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Fuzzy I_{rg}- Continuous Mappings

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Abstract

In this paper we introduce the concept of fuzzy I_{rg} -continuous mappings in fuzzy ideal topological spaces and obtain some of its basic properties and characterizations.

Keywords: Fuzzy ideal topological spaces, fuzzy I_{rg}- closed sets, fuzzy I_{rg}- open sets, *IRGO-compactness and* fuzzy I_{rg}- continuous mappings.

1 Introduction

In 1945 R. Vaidyanathaswamy [22] introduced the concept of ideal topological spaces. Hayashi [6] defined the local function and studied some topological properties using local function in ideal topological spaces in 1964. Since then many mathematicians studied various topological concepts in ideal topological spaces. After the introduction of fuzzy sets by Zadeh [25] in 1965 and fuzzy topology by Chang [2] in 1968, several researches were conducted on the generalization of the notions of fuzzy sets and fuzzy topology. The hybridization of fuzzy topology and fuzzy ideal theory was initiated by Mahmoud [9] and

Sarkar [15] independently in 1997. They [9,15] introduced the concept of fuzzy ideal topological spaces as an extension of fuzzy topological spaces and ideal topological spaces . Recently the concepts of fuzzy semi-I-open sets [5] , fuzzy α-I-open sets [23] , fuzzy γ-I- open sets [4] , fuzzy pre-I-open sets [11] and fuzzy δ-I-open sets [24] fuzzy I_g – closed sets, fuzzy I_g – open sets [17] , fuzzy I_{rg} – closed sets, fuzzy I_{rg} – open [18] sets and fuzzy I_g –continuous mappings [19] have been introduced and studied in fuzzy ideal topological spaces. In the present paper we introduce and study the concept of I_{rg} –continuous mappings in fuzzy ideal topological spaces.

2 Preliminaries

Let X be a nonempty set. A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to τ and τ is closed with respect to any union and finite intersection. If τ is a fuzzy topology on X, then the pair (X, τ) is called a fuzzy topological space [22]. The members of τ are called fuzzy open sets of X and their complements are called fuzzy closed sets. The closure of a fuzzy set A of X denoted by Cl(A), is the intersection of all fuzzy closed sets which contains A. The interior [2] of a fuzzy set A of X denoted by Int(A) is the union of all fuzzy subsets contained in A. A fuzzy set A in (X, τ) is said to be quasi-coincident with a fuzzy set B, denoted by AqB, if there exists a point $x \in X$ such that $A(x) + B(x) > 1$ [4]. A fuzzy set V in (X, τ) is called a Q-neighbourhood of a fuzzy point x_β if there exists a fuzzy open set U of X such that $x_\beta qU \leq V$ [4]. A nonempty collection of fuzzy sets I of a set X satisfying the conditions (i) if $A \in I$ and $B \leq A$, then $B \in I$,(ii) if $A \in I$ and $B \in I$ then $A \cup B \in I$ is called a fuzzy ideal on X. The triplex (X,τ, I) denotes a fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ [8, 11]. The local function for a fuzzy set A of X with respect to τ and I denoted by $A^*(\tau, I)$ (briefly A^*) in a fuzzy ideal topological space (X,τ, I) is the union of all fuzzy points x_β such that if U is a Q-neighbourhood of x_β and $E \in I$ then for at least one point $y \in X$ for which $U(y) + A(y) - 1 > E(y)$ [13]. The ∗–closure operator of a fuzzy set A denoted by $Cl^*(A)$ in (X,τ, I) defined as $Cl^*(A) = A \cup A^*$ [19]. In (X,τ, I), the collection $\tau^*(I)$ is an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta=\{U-E : U \in \tau , E \in I\}$ as a base [13].

