Gen. Math. Notes, Vol. 28, No. 1, May 2015, pp.81-91
ISSN 2219-7184; Copyright ©ICSRS Publication, 2015
www.i-csrs.org
Available free online at http://www.geman.in

Constant Ratio Curves According to Bishop Frame in Euclidean 3-Space $\mathbb{E}^3$

Sezgin Büyükkütük[1] and Günay Öztürk[2]

[1,2]Kocaeli University, Art and Science Faculty
Department of Mathematics, Kocaeli, Turkey
[1]E-mail: sezgin.buyukkutuk@kocaeli.edu.tr
[2]E-mail: ogunay@kocaeli.edu.tr

(Received: 20-2-15 / Accepted: 4-5-15)

Abstract

In the present paper, we consider a curve in Euclidean 3*-space* $\mathbb{E}^3$ *as a curve whose position vector can be written as linear combination of its Bishop frame. We characterize such curves in terms of their curvature functions. Further, we obtain some results of* T*-constant and* N*-constant type curves in Euclidean* 3*-space* $\mathbb{E}^3$*.*

Keywords: *Bishop frame, Position vector, constant-ratio curves.*

1 Introduction

A curve $x : I \subset \mathbb{R} \to \mathbb{E}^3$ in Euclidean 3-space is called a *twisted curve* if it has nonzero Frenet curvatures $\kappa_1(s)$ and $\kappa_2(s)$. From the elementary differential geometry it is well known that a curve $x(s)$ in $\mathbb{E}^3$ lies on a sphere if its position vector (denoted also by x) lies on its normal plane at each point. If the position vector x lies on its rectifying plane then $x(s)$ is called *rectifying curve* [7]. Rectifying curves characterized by the simple equation

$$x(s) = \lambda(s)T(s) + \mu(s)N_2(s), \tag{1}$$

where $\lambda(s)$ and $\mu(s)$ are smooth functions and $T(s)$ and $N_2(s)$ are tangent and binormal vector fields of x respectively [7]. In the same paper B. Y. Chen gave a simple characterization of rectifying curves. In particular it is shown in [10] that there exists a simple relation between rectifying curves and centrodes, which play an important role in mechanics kinematics as well as in differential geometry in defining the curves of constant procession. It is also provide that a twisted curve is congruent to a non constant linear function of s [8]. Further, in the Minkowski 3-space $\mathbb{E}_1^3$, the rectifying curves are investigated in ([11, 16, 17, 18]). In [18] a characterization of the spacelike, the timelike and the null rectifying curves in the Minkowski 3-space in terms of centrodes is given. For the characterization of rectifying curves in three dimensional compact Lee groups or in dual spaces see [23] or [4] respectively. In [1], the authors determined position vector of a general helix with respect to Frenet frame. They deduced the natural representation of a general helix in terms of the curvature and torsion with respect to standard frame of Euclidean 3-space. Furthermore, in [2], the authors studied position vector of a slant helix with respect to standard frame in Euclidean space $\mathbb{E}^3$ in terms of Frenet equations.

For a regular curve $x(s)$, the position vector x can be decompose into its tangential and normal components at each point:

$$x = x^T + x^N. \tag{2}$$

A curve $x(s)$ with $\kappa(s) > 0$ is said to be of *constant ratio* if the ratio $\|x^T\| : \|x^N\|$ is constant on $x(I)$ where $\|x^T\|$ and $\|x^N\|$ denote the length of x^T and x^N, respectively [6].

A curve in $\mathbb{E}^n$ is called *T-constant* (resp. *N-constant*) if the tangential component x^T (resp. the normal component x^N) of its position vector x is of constant length [6]. Recently, in [15], the authors give the necessary and sufficient conditions for twisted curves in Euclidean 3-space $\mathbb{E}^3$ to become T-constant or N-constant.

