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A New Approach of γ-Open Sets in Bitopological Spaces

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Abstract

The main aim of this research paper is to introduce two weaker forms of (1, 2)-γ-open set namely (1, 2)-γ-semi-open set and (1, 2)-semi-γ open set in bitopological space along with their several properties, characterizations and mutual interrelationships. As applications to (1, 2)-γ-semi-open set and (1, 2)-semi-γ open set we introduce (1, 2)-γ-semi-continuous and (1, 2)-semi-γ-continuous functions and obtain some of their basic properties. In this present work it is proved that among the two topologies if one of the topology is weaker than other then every (1, 2)-γ-semi-open set is τ_1-semi-γ-open set. Lastly we show the interrelationships with (1, 2)-γ-semi-continuous and (1, 2)*-semi-γ-continuous functions and the newly defined functions.*

Keywords: *(1, 2)-γ-open set, (1, 2) - γ-semi-open set, (1, 2)-semi-γ-open set, (1, 2) - γ-semi-continuous, (1, 2)-semi-γ-continuous.*

1 Introduction

The study of bitopological spaces first initiated by Kelly [8] and thereafter a large numbers of papers have been done to generalize the topological concepts into bitopological setting. Using the notion of pre-open set in 1990 D. Andrijevic and M. Ganster [1] defined the concept of γ-open set in topological spaces. N.Levine [10] introduced the notion of semi-open set and semi-continuity in topological spaces. Maheshwari and Prasad [11] extended the notion of semi-open sets and semi-continuity to the bitopological setting in1977.Recently the authors [2] introduced γ-open sets in bitopological spaces and studied their properties. The purpose of the present paper is to introduce and study the basic properties of two weaker forms of (1, 2)-γ-open set namely (1, 2)-semi-γ-open set, (1, 2)-γ-semi-open set and also define (1, 2)-semi-γ-continuous and (1, 2)-γ-semi-continuous functions. Suitable examples are provided to illustrate the behavior of these new types of sets and functions. Throughout this paper X and Y will denote the bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) respectively on which no separation axioms are assumed unless explicitly stated.

2 Preliminaries

Definition 2.1 *$A \subset X$ is called [12, 13]*

(i) $\tau_1\tau_2$-open if $A \in \tau_1 \cup \tau_2$,
the complement of $\tau_1\tau_2$-open set is called $\tau_1\tau_2$-closed set

(ii) $\tau_{1,2}$-open if $A = A_i \cup B_i$, where $A_i \in \tau_1$ and $B_i \in \tau_2$,
the complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed set,

(iii) the $\tau_1\tau_2$-closure of A is denoted by $\tau_1\tau_2$-cl(A) and defined as
$\tau_1\tau_2$-cl(A)= $\cap$\{F:A$\subseteq$F and F is $\tau_1\tau_2$-closed set\} and

(iv) $\tau_1\tau_2$-cl(A) $\subseteq \tau_1$-cl(A) and $\tau_1\tau_2$-cl(A) $\subseteq \tau_2$-cl(A).

Definition 2.2 *Let A is a subset of bitopological space X. Then A is called*

(i) (1, 2) pre-open set [6] if $A \subseteq \tau_1$-int (τ_2-cl (A)),
(ii) (1, 2)- γ-open set [2] if for any non empty (1, 2)-pre-open set B such that
$A \cap B \subseteq \tau_1$-int($\tau_2$-cl($A \cap B$),
(iii) $\tau_1\tau_2$-semi-open set [5] if $A \subseteq \tau_2$-cl(τ_1-int(A)),
(iv)(1,2)- γ-semi-open set [3] if $A \subseteq \tau_{1,2}$-cl_γ($\tau_{1,2}$-int_γ(A)) and*
(v) (1, 2)-semi-γ-open set [3] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int_γ(A)).*

Remark 2.3 Note that $\tau_1\tau_2$-open sets of X need not necessarily form a topology on X [13].

3 (1, 2) - γ-Semi-Open Sets

Let A be a subset of a bitopological space (X, τ_1, τ_2).Then τ_1-γ-interior of A denoted by τ_1-int $_\gamma$(A) is defined as the union of all τ_1-γ-open sets contained in A.

Definition 3.1 *A subset A of a bitopological space (X, τ_1, τ_2) is called (1, 2) - γ-semi-open set if A$\subseteq$ τ_2-cl $_\gamma$(τ_1-int $_\gamma$(A)).*

The complement of (1, 2) - γ-semi-open set is called (1, 2) - γ-semi-closed set and is defined as A$\supseteq$ τ_2-int $_\gamma$(τ_1-cl $_\gamma$(A)). The collection of all (1, 2) - γ-semi-open sets of (X, τ_1, τ_2) is denoted by (1, 2)- γSO(X).