Definition 2.1: *A fuzzy set A of a fuzzy topological space (X,τ) is called:*

(a) *fuzzy regular open if A=Int(Cl(A))[1] .*
(b) *fuzzy regular closed if 1-A is fuzzy regular open[1].*
(c) *fuzzy g- closed if $Cl(A) \leq O$ whenever $A \leq O$ and O is fuzzy open [20].*
(d) *Fuzzy g-open if 1-A is fuzzy g-closed [20].*
(e) *fuzzy rg–closed if $Cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy regular open[16].*
(f) *fuzzy rg–open if 1-A is fuzzy rg–closed[16]*

Remark 2.1: Every fuzzy regular open (resp. fuzzy regular closed) set is fuzzy open (resp. fuzzy regular closed), every fuzzy open set is fuzzy g-open (resp. fuzzy g-closed) and every fuzzy g-open (resp. fuzzy g-closed) set is fuzzy rg-open (resp. fuzzy rg-closed). But the converse may not be true [16].

Definition 2.2: *A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called:*

(a) *fuzzy I_g –closed if $A^* \leq U$, whenever $A \leq U$ and U is fuzzy open [17].*

(b) *fuzzy I_g –open if its complement $1-A$ is fuzzy I_g –closed [17].*

(c) *fuzzy I_{rg} –closed if $A^* \leq U$, whenever $A \leq U$ and U is fuzzy regular open [18].*

(d) *fuzzy I_{rg} –open if its complement $1-A$ is fuzzy I_{rg} –closed [18].*

Remark 2.2: Every fuzzy I_g–closed set is fuzzy I_{rg} –closed. But the converse may not be true [18].

Definition 2.3: *A mapping f from a fuzzy ideal topological space (X, τ, I) to a fuzzy topological space (Y, σ) is said to be fuzzy I_g -continuous] if the inverse image of every fuzzy closed set of Y is fuzzy I_g -closed in [19].*

3 Fuzzy I_{rg} –Continuous Mappings

Definition 3.1: *A mapping f from a fuzzy ideal topological space (X, τ, I) to a fuzzy topological space (Y, σ) is said to be fuzzy I_{rg} -continuous if the inverse image of every fuzzy closed set of Y is fuzzy I_{rg} -closed in X.*

Theorem 3.1: *A mapping $f:(X, \tau, I)\to(Y, \sigma)$ is fuzzy I_{rg} -continuous if and only if the inverse image of every fuzzy open set of Y is fuzzy I_{rg} -open in X.*

Proof: The proof is obvious because $f^{-1}(1-U) = 1-f^{-1}(U)$ for every fuzzy set U of Y.

Remark 3.1: Every fuzzy I_g-continuous mapping is fuzzy I_{rg}-continuous, but the converse may not be true. For-

Example 3.1: Let X = {a, b} , Y = {x ,y} and the fuzzy sets U and V are defined as follows:

$$U(a) = 0.5 \ , \ U(b) = 0.7;$$
$$V(a) = 0.3 \ , \ V(b) = 0.2;$$

Let τ = {0, U, 1}, σ = {0, V, 1} be the topologies on X and Y respectively and I = {0} be the fuzzy ideal on X. Then the mapping $f:(X, \tau, I)\to(Y, \sigma)$ defined by f(a) = x and f(b) = y is fuzzy I_{rg} -continuous, but not fuzzy I_g-continuous.

Theorem 3.2: *If $f:(X, \tau, I)\to(Y, \sigma)$ is fuzzy I_{rg} -continuous then for each fuzzy point x_β of X and each fuzzy open set V, $f(x_\beta) \in V$ there exists a fuzzy I_{rg} -open set U such that $x_\beta \in U$ and $f(U) \leq V$.*

Proof: Let x_β be a fuzzy point of X and V be a fuzzy open set such that $f(x_\beta) \in V$ .Put $U = f^{-1}(V)$ then by hypothesis U is a fuzzy I_{rg} -open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \leq V$.

