, y \in X$ and $t > 0$ and if for a number $k \in (0, 1)$, $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$. Then $x = y$.*

Lemma 2.2 [1] *Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in [0, 1]$ such that $M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$, $N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t)$ for all $t>0$ and $n \in N$, then $\{y_n\}$ is a Cauchy sequence in X.*

3 Main Result

Theorem 3.1 *Let A, B, S, T, P and Q be self maps on an intuitionistic fuzzy metric space* $(X, M, N, *, \diamond)$ *with* $t * t \geq t$ *and* $(1-t) \diamond (1-t) \leq (1-t)$ *for some* $t \in [0,1]$ *such that*

(i) There exists a number $k \in (0,1)$ such that

$$M^2(Px, Qy, kt) * [M(ABx, Px, kt).\,M(STy, Qy, kt)] \geq [pM(ABx, Px, t) + qM(ABx, STy, t)].\,M(ABx, Qy, 2kt)$$

$$\text{and } N^2(Px, Qy, kt) \diamond [N(ABx, Px, kt).\,N(STy, Qy, kt)] \leq [pN(ABx, Px, t) + qN(ABx, STy, t)].\,N(ABx, Qy, 2kt)$$

for all x, y ϵX and t>0, where 0<p, q<1 such that p+q=1.

(ii) AB=BA, ST=TS, PB=BP, QT=TQ
(iii) AB is continuous.
(iv) The pair (P, AB) is subcompatible and (Q, ST) is occasionally weakly compatible (owc).

Then A , B , S, T, P and Q have a unique common fixed point in X.

Proof: Since the pair (P, AB) is subcompatible, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = z$ for some $z \in X$ and satisfy $\lim_{n\to\infty} P(AB)x_n = \lim_{n\to\infty} AB(P)x_n$.

Since AB is continuous, $AB(AB)x_n \to ABz$ and $(AB)Px_n \to ABz$.
Since (P, AB) is subcompatible, $P(AB)x_n \to ABz$.

Since (Q, ST) is occasionally weakly compatible, then there exists a point $v \in X$ such that $Qv = STv$ and $QSTv = STQv$.

Step-1: By taking $x = x_n$ and $y = v$ in (i), we have

$$M^2(Px_n, Qv, kt) * [M(ABx_n, Px_n, kt).\,M(STv, Qv, kt)] \geq [pM(ABx_n, Px_n, t) + qM(ABx_n, STv, t)].\,M(ABx_n, Qv, 2kt)$$

$$\text{and } N^2(Px_n, Qv, kt) \diamond [N(ABx_n, Px_n, kt).\,N(STv, Qv, kt)] \leq [pN(ABx_n, Px_n, t) + qN(ABx_n, STv, t)].\,N(ABx_n, Qv, 2kt)$$

Taking limit as $n \to \infty$ and using $Qv = STv$, we have

$$M^2(z, Qv, kt) * [M(z, z, kt).\,M(Qv, Qv, kt)] \geq [pM(z, z, t) + qM(z, Qv, t)].\,M(z, Qv, 2kt)$$

$$\Rightarrow M^2(z, Qv, kt) \geq [p + qM(z, Qv, t)].M(z, Qv, 2kt)$$

$$\Rightarrow M^2(z, Qv, kt) \geq [p + qM(z, Qv, t)].M(z, Qv, kt)$$

$$\Rightarrow M(z, Qv, kt) \geq \frac{p}{1-q} = 1.$$

$$\text{and } N^2(z, Qv, kt) \diamondsuit [N(z, z, kt).N(Qv, Qv, kt)] \\ \geq [pN(z, z, t) + qN(z, Qv, t)].N(z, Qv, 2kt)$$

$$\Rightarrow N^2(z, Qv, kt) \leq qN(z, Qv, t).N(z, Qv, 2kt)$$

$$\leq qN(z, Qv, t).N(z, Qv, kt)$$

$$\Rightarrow N(z, Qv, kt) \leq 0 \text{ for } k \in (0,1) \, and \ all \ t > 0.$$

Therefore, we have $z = Qv$ and so $z = Qv = STv$, then we get $Qz = STz$.