The ability to "ride" along a three-dimensional space curve and illustrate the properties of the curve, such as curvature and torsion, would be a great asset to Mathematicians. The classic Serret-Frenet frame provides such ability, however the Frenet-Serret frame is not defined for all points along every curve. A new frame is needed for the kind of Mathematical analysis that is typically done with computer graphics. The Relatively Parallel Adapted Frame or Bishop Frame could provide the desired means to ride along any given space curve [3]. The Bishop Frame has many properties that make it ideal for mathematical research. Another area of interested about the Bishop Frame is so-called Normal Development, or the graph of the twisting motion of Bishop Frame. This information along with the initial position and orientation of the Bishop Frame provide all of the information necessary to define the curve. The Bishop frame may have applications in the area of Biology and Computer Graphics. For example it may be possible to compute information about the shape of sequences of DNA using a curve defined by the Bishop frame. The Bishop frame may also provide a new way to control virtual cameras in computer animations. In [19] authors studied natural curvatures of Bishop frame. In [5], the same authors considered slant helix according to Bishop frame in Euclidean 3-Space. In [21], authors researched the spinor formulations of curves according to Bishop frames in $\mathbb{E}^3$.

In [22], the authors investigated position vectors of some special spacelike curves with respect to Bishop frame in $\mathbb{E}^3_1$. They presented some characterizations of curves with the components of position vector on the Bishop axis. Furthermore, in [20], the authors studied classical differential geometry of the curves according to type-2 Bishop trihedra. Also, they investigated position vector of a regular curve by a system of ordinary differential equations whose solution gives the components of the position vector with respect to type-2 Bishop frame. They presented some special characterizations introducing special planes of three dimensional Euclidean space.

In [13], the authors gave parallel transport frame of a curve and they introduce the relations between the Bishop frame and Frenet frame of the curve in 4-dimensional Euclidean space. They characterized curves whose position vectors lie in their normal, rectifying and osculating planes in 4-dimensional Euclidean space $\mathbb{E}^4$.

In the present study, we consider a curve in Euclidean 3-space $\mathbb{E}^3$ as a curve whose position vector can be written as linear combination of its Bishop frame. Then its position vector satisfies the parametric equation

$$x(s) = m_0(s)T(s) + m_1(s)M_1(s) + m_2(s)M_2(s), \tag{3}$$

for some differentiable functions, $m_i(s)$, $0 \leq i \leq 2$, where $\{T, M_1, M_2\}$ is its Bishop frame. We characterize such curves in terms of their curvature functions $m_i(s)$ and give the necessary and sufficient conditions for such curves to become T-constant or N-constant.

2 Basic Notation

Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in Euclidean 3-space $\mathbb{E}^3$. Let us denote $T(s) = x'(s)$ and call $T(s)$ as a unit tangent vector of x at s. We denote the curvature of x by $\kappa(s) = \|x''(s)\|$. If $\kappa(s) \neq 0$, then the unit principal normal vector $N_1(s)$ of the curve x at s is given by $x''(s) = \kappa(s)N_1(s)$. The unit vector $N_2(s) = T(s) \times N_1(s)$ is called the unit binormal vector of x at s. Then we have the Serret-Frenet formulae:

$$\begin{aligned} T'(s) &= \kappa(s)N_1(s), \\ N_1'(s) &= -\kappa(s)T(s) + \tau(s)N_2(s), \\ N_2'(s) &= \tau(s)N_1(s), \end{aligned} \tag{4}$$

where $\tau(s)$ is the torsion of the curve x at s (see, [12] and [14]).

For a space curve $x : I \subset \mathbb{R} \to \mathbb{E}^3$, the planes at each point of $x(s)$ the spanned by $\{T, N_1\}$, $\{T, N_2\}$ and $\{N_1, N_2\}$ are known as the *osculating plane*, the *rectifying plane* and *normal plane* respectively. If the position vector x lies on its rectifying plane then $x(s)$ is called *rectifying curve*. Similarly, the curve for which the position vector x always lies in its osculating plane is called *osculating curve*. Finally, x is called *normal curve* if its position vector x lies in its normal plane.