Example 3.2 Let X = {a, b, c},

τ_1={{a} ,{a, b}, φ, X } and τ_2 = {{ b}, φ, X }.

Thus (1, 2) - γSO(X) = {{a}, {a, b}, {a, c}, φ, X }.

Proposition 3.3 *In a bitopological space X, the union of any two (1, 2) - γ-semi-open sets is always a (1, 2) - γ-semi-open set.*

Proof: Let A and B be any two (1, 2) - γ-semi-open sets in X.
Now A$\cup$ B$\subseteq$ τ_2-cl $_\gamma$(τ_1-int $_\gamma$(A)) $\cup$ τ_2-cl $_\gamma$(τ_1-int $_\gamma$(B))
$\Rightarrow$ A$\cup$ B$\subseteq$ τ_2-cl $_\gamma$(τ_1-int $_\gamma$(A$\cup$ B)).Hence A$\cup$ B is (1, 2) - γ-semi-open.

Remark 3.4 Intersection of any two (1, 2) - γ-semi-open sets may not be a (1, 2) - γ-semi-open set as shown in the following example.

Example 3.5 Let X = {a, b, c},
τ_1={{a} ,{b}, {a, b}, {b, c}, φ, X } and τ_2 = {{a}, { b},{a,b}, φ, X }.
Here {a, c}$\cap${b, c} = {c}$\notin$ (1, 2) - γSO(X).

Proposition 3.6 *If A is τ_1-γ-open set then A is (1, 2) - γ-semi-open set.*

Proof: Given A is τ_1-γ-open set.
Therefore A = τ_1-int $_\gamma$(A).
Now A$\subseteq$ τ_2-cl $_\gamma$(A) = τ_2-cl $_\gamma$(τ_1-int $_\gamma$(A)).Hence A is (1, 2) - γsemi-open set.

Remark 3.7 Converse of the above proposition may not be true as explained in the following example:

Example 3.8 From the above example (3.5) {a, c}$\in$ (1, 2) - γSO(X) but {a, c}$\notin$ (1, 2) - τ_1 - γO(X).

Proposition 3.9 *Let A and B be subsets of X such that B$\subseteq$A$\subseteq$ τ_2-cl (B). If B is (1, 2) - γ-semi-open set then A is also (1, 2) - γ-semi-open set.*

Proof: Given B is (1, 2) - γ-semi-open set.
So we have B $\subseteq \tau_2$-cl $_\gamma$(τ_1-int $_\gamma$(B)) $\subseteq \tau_2$-cl $_\gamma$(τ_1-int $_\gamma$(A)).
Thus τ_2-cl (B) $\subseteq \tau_2$-cl $_\gamma$(τ_1-int $_\gamma$(A)). Hence A is also (1, 2) - γ-semi-open set.

From the literature [4, 7, 9] we studied various kinds of (i, j) continuous function in bitopological spaces which has been introduced by several authors. Analogously we define the followings:

Definition 3.10 *Let (X, τ_1,τ_2) and (Y, σ_1,σ_2) be two bitopological spaces. A function f: (X, τ_1,τ_2)$\rightarrow$ (Y, σ_1,σ_2) is called (1, 2)-continuous function if the inverse image of each σ_1-open set in Y is $\tau_1\tau_2$-open set in X.*

Example 3.11 Let X = Y = {a, b, c},
τ_1= {{b}, {a, b}, φ, X }, τ_2 = {{b, c}, φ, X },
σ_1 = {{b}, {a, b}, φ, Y} and σ_2 = {{ a, c}, φ, Y }.

If we consider the function f: (X, τ_1,τ_2) $\rightarrow$ (Y, σ_1,σ_2) defined as an identity function then f is a (1, 2)-continuous, since the inverse image of each σ_1-open set in Y under f is $\tau_1\tau_2$-open set in X.

Definition 3.12 *Let (X, τ_1,τ_2) and (Y, σ_1,σ_2) be two bitopological spaces. A function f: (X, τ_1,τ_2)$\rightarrow$ (Y, σ_1,σ_2) is called (1, 2) - γ-continuous function if the inverse image of each σ_1-open set in Y is (1, 2) - γ-open set in X.*

Example 3.13 Let X = Y = {a, b, c},
τ_1= {{a}, φ, X }, τ_2 = {{ b},{a, b}, φ, X },
σ_1 = {{a}, {b, c}, φ, Y } and σ_2 = {{ a, b}, φ, Y }.