Theorem 3.3: *If f:(X, τ, I)→(Y, σ) is fuzzy I_{rg} -continuous then for each fuzzy point x_β of X and each fuzzy open set V of Y such that $f(x_\beta)qV$, there exists a fuzzy I_{rg} -open set U of X such that $x_\beta qU$ and f(U)≤V.*

Proof: Let x_β be a fuzzy point of X and V be a fuzzy open set such that $f(x_\beta)qV$. Put $U = f^{-1}(V)$.Then by hypothesis U is a fuzzy I_{rg} -open set of X such that $x_\beta qU$ and $f(U) = f(f^{-1}(V)) \leq V$.

Definition 3.2: *Let (X,τ,I) be a fuzzy ideal topological space. The regular generalized-I-closure of a fuzzy set A of X denoted by rgIcl (A) is defined as follows:*

$$rgIcl(A) = \inf\{B: B \geq A, B \text{ is fuzzy } I_{rg}\text{-closed set of } (X, \tau, I)\}.$$

Remark 3.2: A≤rgIcl(A) ≤cl(A) for any fuzzy set A of X.

Theorem 3.4: *If f: (X, τ, I) → (Y, σ) is fuzzy I_{rg}-continuous then f(rgIcl(A)) ≤cl(f(A)) for every fuzzy set A of X.*

Proof: Let A be a fuzzy set of X. Then cl(f(A)) is a fuzzy closed set of Y. Since f is fuzzy I_{rg} -continuous, $f^{-1}(cl(f(A)))$ is fuzzy I_{rg} -closed in X. Clearly $A \leq f^{-1}(cl(f(A)))$. Therefore $rgIcl(A) \leq rgIcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Hence $f(rgIcl(A)) \leq cl(f(A))$.

Remark 3.3: The converse of Theorem3.4 may not be true. For,

Example 3.2: Let X = {a, b, c}, Y = {x, y, z} and the fuzzy set U and V are defined as:

$$U(a) = 1,\ U(b) = 0,\ U(c) = 0;$$
$$V(x) = 1,\ V(y) = 0,\ V(z) = 1.$$

Let τ ={0, U, 1} and σ ={0, V, 1} be fuzzy topologies on X and Y respectively and I = {0} be a fuzzy ideal on X. Consider the mapping f: (X, τ, I)→(Y, σ) defined by f(a) = y, f(b) = x, f(c) = z. Then f(rgIcl(A)) ≤cl(f(A)) holds for every fuzzy set A of X, but f is not fuzzy I_{rg} -continuous.

Definition 3.3: *A fuzzy ideal topological space (X, τ, I) is said to be fuzzy I- $T^*_{1/2}$ if every fuzzy I_{rg} -closed set in X is fuzzy closed in X.*

Theorem 3.5: *A mapping f from a fuzzy I- $T^*_{1/2}$ -space (X, τ, I) to a fuzzy topological space (Y, σ) is fuzzy continuous if and only if it is fuzzy I_{rg} -continuous.*

Proof: Obvious.

Remark 3.4: The composition of two fuzzy I_{rg} -continuous mappings may not be fuzzy I_{rg} -continuous.

Example 3.3: Let X = {a, b}, Y = {x, y} and Z = {p, q} and the fuzzy sets U, V and W are defined as follows:

U(a) = 0.5, U(b) = 0.7;
V(x) = 0.3, V(y) = 0.2;
W(p) = 0.6, W(q) = 0.4.

Let $\tau = \{0, U, 1\}$, $\sigma = \{0, V, 1\}$ and $\eta=\{0, W, 1\}$ be fuzzy topologies on X , Y and Z respectively and $I_1 = \{0\}$ be the fuzzy ideal on X and $I_2=\{0\}$ fuzzy ideal on Y. Then the mapping f: $(X, \tau, I_1) \to (Y, \sigma)$ defined by f(a) = x, f(b) = y and the mapping g: $(Y, \sigma, I_2) \to (Z, \eta)$ defined by g(x) = p and g(y) = q are fuzzy I_{rg} -continuous but gof is not fuzzy I_{rg} -continuous.