From elementary differential geometry it is well known that a curve in $\mathbb{E}^3$ lies in a plane if its position vector lies in its osculating plane at each point, and lies on a sphere if its position vector lies in its normal plane at each point [7].

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well-defined even when the curve has vanishing second derivative. One can express parallel transport of an orthonormal frame along a curve simply by parallel transporting each component of the frame. The tangent vector and any convenient arbitrary basis for the remainder of the frame are used. Therefore, the Bishop (frame) formulas are expressed as

$$\begin{bmatrix} T' \\ M_1' \\ M_2' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} T \\ M_1 \\ M_2 \end{bmatrix}$$

where $\{T, M_1, M_2\}$ is the Bishop Frame and k_1, k_2 are called first and second Bishop curvatures, respectively [3].

The relation between Frenet frame and Bishop frame is given as follows:

$$\begin{bmatrix} T \\ N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} T \\ M_1 \\ M_2 \end{bmatrix}$$

where $\theta(s) = \arctan\left(\frac{k_2}{k_1}\right)$, $\tau(s) = \left(\frac{d\theta(s)}{ds}\right)$ and $\kappa(s) = \sqrt{k_1^2 + k_2^2}$. Here Bishop curvatures are defined by $k_1 = \kappa\cos\theta$, $k_2 = \kappa\sin\theta$.

3 Characterization of Curves According to its Bishop Frame in $\mathbb{E}^3$

In the present section we characterize the curves in $\mathbb{E}^3$ in terms of their curvature functions according to Bishop Frame.

Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve with Bishop curvatures $k_1(s)$ and $k_2(s)$. By definition of the position vector of the curve (also defined by x) satisfies the vectorial equation (3), for some differential functions $m_i(s)$,

$0 \leq i \leq 2$. By taking the derivative of (3) with respect to arclength parameter s and using the Bishop formulas (4), we obtain

$$\begin{aligned} x^{'}(s) &= (m_0^{'}(s) - k_1(s)m_1(s) - k_2(s)m_2(s))T(s) \\ &\quad +(m_1^{'}(s) + k_1(s)m_0(s))M_1(s) \\ &\quad +(m_2^{'}(s) + k_2(s)m_0(s))M_2(s) \end{aligned} \tag{5}$$

It follows that,

$$\begin{aligned} m_0^{'} - k_1m_1 - k_2m_2 &= 1 \\ m_1^{'} + k_1m_0 &= 0 \\ m_2^{'} + k_2m_0 &= 0 \end{aligned} \tag{6}$$

Lemma 3.1 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$ with the vectorial equation (3). Then position vector x satisfies the curvature conditions in the equation (6).*

3.1 Curves of Constant-Ratio

Definition 3.2 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^n$ be a unit speed curve in $\mathbb{E}^3$. Then the position vector x can be decompose into its tangential and normal components at each point as in (2). If the ratio $\left\|x^T\right\| : \left\|x^N\right\|$ is constant on $x(I)$ then x is said to be of constant-ratio [7].*

For a unit speed curve x in $\mathbb{E}^n$, the gradient of the distance function $\rho = \|x(s)\|$ is given by

$$grad\rho = \frac{d\rho}{ds}x'(s) = \frac{< x(s), x'(s) >}{\|x(s)\|}x'(s) \tag{7}$$

where T is the tangent vector field of x.

Theorem 3.3 *[9] Let $x : I \subset \mathbb{R} \to \mathbb{E}^n$ be a unit speed regular curve in $\mathbb{E}^n$. Then $\|grad\rho\| = c$ holds for a constant c if and only if one of the following three cases occurs:*

(i) $x(I)$ is contained in a hypersphere centered at the origin.

(ii) $x(I)$ is an open portion of a line through the origin.