We get (1, 2) - γO(X) = {{a}, {b}, {a, b}, {b, c} φ, X}.

If we consider the function f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is defined as f (a) = a, f (b) = b, f(c) = b then f is a (1, 2) - γ-continuous, since the inverse image of each σ_1-open set in Y under f is (1, 2) - γ-open set in X.

Definition 3.14 *Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces. A function f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called (1, 2) - γ-semi-continuous if the inverse image of each σ_1-open set in Y is (1, 2) - γ-semi-open set in X.*

Example 3.15 Let X = Y = {a, b, c},
$\tau_1 = \{\{b\}, \{a, b\}, \varphi, X\}$, $\tau_2 = \{\{b, c\}, \varphi, X\}$,
$\sigma_1 = \{\{b\}, \{a, b\}, \varphi, Y\}$ and $\sigma_2 = \{\{a, c\}, \varphi, Y\}$.

We get (1, 2) - $\gamma O(X) = \{\{a\}, \{b\}, \{a, b\}, \{b, c\}\, \varphi, X\}$.

If we consider the function f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is defined as f (a) = a, f (b) = c, f(c) = b then f is a (1, 2) - γ-semi-continuous, since the inverse image of each σ_1-open set in Y under f is (1, 2) - γ-semi-open set in X.

Remark 3.16 If f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two (1, 2) - γ-semi-continuous functions then (gof): $(X,\tau_1,\tau_2)\to(Z,\eta_1,\eta_2)$ may not be a (1, 2) - γ-semi-continuous function as shown in the following example:

Example 3.17 Let X = Y = Z = {a, b, c},
$\tau_1 = \{\{a\}, \varphi, X\}$, $\tau_2 = \{\{a, b\}, \varphi, X\}$,
$\sigma_1 = \{\{a, c\}, \varphi, Y\}$, $\sigma_2 = \{\varphi, Y\}$,
$\eta_1 = \{\{c\}, \{a, c\}, \varphi, Z\}$, $\eta_2 = \{\{a\}, \varphi, Z\}$.

If we consider the function f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ defined as an identity function then f is a (1, 2) - γ-semi-continuous function, since the inverse image of σ_1-open set in Y under f are: {a, c}, φ, X which are (1, 2) - γ-semi-open set in X. Again, consider the function g: $(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ defined as an identity function which is (1, 2)- γ-semi-continuous function , since the inverse image of the η_1-open set in Z under g are {c},{a, c}, φ , Y which are (1, 2)- γ-semi-open set in Y.

Now the mapping (gof): $(X,\tau_1,\tau_2)\to(Z,\eta_1,\eta_2)$ is not a (1, 2)- γ-semi-continuous function as the inverse image of η_1-open set in Z under (gof) is {c} which is not (1, 2)- γ-semi-open set in X.

Proposition 3.18 *Every (1, 2) - γ-continuous function is (1, 2) - γ-semi continuous function.*

Proof: Proof is obvious from the definition.

Remark 3.19 Converse of the above proposition may not be true as shown in the following example:

Example 3.20 Let X = Y = {a, b, c},
τ_1 = {{a}, {a, b}, φ, X }, τ_2 = {{ b} , φ, X },
σ_1 = {{a, c}, φ, Y }, σ_2 = {{a}, φ, Y }.

Now if we consider f: X $\rightarrow$ Y as an identity function then f is a (1, 2)- γ-semi-continuous function but f is not (1, 2)- γ-continuous, since the inverse image of the σ_1-open set {a, c} in Y under f is{a, c} which is not a (1, 2)- γ-open set in X.

4 (1, 2) -Semi- γ -Open Sets

In this section we study another generalization of (1, 2) - γ open set namely (1, 2)-semi-γ-open sets with the help of τ_2-closure operator in a bitopological space.

Definition 4.1 *A subset A of a bitopological space (X, τ_1, τ_2) is called (1, 2)-semi-γ-open set if A $\subseteq$ τ_2 -cl (τ_1-int$_\gamma$ (A)).*

The complement of (1, 2)-semi-γ-open set is called (1, 2)-semi-γ-closed set and is defined as A $\supseteq$ τ_2-int (τ_1-cl$_\gamma$ (A)).

The collection of all (1, 2)-semi-γ-open sets of (X, τ_1, τ_2) is denoted by (1, 2)-SγO(X).

Example 4.2 Let X = {a, b, c},
τ_1 = {{a}, {a, b}, φ, X} and τ_2 ={{b}, φ, X},

Thus (1, 2)-SγO(X) = {{a}, {a, b}, {a, c}, φ , X}.