Theorem 3.6: *If f: $(X, \tau, I)\to(Y, \sigma)$ is fuzzy I_{rg} -continuous and g:$(Y, \sigma) \to(Z, \eta)$ is fuzzy continuous. Then gof: $(X, \tau, I) \to (Z, \eta)$ is fuzzy I_{rg} -continuous.*

Proof: If A is fuzzy closed in Z, then $f^{-1}(A)$ is fuzzy closed in Y because g is fuzzy continuous. Therefore $(gof)^{-1}(A)= f^{-1}(g^{-1}(A))$ is fuzzy I_{rg} -closed in X. Hence gof is fuzzy I_{rg} -continuous.

Theorem 3.7: *If f: $(X, \tau, I)\to(Y, \sigma)$ and g:$(Y, \sigma) \to(Z, \eta)$ are two fuzzy I_{rg} -continuous mappings and (Y, σ) is fuzzy I- $T^*_{1/2}$-space then gof: $(X, \tau, I)\to(Z, \eta)$ is fuzzy I_{rg} -continuous.*

Proof: The proof is obvious.

Definition 3.4: *A collection $\{A_i: i\in \Lambda\}$ of fuzzy I_{rg} -open sets in a fuzzy ideal topological space (X, τ, I) is called a fuzzy I_{rg} -open cover of a fuzzy set B of X if $B\leq\cup\{A_i: i\in \Lambda\}$.*

Definition 3.5: *A fuzzy ideal topological space (X, τ, I) is said to be fuzzy IRGO-compact if every fuzzy I_{rg} -open cover of X has a finite sub cover.*

Definition 3.6: *A fuzzy set B of a fuzzy ideal space (X, τ, I) is said to be fuzzy IRGO-compact if for every collection $\{A_i: i\in \Lambda\}$ of fuzzy I_{rg} -open subsets of X such that $B\leq\cup\{A_i: i\in \Lambda\}$ there exists a finite subset Λ_0 and Λ such that $B\leq\cup\{A_i: i\in \Lambda_0\}$.*

Definition 3.7: *A crisp subset B of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy IRGO-compact if B is fuzzy IRGO-compact as a fuzzy ideal subspace of X.*

Theorem 3.8: *A fuzzy I_{rg} -closed crisp subset of fuzzy IRGO-compact space (X, τ, I) is fuzzy IRGO-compact relative to X.*

Proof: Let A be a fuzzy I_{rg} -closed crisp set of fuzzy IRGO-compact space (X, τ, I).Then 1−A is fuzzy I_{rg} -open in X. Let M be a cover of A by fuzzy I_{rg} -open sets in X. Then {M, 1−A} is a fuzzy I_{rg} -open cover of X. Since X is fuzzy IRGO-compact, it has a finite sub cover say $\{G_1, G_2, G_3, \ldots\ldots, G_n\}$, if this subcover contains 1−A, we discard it. Otherwise leave the subcover as it is, true we have obtained a finite fuzzy I_{rg} -open sub cover of A. Therefore A is fuzzy IRGO-compect relative to X.

Theorem 3.9: *A fuzzy I_{rg} -continuous image of a fuzzy IRGO-compact fuzzy ideal topological space is fuzzy compact.*

Proof: Let f: (X, τ, I)→(Y, σ) be a fuzzy I_{rg} -continuous mapping from a fuzzy IRGO-compact space (X, τ, I) onto a fuzzy topological space (Y, σ). Let $\{A_i : i \in \Lambda\}$ be a fuzzy open cover of Y then $\{f^{-1}(A_i) : i \in \Lambda\}$ is a fuzzy I_{rg} -open cover of X. Since X is fuzzy IRGO-compact it has finite sub cover say $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3), \ldots\ldots, f^{-1}(A_n)\}$of X. Since $f(f^{-1}(A_i)) = A_i$ for each i, it follows that $\{A_1, A_2, A_3, \ldots\ldots, A_n\}$ is a finite sub cover of Y. Hence (Y, σ) is fuzzy compact.

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