Step-2: By taking $x = ABx_n$ and $y = v$ in (i), we have

$$M^2(P(AB)x_n, Qv, kt) * [M(AB(AB)x_n, P(AB)x_n, kt).M(STv, Qv, kt)] \\ \geq [pM(AB(AB)x_n, P(AB)x_n, t) \\ + qM(AB(AB)x_n, STv, t)].M(AB(AB)x_n, Qv, 2kt)$$

$$\text{and } N^2(P(AB)x_n, Qv, kt) \diamondsuit [N(AB(AB)x_n, P(AB)x_n, kt).N(STv, Qv, kt)] \\ \leq [pN(AB(AB)x_n, P(AB)x_n, t) \\ + qN(AB(AB)x_n, STv, t)].N(AB(AB)x_n, Qv, 2kt)$$

Taking limit as $n \to \infty$ and using $z = Qv = STv$, we have

$$M^2(ABz, z, kt) * [M(ABz, ABz, kt).M(z, z, kt)] \\ \geq [pM(ABz, ABz, t) + qM(ABz, z, t)].M(ABz, z, 2kt)$$

$$\Rightarrow M^2(ABz, z, kt) \geq [p + qM(ABz, z, t)].M(ABz, z, 2kt)$$

$$\geq [p + qM(ABz, z, t)].M(ABz, z, kt)$$

$$\Rightarrow M(ABz, z, kt) \geq [p + qM(ABz, z, t)]$$

$$\geq [p + qM(ABz, z, kt)]$$

$$\Rightarrow M(ABz, z, kt) \geq \frac{p}{1-q} = 1.$$

$$\text{and} N^2(ABz, z, kt) \diamondsuit [N(ABz, ABz, kt).N(z, z, kt)] \\ \leq [pN(ABz, ABz, t) + qN(ABz, z, t)].N(ABz, z, 2kt)$$

$$\mathrm{N}^2(\mathrm{ABz},\mathrm{z},\mathrm{kt}) \leq \mathrm{qN}(\mathrm{ABz},\mathrm{z},\mathrm{t}).\mathrm{N}(\mathrm{ABz},\mathrm{z},2\mathrm{kt})$$
$$\leq \mathrm{qN}(\mathrm{ABz},\mathrm{z},\mathrm{t}).\mathrm{N}(\mathrm{ABz},\mathrm{z},\mathrm{kt})$$

$$\mathrm{N}(\mathrm{ABz},\mathrm{z},\mathrm{kt}) \leq \mathrm{qN}(\mathrm{ABz},\mathrm{z},\mathrm{t}) \leq \mathrm{qN}(\mathrm{ABz},\mathrm{z},\mathrm{kt})$$

$$\mathrm{N}(\mathrm{ABz},\mathrm{z},\mathrm{kt}) \leq 0 \text{ for } \mathrm{k} \in (0,1) \, and \; all \; t > 0.$$

Thus, we have $z = ABz$.

Step-3: By taking $x = z$ and $y = v$ in (i), we have

$$\mathrm{M}^2(\mathrm{Pz},\mathrm{Qv},\mathrm{kt}) * [\mathrm{M}(\mathrm{ABz},\mathrm{Pz},\mathrm{kt}).\mathrm{M}(\mathrm{STv},\mathrm{Qv},\mathrm{kt})]$$
$$\geq [\mathrm{pM}(\mathrm{ABz},\mathrm{Pz},\mathrm{t}) + \mathrm{qM}(\mathrm{ABz},\mathrm{STv},\mathrm{t})].\mathrm{M}(\mathrm{ABz},\mathrm{Qv},2\mathrm{kt})$$

$$\text{and } \mathrm{N}^2(\mathrm{Pz},\mathrm{Qv},\mathrm{kt}) \diamond [\mathrm{N}(\mathrm{ABz},\mathrm{Pz},\mathrm{kt}).\mathrm{N}(\mathrm{STv},\mathrm{Qv},\mathrm{kt})]$$
$$\leq [\mathrm{pN}(\mathrm{ABz},\mathrm{Pz},\mathrm{t}) + \mathrm{qN}(\mathrm{ABz},\mathrm{STv},\mathrm{t})].\mathrm{N}(\mathrm{ABz},\mathrm{Qv},2\mathrm{kt})$$