Proposition 4.3 *In a bitopological space (X, τ_1, τ_2) every τ_1 - γ -open set is (1, 2)-semi-γ-open set.*

Proof: Let A be any τ_1 - γ-open set in X i.e. A = τ_1 -int $_\gamma$(A).
Now A $\subseteq$ τ_2-clA = τ_2 -cl (τ_1-int $_\gamma$(A)). Therefore A is (1, 2)-semi-γ-open set.

Remark 4.4 Converse of the above proposition need not be true as seen in the following example:

Example 4.5 Let X = {a, b, c},
τ_1 = {{a}, {b}, {a, b}, φ, X} and τ_2 ={{a, b}, φ, X}.

It is obvious that {a, c}$\in$ (1, 2)-SγO(X) but {a, c}$\notin$ τ_1-γO(X).

Proposition 4.6 *In a bitopological space X the following results are equivalent:*

(i) *A is (1, 2)-semi-γ-open set,*

(ii) *for any τ_1-γ-open set O such that O$\subseteq$A$\subseteq$ τ_2-(τ_1 -int$_\gamma$(A)),*

(iii) *τ_2 -cl(A)$-$A does not contain any τ_1 - γ-open set and*

(iv) *A is τ_1 - γ-open set if A$\cap$ τ_2-cl (τ_1-int$_\gamma$ (A)) is a τ_1 - γ-open set.*

Proof: Proof is straight forward.

It is established that, some special results in a bitopological space are obtained by many authors considering the topology τ_1 is weaker than τ_2 i.e. $\tau_1 \subseteq \tau_2$ and such bitopological space are denoted by (X, $\tau_1 \leq \tau_2$). Using these types of bitopology we study the following result:

Proposition 4.7 *In a bitopological space (X, $\tau_1 \leq \tau_2$), every (1, 2)-semi-γ-open set is τ_1-semi-γ-open set.*

Proof: Let A be any (1, 2) - semi-γ-open set in X. i.e. A$\subseteq$ τ_2-cl (τ_1-int$_\gamma$ (A)) $\subseteq$ τ_1-cl (τ_1-int $_\gamma$(A)). Hence the proof.

Remark 4.8 Converse of the above proposition need not be true as seen in the following example:

Example 4.9 Let X = {a, b, c},
τ_1 = {{a, c}, φ, X} and τ_2 ={{a, c}, {b}, φ , X}.

It is obvious that {b, c} is τ_1-semi-γ-open set but {b, c}$\notin$ (1, 2)-SγO(X).

Proposition 4.10 *In a bitopological space X, the union of any two (1, 2)-semi-γ open sets is again a (1, 2)-semi-γ-open set in X.*

Proof: Let A and B be any two (1, 2)-semi-γ-open set in X.
Now A$\cup$B$\subseteq$ τ_2-cl (τ_1-int$_\gamma$ (A))$\cup$ τ_2-cl (τ_1-int $_\gamma$(B))

$\Rightarrow A \cup B \subseteq \tau_2$-cl ($\tau_1$-int $_\gamma (A \cup B)$).

Hence $A \cup B$ is (1, 2)-semi-γ-open set in X.

Remark 4.11 In a bitopological space X, the intersection of any two (1, 2)-semi-γ-open sets may not be a (1, 2)-semi-γ-open set as seen in the following example:

Example 4.12 Let X = {a, b, c},
τ_1 = {{a}, {b}, {a, b}, φ, X} and τ_2 ={{a, b}, φ, X}.

Thus (1, 2)-SγO(X) = {{a}, {b}, {a, b}, {a, c}, {b, c}, φ, X}.

Here {a, c} $\cap$ {b, c} ={c} $\notin$ (1, 2)-SγO(X).

Thus in a bitopological space X the collection of all (1, 2)-semi-γ-open sets do not be form a topology.

Proposition 4.13 *If B is (1, 2)-semi-γ-open set in X and $B \subseteq A \subseteq \tau_2$-cl (B) then A is also (1, 2)-semi-γ-open set.*

Proof: Given B is (1, 2)-semi-γ-open set.
So we have $B \subseteq \tau_2$-cl (τ_1-int $_\gamma$(B)) $\subseteq \tau_2$-cl (τ_1-int $_\gamma$(A)).
Thus τ_2 -cl (B) $\subseteq \tau_2$-cl (τ_1-int $_\gamma$(A).Hence A is also (1, 2)-semi-γ-open set.

Proposition 4.14 *In a bitopological space X every τ_1 τ_2 -semi-open set is a (1, 2)-semi-γ-open set.*

Proof: Let A be any τ_1 τ_2 -semi-open set in X.
So $A \subseteq \tau_2$-cl (τ_1-int (A).