(iii) $x(s) = csy(s)$, $c \in (0, 1)$, where $y = y(u)$ is a unit curve on the unit sphere of $\mathbb{E}^n$ centered at the origin and $u = \frac{\sqrt{1-c^2}}{c} \ln s$.

Corollary 3.4 *[9] Let $x : I \subset \mathbb{R} \to \mathbb{E}^n$ be a unit speed regular curve in $\mathbb{E}^n$. Then up to a translation of the arc length function s, we have*

i) $\|grad\rho\| = 0 \iff x(I)$ is contained in a hypersphere centered at the origin.

ii) $\|grad\rho\| = 1 \iff x(I)$ is an open portion of a line through the origin.

iii) $\|grad\rho\| = c \iff \rho = \|x(s)\| = cs$, for $c \in (0,1)$.

iv) If $n = 2$ and $\|grad\rho\| = c$ for $c \in (0,1)$, then the curvature of x satisfies

$$\kappa^2 = \frac{1-c^2}{c^2\left(s^2+b\right)},$$

for some real constant b.

The following result characterize constant-ratio curves in $\mathbb{E}^3$ with its Bishop frame and Bishop curvatures.

Proposition 3.5 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$. If x is of constant-ratio then the position vector of the curve has the parametrization of the form*

$$\begin{aligned} x(s) &= \left(c^2 s\right) T(s) + \left(\frac{c^2 k_2(k_1^2+k_2^2)s - k_2'(c^2-1)}{k_1' k_2 - k_2' k_1}\right) M_1(s) \\ &+ \left(\frac{c^2-1}{k_2} - \frac{k_1}{k_2}\left(\frac{c^2 k_2(k_1^2+k_2^2)s - k_2'(c^2-1)}{k_1' k_2 - k_2' k_1}\right)\right) M_2(s) \end{aligned}$$

for some differentiable functions c,d $\in [0,1)$.

Proof. Let x be a curve of constant-ratio given with arclength function s. Then, from the previous result the distance function ρ of x satisfies the equality $\rho = \|x(s)\| = cs$ for some real constant c. Further, using (7) we get

$$\|grad\rho\| = \frac{< x(s), x'(s) >}{\|x(s)\|} = c. \tag{8}$$

Since, x is a curve of $\mathbb{E}^3$, then it satisfies the equality (3). So, we get $m_0 = c^2 s$. Hence, substituting this value into the equations in (6) one can get,

$$\begin{aligned} m_1(s) &= \frac{c^2 k_2(k_1^2+k_2^2)s - k_2'(c^2-1)}{k_1' k_2 - k_2' k_1}, \\ m_2(s) &= \frac{c^2-1}{k_2} - \frac{k_1}{k_2}\left(\frac{c^2 k_2(k_1^2+k_2^2)s - k_2'(c^2-1)}{k_1' k_2 - k_2' k_1}\right). \end{aligned} \tag{9}$$

Substituting these values into (3), we obtain the desired result. ∎

As a consequence of (9) with the (6), we get the following result.

Corollary 3.6 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$. Then x is of constant-ratio if and only if*

$$\left(\frac{c^2 k_2(k_1^2+k_2^2)s + (1-c^2)\, k_2'}{k_1' k_2 - k_2' k_1}\right)' + c^2 k_1 s = 0.$$

3.2 T-Constant Curves

Definition 3.7 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^n$ be a unit speed curve in $\mathbb{E}^n$. If $\left\| x^T \right\|$ is constant then x is called a T-constant curve [7]. For a T-constant curve x, either $\left\| x^T \right\| = 0$ or $\left\| x^T \right\| = \lambda$ for some non-zero smooth function λ. Further, a T-constant curve x is called first kind if $\left\| x^T \right\| = 0$, otherwise second kind.*

As a consequence of (6), we get the following result.

Theorem 3.8 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$ with the position vector which satisfies the equation (3). Then x is a T-constant curve of first kind, if and only if*

$$c_1 k_1 + c_2 k_2 + 1 = 0 \tag{10}$$

where c_1, c_2 are integral constants.