Using $\mathrm{z} = \mathrm{Qv} = \mathrm{STv} = \mathrm{ABz}$; we have

$$\mathrm{M}^2(\mathrm{Pz},\mathrm{z},\mathrm{kt}) * [\mathrm{M}(\mathrm{z},\mathrm{Pz},\mathrm{kt}).\mathrm{M}(\mathrm{z},\mathrm{z},\mathrm{kt})]$$
$$\geq [\mathrm{pM}(\mathrm{z},\mathrm{Pz},\mathrm{t}) + \mathrm{qM}(\mathrm{z},\mathrm{z},\mathrm{t})].\mathrm{M}(\mathrm{z},\mathrm{z},2\mathrm{kt})$$

$$\Rightarrow \mathrm{M}^2(\mathrm{z},\mathrm{Pz},\mathrm{kt}) * \mathrm{M}(\mathrm{z},\mathrm{Pz},\mathrm{kt}) \geq [\mathrm{pM}(\mathrm{z},\mathrm{Pz},\mathrm{t}) + \mathrm{q}]$$

Since $\mathrm{M}^2(\mathrm{Pz},\mathrm{z},\mathrm{kt}) \leq 1$ and using (iii) in definition 2.1, we have

$$\mathrm{M}(\mathrm{z},\mathrm{Pz},\mathrm{kt}) \geq [\mathrm{pM}(\mathrm{z},\mathrm{Pz},\mathrm{t}) + \mathrm{q}] \geq \mathrm{pM}(\mathrm{z},\mathrm{Pz},\mathrm{kt}) + \mathrm{q}$$

$$\Rightarrow \mathrm{M}(\mathrm{z},\mathrm{Pz},\mathrm{kt}) \geq \frac{\mathrm{q}}{1-\mathrm{p}} = 1.$$

$$\text{and } \mathrm{N}^2(\mathrm{Pz},\mathrm{z},\mathrm{kt}) \diamond [\mathrm{N}(\mathrm{z},\mathrm{Pz},\mathrm{kt}).\mathrm{N}(\mathrm{z},\mathrm{z},\mathrm{kt})]$$
$$\leq [\mathrm{pN}(\mathrm{z},\mathrm{Pz},\mathrm{t}) + \mathrm{qN}(\mathrm{z},\mathrm{z},\mathrm{t})].\mathrm{N}(\mathrm{z},\mathrm{z},2\mathrm{kt})$$

$$\Rightarrow \mathrm{N}^2(\mathrm{Pz},\mathrm{z},\mathrm{kt}) \leq 0$$

$$\Rightarrow \mathrm{N}(\mathrm{Pz},\mathrm{z},\mathrm{kt}) \leq 0 \text{ for } \mathrm{k} \in (0,1) \, and \; all \; t > 0.$$

Thus, we have $z = Pz = ABz$.