Now τ_2-cl (τ_1-int (A) $\subseteq$ τ_2-cl (τ_1-int $_\gamma$(A).
Hence A is a (1, 2)-semi-γ-open set.

Remark 4.15 Converse of the above proposition may not be true as shown in the following example:

Example 4.16 Let X = {a, b, c},
τ_1= {{a, b}, φ, X} and τ_2 = {{a, c}, φ, X}.

Thus (1, 2)-SγO(X) = {{a}, {b}, {a, c}, {a, b}, φ, X} and

(1, 2)-SO(X) = {{a, b}, φ, X}.

It is obvious that {a, c} is (1, 2)-semi-γ-open set but {a, c}$\notin$ (1, 2)-SO(X).

Proposition 4.17 *In a bitopological space X every (1, 2) - γ-semi-open set is (1, 2)-semi-γ-open set.*

Proof: Let A be any (1, 2) - γ-semi-open set. So A$\subseteq \tau_2$-cl $_\gamma$(τ_1-int $_\gamma$(A). Now for any τ_2 - γ-closed set containing τ_1-int $_\gamma$(A) there exist τ_2-closed set greater than or equal to the given τ_2 - γ-closed set.
Thus A$\subseteq \tau_2$-cl $_\gamma$(τ_1-int $_\gamma$(A)) $\subseteq \tau_2$-cl (τ_1-int $_\gamma$(A)). Hence the proof.

Remark 4.18 Converse of the above proposition may not be true as shown in the following example:

Example 4.19 Let X = {a, b, c},
τ_1= {{b, c}, φ, X} and τ_2 = {{a, c}, φ, X}.

Thus (1, 2)-SγO(X) = {{b}, {c}, {a, c} {b, c}, φ, X} and
(1, 2)- γSO(X) = {{b}, {c}, {b, c}, φ, X}.

Remark 4.20 In a bitopological space X the concept of (1, 2)-semi-γ-open set and (1, 2)*- γ-semi-open set are independent of each other as seen in the following example:

Example 4.21 (i) Let X = {a, b, c},
τ_1= {{a}, {a, b}, φ, X} and τ_2 = {{b }, φ, X}.

Thus (1, 2)-SγO(X) = {{a}, {a, b}, {a, c}, φ, X} and
(1, 2)*- γSO(X) = {{a}, {b}, {a, b}, {b, c}, φ, X}.

(ii) Let X = {a, b, c}, τ_1= {{a, c}, φ, X} and τ_2 = {{b, c}, φ, X}.

Thus (1, 2)-SγO(X) = {{a}, {c}, {a, c}, {b, c}, φ, X} and
(1, 2)*- γSO(X) = {{c}, {a, c}, {b, c}, φ, X}.