Step-4: By taking $x = x_n$ and $y = z$ in (i), we have

$$\mathrm{M}^2(\mathrm{Px_n},\mathrm{Qz},\mathrm{kt}) * [\mathrm{M}(\mathrm{ABx_n},\mathrm{Px_n},\mathrm{kt}).\mathrm{M}(\mathrm{STz},\mathrm{Qz},\mathrm{kt})]$$
$$\geq [\mathrm{pM}(\mathrm{ABx_n},\mathrm{Px_n},\mathrm{t}) + \mathrm{qM}(\mathrm{ABx_n},\mathrm{STz},\mathrm{t})].\mathrm{M}(\mathrm{ABx_n},\mathrm{Qz},2\mathrm{kt})$$

$$\text{and } N^2(Px_n, Qz, kt) \diamondsuit [N(ABx_n, Px_n, kt). N(STz, Qz, kt)] \\ \leq [pN(ABx_n, Px_n, t) + qN(ABx_n, STz, t)]. N(ABx_n, Qz, 2kt)$$

Taking limit as $n \to \infty$ and using $Qz = STz$, we have

$$M^2(z, Qz, kt) * [M(z, z, kt). M(Qz, Qz, kt)] \\ \geq [pM(z, z, t) + qM(z, Qz, t)]. M(z, Qz, 2kt)$$

$$\Rightarrow M^2(z, Qz, kt) \geq [p + qM(z, Qz, t)]. M(z, Qz, 2kt)$$

$$\geq [p + qM(z, Qz, t)]M(z, Qz, kt)$$

$$\Rightarrow M(z, Qz, kt) \geq [p + qM(z, Qz, t)] \geq [p + qM(z, Qz, kt)].$$

$$\Rightarrow M(z, Qz, kt) \geq \frac{p}{1 - q} = 1.$$

$$\text{and } N^2(z, Qz, kt) \diamondsuit [N(z, z, kt). N(Qz, Qz, kt)] \\ \geq [pN(z, z, t) + qN(z, Qz, t)]. N(z, Qz, 2kt)$$

$$\Rightarrow N^2(z, Qz, kt) \leq qN(z, Qz, t). N(z, Qz, 2kt)$$

$$\leq qN(z, Qz, t). N(z, Qz, kt)$$

$$\Rightarrow N(z, Qz, kt) \leq qN(z, Qz, t) \leq qN(z, Qz, kt)$$

$$\Rightarrow N(z, Qz, kt) \leq 0 \text{ for } k \in (0,1) \text{and all } t > 0.$$

Thus, we have $z = Qz$ and therefore $z = ABz = Pz = Qz = STz$.

Step-5: By taking $x = Bz$ and $y = z$ in (i), we have

$$M^2(P(B)z, Qz, kt) * [M(AB(B)z, P(B)z, kt). M(STz, Qz, kt)] \\ \geq [pM(AB(B)z, P(B)z, t) \\ + qM(AB(B)z, STz, t)]. M(AB(B)z, Qz, 2kt)$$

$$\text{and } N^2(P(B)z, Qz, kt) \diamondsuit [N(AB(B)z, P(B)z, kt). N(STz, Qz, kt)] \\ \leq [pN(AB(B)z, P(B)z, t) \\ + qN(AB(B)z, STz, t)]. N(AB(B)z, Qz, 2kt)$$

Since AB=BA and PB=BP, we have $P(B)z = B(P)z = Bz$ and $AB(B)z = B(AB)z = Bz$ and using $Qz = STz$ =z; we have

$$M^2(Bz, z, kt) * [M(Bz, Bz, kt). M(z, z, kt)] \\ \geq [pM(Bz, Bz, t) + qM(Bz, z, t)]. M(Bz, z, 2kt)$$

$$\Rightarrow M^2(Bz, z, kt) \geq [p + qM(Bz, z, t)].M(Bz, z, 2kt)$$

$$\geq [p + qM(Bz, z, t)].M(Bz, z, kt)$$

$$M(Bz, z, kt) \geq [p + qM(Bz, z, t)] \geq [p + qM(Bz, z, kt)]$$

$$\Rightarrow M(Bz, z, kt) \geq \frac{p}{1-q} = 1.$$

$$\text{and } N^2(Bz, z, kt) \diamondsuit [N(Bz, Bz, kt).N(z, z, kt)] \\ \leq [pN(Bz, Bz, t) + qN(Bz, z, t)].N(Bz, z, 2kt)$$

$$\Rightarrow N^2(Bz, z, kt) \leq qN(Bz, z, t).N(Bz, z, 2kt)$$

$$\leq qN(Bz, z, t).N(Bz, z, kt)$$

$$\Rightarrow N(Bz, z, kt) \leq qN(Bz, z, t) \leq qN(Bz, z, kt)$$

$$\Rightarrow N(Bz, z, kt) \leq 0 \text{ for } k \in (0,1) \, and \; all \; t > 0.$$

Thus, we have $z = Bz$. since $z = ABz$, we also have $z = Az$, therefore $z = Az = Bz = Pz = Qz = STz$.

Step-6: By taking $x = x_n$ and $y = Tz$ in (i), we have

$$M^2(Px_n, Q(Tz), kt) * [M(ABx_n, Px_n, kt).M(ST(Tz), Q(Tz), kt)] \\ \geq [pM(ABx_n, Px_n, t) \\ + qM(ABx_n, ST(Tz), t)].M(ABx_n, Q(Tz), 2kt)$$

$$\text{and } N^2(Px_n, Q(Tz), kt) \diamondsuit [N(ABx_n, Px_n, kt).N(ST(Tz), Q(Tz), kt)] \\ \leq [pN(ABx_n, Px_n, t) + qN(ABx_n, ST(Tz), t)].N(ABx_n, Q(Tz), 2kt)$$

Since $QT = TQ$ and $ST = TS$, we have $QTz = TQz = Tz$ and $ST(Tz) = T(STz) = Tz$.

Letting $n \to \infty$, we have

$$M^2(z, Tz, kt) * [M(z, z, kt).M(Tz, Tz, kt)] \\ \geq [pM(z, z, t) + qM(z, Tz, t)].M(z, Tz, 2kt)$$

$$\Rightarrow M^2(z, Tz, kt) \geq [p + qM(z, Tz, t)].M(z, Tz, 2kt)$$

$$\geq [p + qM(z, Tz, t)]M(z, Tz, kt)$$

$$\Rightarrow M(z, Tz, kt) \geq [p + qM(z, Tz, t)] \geq [p + qM(z, Tz, kt)].$$

$$\Rightarrow M(z,Tz,kt) \geq \frac{p}{1-q} = 1.$$

$$\text{and } N^2(z,Tz,kt) \diamond [N(z,z,kt).N(Tz,Tz,kt)] \geq [pN(z,z,t) + qN(z,Tz,t)].N(z,Tz,2kt)$$

$$\Rightarrow N^2(z,Tz,kt) \leq qN(z,Tz,t).N(z,Tz,2kt)$$

$$\leq qN(z,Tz,t).N(z,Tz,kt)$$

$$\Rightarrow N(z,Tz,kt) \leq qN(z,Tz,t) \leq qN(z,Tz,kt)$$

$$\Rightarrow N(z,Tz,kt) \leq 0 \text{ for } k \in (0,1) \, and\ all\ t > 0.$$

Thus, we have z = Tz. Since Tz = STz, we also have z = Sz. Therefore z = Az = Bz = Pz = Qz = Sz = Tz, that is, z is the common fixed point of the six maps.

Step-7: For uniqueness, let w, $(w \neq z)$ be another common fixed point of A, B, S, T, P and Q.

By taking $x = z$ and $y = w$ in (i), we have

$$M^2(Pz,Qw,kt) * [M(ABz,Pz,kt).M(STw,Qw,kt)] \geq [pM(ABz,Pz,t) + qM(ABz,STw,t)].M(ABz,Qw,2kt)$$

$$\text{and } N^2(Pz,Qw,kt) \diamond [N(ABz,Pz,kt).N(STw,Qw,kt)] \leq [pN(ABz,Pz,t) + qN(ABz,STw,t)].N(ABz,Qw,2kt)$$