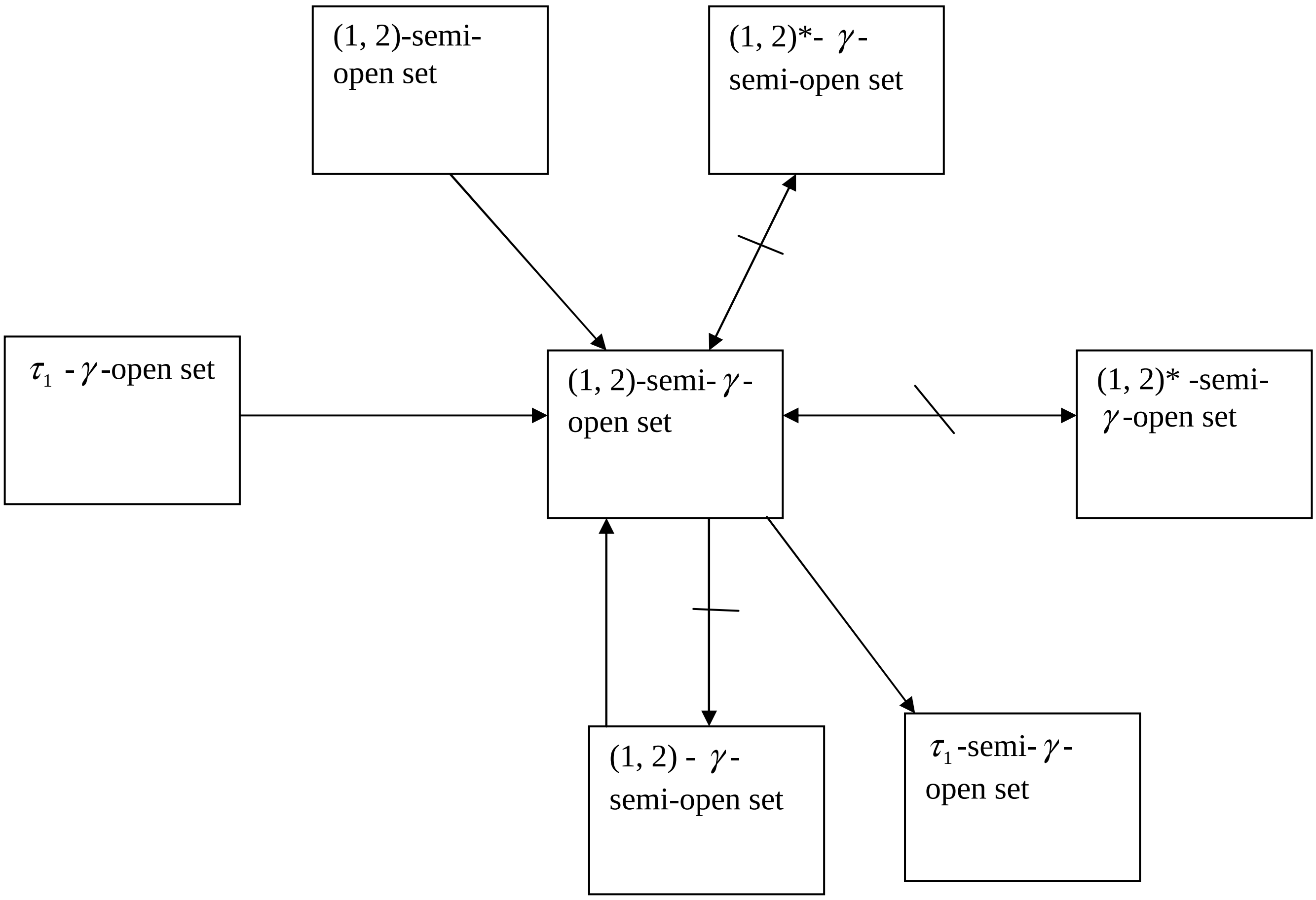
Remark 4.22 In a bitopological space X the concept of (1, 2)-semi-γ-open set and (1, 2)* -semi-γ-open set are independent of each other as shown in the following example:

Example 4.23 If we follow the above example (4.21) then we get

(i) (1, 2)-SγO(X) = {{a}, {a, b}, {a, c}, φ, X} and
(1, 2)*- SγO(X) = {{a}, {b}, {a, b}, φ, X}.

(ii) (1, 2)-SγO(X) = {{a}, {c}, {a, c}, {b, c}, φ, X} and
(1, 2)*- SγO(X) = {{c}, {a, c}, {b, c}, φ, X}.

From the above study we can draw the following diagram:



5 (1, 2) -Semi-γ-Continuity

Definition 5.1 *A function f: $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is called (1, 2)-semi-γ-continuous if the inverse image of each σ_1-open set in Y is (1, 2) -semi-γ-open set in X.*

Example 5.2 Let X = Y = {a, b, c},
τ_1 = {{a, b}, φ, X }, τ_2 = {{a, b}, φ, X },

σ_1 = {{a}, φ, Y } and σ_2 = {{a, b}, φ, Y }.

We get τ_1- γO(X) = {{a}, {b}, {a, b}, φ, X } and
(1, 2)- S γO(X) = {{a}, {b}, {a, b}, {b, c}, {a, c}, φ, X }.

Now for the identity function f: X$\rightarrow$ Y we can prove that f is an (1, 2)-semi-γ-continuous function.

Proposition 5.3 *Every (1, 2) - γ-continuous function is (1, 2)-semi-γ-continuous function.*

Proof: Proof is obvious from the definition.

Remark 5.4 Converse of the above proposition may not be true as shown in the following example:

Example 5.5 Let X =Y = {a, b, c},
τ_1= {{a}, {b}, {a, b}, φ, X}, τ_2 = {{b}, φ, X },
σ_1 = {{a}, {a, c}, φ, Y } and σ_2 = {{ a, b}, φ, Y }.

Thus (1, 2) - S γO(X) = {{a}, {b}, {a, b}, {b, c}, {a, c}, φ, X }.

Now for the identity function f: X$\rightarrow$ Y we can prove that f is an (1, 2)-semi-γ-continuous function but f $^{-1}${a, c} = {a, c}$\notin$(1, 2) - γ-O(X). Hence f is not an (1, 2) - γ-continuous function.