Which implies that

$$M^2(z,w,kt) * [M(z,z,kt).M(w,w,kt)] \geq [pM(z,z,t) + qM(z,w,t)].M(z,w,2kt)$$

$$\Rightarrow M^2(z,w,kt) \geq [p + qM(z,w,t)].M(z,w,2kt)$$

$$\geq [p + qM(z,w,t)].M(z,w,kt)$$

$$\Rightarrow M(z,w,kt) \geq p + qM(z,w,t) \geq p + qM(z,w,kt)$$

$$\Rightarrow M(z,w,kt) \geq \frac{p}{1-q} = 1.$$

$$\text{and } N^2(z,w,kt) \diamond [N(z,z,kt).N(w,w,kt)] \geq [pN(z,z,t) + qN(z,w,t)].N(z,w,2kt)$$

$$\Rightarrow N^2(z,w,kt) \leq qN(z,w,t).N(z,w,2kt)$$

$$\leq qN(z,w,t).N(z,w,kt)$$

$$\Rightarrow N(z,w,kt) \leq qN(z,w,t)q \leq N(z,w,kt)$$

$$\Rightarrow N(z,w,kt) \leq 0 \text{ for } k \in (0,1) \text{and all } t > 0.$$

Thus, we have $z = w$. This completes the proof of the theorem.

If we take B=T=I$_X$ (the identity map on X) in the main theorem, we have the following:

Corollary 3.2: *Let A, S, P and Q be self maps on an intuitionistic fuzzy metric space* $(X, M, N, *, \diamond)$ *with* $t * t \geq t$ *and* $(1-t) \diamond (1-t) \leq (1-t)$ *for some* $t \in [0,1]$ *such that*

(i) There exists a number $k \in (0,1)$ such that

$$M^2(Px,Qy,kt) * [M(Ax,Px,kt).M(Sy,Qy,kt)] \geq [pM(Ax,Px,t) + qM(Ax,Sy,t)].M(Ax,Qy,2kt)$$

$$N^2(Px,Qy,kt) \diamond [N(Ax,Px,kt).N(Sy,Qy,kt)] \leq [pN(Ax,Px,t) + qN(Ax,Sy,t)].N(Ax,Qy,2kt)$$

for all x, y ϵX and t>0, where 0<p, q<1 such that p+q=1.

(ii) A is continuous.

(iii) The pair (P, A) is subcompatible and (Q, S) is occasionally weakly compatible (owc).

Then A, S, P and Q have a unique common fixed point in X.

Example 3.3 Let $X = \{\frac{1}{n} : n \in N\} \cup \{0\}$ with metric d defined by $d(x,y) = |x-y|$. For all $x, y \in X$ and $t \in (0,\infty)$, define

$$M(x,y,t) = \frac{t}{t+|x-y|}, N(x,y,t) = \frac{|x-y|}{t+|x-y|}, M(x,y,0) = 0, N(x,y,0) = 1.$$

Clearly $(X, M, N, *, \diamond)$ is an Intuitionistic Fuzzy metric space, where $*$ and $\diamond$ are defined by $a * b = \min\{a,b\}$ and $a \diamond b = \min\{1, a+b\}$.

Let A, B, S, T, P and Q be maps from X into itself defined as $Ax = x, Bx = \frac{x}{2}, Sx = \frac{x}{5}, Tx = \frac{x}{3}, Px = 0, Qx = \frac{x}{6}$ for all $x \in X$.

Clearly AB=BA, ST=TS, PB=BP, QT=TQ and AB is continuous. If we take $k = 0.5$ and $t = 1$, we see that the condition (i) of the main theorem is also satisfied.

Moreover, the maps P and AB are subcompatible if $\lim_{n\to\infty} x_n = 0$, where $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = 0$ for $0 \in X$. The maps Q and ST are occasionally weakly compatible at 0. Thus, all conditions of the main theorem are satisfied and 0 is the unique common fixed point of A, B, S, T, P and Q.

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