Definition 5.6 *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces and f: X$\rightarrow$Y is any function. Then f is said to be (1, 2)-semi-γ-open mapping if the image of each τ_1-open set in X is (1, 2)-semi-γ-open set in Y.*

Remark 5.7 Composition of any two (1, 2)-semi-γ-continuous functions may not be an (1, 2)-semi-γ-continuous function as shown in the following example:

Example 5.8 Let X = Y = Z = {a, b, c},
τ_1= {{a}, φ, X }, τ_2 = {{ a, b} , φ, X }, σ_1 ={{a, c}, φ, Y } , σ_2 = { φ, Y },
η_1 = {{c}, {a, c}, {a}, φ, Z} and η_2 = {{a, b}, φ, Z }.

Thus (1, 2) - S γO(X) = {{a}, {a, b}, {a, c}, φ, X } and
(1, 2) - S γO(Y) = {{a}, {c}, {a, c}, {a, b}, {b, c}, φ, Y }.

Now for the identity functions f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z we can prove that f and g are (1, 2)-semi-γ-continuous function.

But $\{gof\}^{-1}\{c\} = \{c\} \notin$ (1, 2) - SγO(X).

Thus composition of any two (1, 2)-semi-γ-continuous functions may not be an (1, 2)-semi-γ-continuous function.

Proposition 5.9 *Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2) \rightarrow (Z,\eta_1,\eta_2)$ be two maps. Then if*

(i) (gof) is (1, 2)-semi-γ-open map and f is (1, 2)-continuous and injective then g is (1, 2) -semi-γ-open map.

(ii) Every τ_1-open set under (gof) is $\eta_1\eta_2$-open set and g is (1, 2)-semi-γ-continuous and injective then f is (1, 2)-semi - γ-open.

Proof:

(i) Let A be any σ_1-open set in Y. Since f is (1, 2)-continuous f^{-1} (A) is (1, 2)-open set in X. Now (gof) (f^{-1} (A)) = g (f (f^{-1} (A)) = g (A) (since f is injevtive) is (1, 2)-semi-γ-open map. Hence g is (1, 2) - semi-γ -open map.

(ii) Let A be any τ_1-open set in X. Then (gof) (A) is $\eta_1\eta_2$-open set in Z.
Now since g is (1, 2)-semi-γ -continuous so g^{-1} ((gof) (A)) = g^{-1}(f (A))) = f (A) (since g is injective) is (1, 2)-semi-γ-open set in Y. Hence f is (1, 2)-semi-γ-open map.

Proposition 5.10 *Let f^{-1} : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ be bijective. Then the following conditions are equivalent:*

(i) f is a (1, 2) semi-γ-open map,
(ii) f is (1, 2)-semi-γ-closed map and
(iii) f^{-1} is (1, 2)-semi-γ-continuous map.

Proof: (i) $\rightarrow$ (ii) Suppose B is a τ_1-closed set in X. Then (X $-$ B) is an τ_1-open set in X. Now by (i) f(X $-$ B) is an (1, 2)-semi-γ-open set in Y. Now since f^{-1} is bijective so f(X $-$ B) =Y $-$ f (B) .Hence f (B) is an (1, 2)-semi-γ-closed set in Y.Therefore f is a (1, 2)-semi-γ-closed map.

(ii)$\rightarrow$ (iii). Let f is an (1, 2)-semi-γ-closed map and B be τ_1-closed set of X. Since f^{-1} is bijective so $(f^{-1})^{-1}$(B) which is an (1, 2) semi-γ-closed set in Y. Hence f^{-1} is (1, 2)-semi-γ -continuous map.

(iii)$\rightarrow$ (i). Let A be an τ_1-open set in X. Since f^{-1} is a (1, 2)-semi-γ -continuous map so $(f^{-1})^{-1}$(A) = f (A) is a (1, 2)-semi-γ-open set in Y. Hence f is (1, 2)-semi-γ-open map.

Proposition 5.11 *Let X and Y are two bitopological spaces. Then f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is (1, 2)-semi-γ-continuous if one of the followings hold:*

(i) $f^{-1}(\tau_1\text{-}int\,(B))\subseteq \tau_1\text{-}int_{\gamma}(f^{-1}(B))$*, for every* τ_1*-open set B in Y.*

(ii) $\tau_1\text{-}cl_{\gamma}(f^{-1}(B))\subseteq f^{-1}(\tau_1\text{-}cl\,(B))$*, for every* τ_1*-closed set B in Y.*

Proof: Let B be any τ_1-open set in Y and if condition (i) is satisfied then $f^{-1}(\tau_1\text{-int}(B))\subseteq \tau_1\text{-int}_{\gamma}(f^{-1}(B))$.

We get $f^{-1}(B) \subseteq \tau_1\text{-int}_{\gamma}(f^{-1}(B))$. Therefore f^{-1}(B) is a τ_1-open set in X. Hence f is (1, 2) - γ-semi-continuous.

Similarly we can prove (ii).

Proposition 5.12 *A map f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is called (1, 2)-semi-γ-open map iff* $f(\tau_1\text{-}int\,(A))\subseteq \tau_1\text{-}int_{\gamma}(f(A))$*, for every* τ_1*-open set A in X.*

Proof: Suppose that f is an (1, 2)-semi-γ-open map.
Since τ_1-int (A) $\subseteq$ A so f (τ_1-int (A)) $\subseteq$ f (A).

By hypothesis f (τ_1-int (A)) is an (1, 2)-semi-γ-open set and $\tau_1\text{-int}_{\gamma}$(f (A)) is largest (1, 2)-semi-γ-open set contained in f (A) so f (τ_1-int (A)) $\subseteq \tau_1\text{-int}_{\gamma}$ f ((A)).

Conversely suppose A is an τ_1-open set in X. So
f (τ_1-int (A)) $\subseteq \tau_1\text{-int}_{\gamma}$ f ((A)).

Now since A = τ_1-int (A) so f (A) $\subseteq \tau_1\text{-int}_{\gamma}$ f ((A)).Therefore f (A) is an (1, 2)-semi-γ-open set in Y and f is (1, 2)-semi-γ-open map.

Proposition 5.13 *A map f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1, 2)-semi-γ-closed map iff τ_1-cl $_\gamma$(f (A))$\subseteq$ f (τ_1-cl (A)), for every τ_1-closed set A in X.*

Proof: Suppose that f is a (1, 2)-semi-γ-closed map.
Since A $\subseteq$ τ_1-cl A so f (A) $\subseteq$ f (τ_1-cl (A)).

By hypothesis, f (τ_1-cl (A)) is a (1, 2)-semi-γ-closed set and τ_1-cl $_\gamma$(f (A)) is smallest (1, 2)-semi-γclosed set containing f (A) so τ_1-cl $_\gamma$(f (A)) $\subseteq$ f (τ_1-cl (A)).
Conversely suppose A is an τ_1-closed set in X.
So τ_1-cl $_\gamma$(f (A))$\subseteq$ f (τ_1-cl (A)).

Since A = τ_1-cl (A) so τ_1-cl $_\gamma$(f (A))$\subseteq$ f (A).Therefore f (A) is a (1, 2)-semi-γ-closed set in Y and f is (1, 2)-semi-γ-closed map.

Remark 5.14 Let f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and g: $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be any two functions then (gof) is (1, 2)-semi-γ- continuous function if g is (1, 2)-continuous and f is (1, 2)-semi-γ-continuous.

Proposition 5.15 *Every τ_1-semi-γ-continuous is (1, 2)-γ-continuous.*

Proof: Proof is obvious.

Remark 5.16 Converse of the above proposition may not be true in general as shown in the following example:

Example 5.17 Let X = Y = {a, b, c},

τ_1= {{a, c}, φ, X }, τ_2 = { φ, X },
σ_1 = {{a}, {b, c}, φ, Y} and σ_2 = {{b}, {b, c}, φ, Y }.

We get τ_1-SγO(X) = {{a}, {c}, {a, c}, φ, X} and

(1, 2) - SγO(X) = {{a}, {c}, {a, c}, {a, b}, {b, c}, φ, X }.

Now if the function f: X$\rightarrow$Y be defined as identity function then one can show that f is (1, 2)-semi-γ-continuous but not τ_1-semi-γ- continuous since f^{-1}{b, c} = {b, c}$\notin$ τ_1-SγO(X).

6 Conclusion

We studied new weaker forms of γ-open sets in bitopological spaces. In general it is established that in bitopological space a (1, 2)-semi-γ-open set may not be a τ_1- γ-open set, but from the present work we conclude that every (1, 2)-semi-γ-open set is a τ_1-γ-open set if there exist a τ_1-γ-open set such that it can be expressed as an intersection of given (1, 2)-semi-γ-open set and smallest τ_2-closed set which contains greatest τ_1-γ-open set contained in given (1, 2)-semi-γ-open set. Further it is clear that the collection of all (1, 2)-semi-γ-open set and (1, 2) - γ-semi-open set forms a supra topology. It is also shown that for any two functions the composition is (1, 2)-semi-γ-continuous if the second function is (1, 2)-continuous and the first function is (1, 2)-semi-γ-continuous even composition of two (1, 2)-semi-γ-continuous functions may not be (1, 2)-semi-γ-continuous function.

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