Proof. Let x is a T-constant curve of first kind. Then, from the second and third equalities in (6) we get $m_1' = 0$ and $m_2' = 0$. Further substituting $m_1 = c_1$ and $m_2 = c_2$ into the first equation we get the result. ∎

Theorem 3.9 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$. If x is a T-constant curve of second kind, then the position vector of the curve has the parametrization of the form*

$$x(s) = m_0 T(s) + \frac{k_2(k_1^2 + k_2^2)m_0 - k_2'}{k_1' k_2 + k_1 k_2'} M_1(s) + \left[\frac{k_1 \left(k_2(k_1^2 + k_2^2)m_0 - k_2' \right)}{k_2(k_1' k_2 + k_1 k_2')} + \frac{1}{k_2} \right] M_2(s), \tag{11}$$

where m_0 is a constant function.

Proof. Suppose that x is a T-constant curve of second kind. Then, by the use of (6), we get

$$k_1' m_1 + k_2' m_2 = (k_1^2 + k_2^2) m_0 \tag{12}$$

Also substituting $m_2 = \frac{k_1 m_1 + 1}{k_2}$ into (12), we get

$$m_1 = \frac{k_2(k_1^2 + k_2^2)m_0 - k_2'}{k_1' k_2 + k_1 k_2'},$$

and

$$m_2 = \frac{k_1 \left(k_2(k_1^2 + k_2^2)m_0 - k_2' \right)}{k_2(k_1' k_2 + k_1 k_2')} + \frac{1}{k_2}.$$

Substituting this values into (3), we obtain the result.

As a consequence of (6), we get the following result. ∎

Corollary 3.10 *Let $x : I \subset \mathbb{R} \rightarrow \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$. If x is a T-constant curve of second kind, then the curvatures functions m_i of the curve x satisfy the equation*

$$2m_0 s + c = m_1^2 + m_2^2 \tag{13}$$

where c is a integral constant.

Proof. Let x is a T-constant curve of second kind, from the second and third equalities in (6), we get

$$k_1 = -\frac{m_1'}{m_0}, \quad k_2 = -\frac{m_2'}{m_0}.$$

Substituting this values into first equation in (6), we obtain the differantial equation

$$m_1' m_1 + m_2' m_2 = m_0$$

which has the solution (13). ■

3.3 N-Constant Curves

Definition 3.11 *Let $x : I \subset \mathbb{R} \rightarrow \mathbb{E}^n$ be a unit speed curve in $\mathbb{E}^n$. If $\|x^N\|$ is constant then x is called a N-constant curve. For a N-constant curve x, either $\|x^N\| = 0$ or $\|x^N\| = \mu$ for some non-zero smooth function μ [7]. Further, a N-constant curve x is called first kind if $\|x^N\| = 0$, otherwise second kind.*

So, for a N-constant curve x

$$\|x^N(s)\|^2 = m_1^2(s) + m_2^2(s) \tag{14}$$

becomes a constant function.

As a consequence of (3) and (6) with (14) we get the following result.

Lemma 3.12 *Let $x : I \subset \mathbb{R} \rightarrow \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$. Then x is a N-constant curve if and only if*

$$\begin{aligned} m_0' &= k_1 m_1 + k_2 m_2 + 1 \\ m_1' &= -k_1 m_0 \\ m_2' &= -k_2 m_0 \\ 0 &= m_1 m_1' + m_2 m_2' \end{aligned} \tag{15}$$

hold, where $m_0(s)$, $m_1(s)$ and $m_2(s)$ are differentiable functions.

Theorem 3.13 *Let $x : I \subset \mathbb{R} \rightarrow \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$. Then x is a N-constant curve of first kind if and only if $x(I)$ is an open portion of a straight line.*

Proof. Suppose that x is N-constant curve of $\mathbb{E}^3$, then the equality (14) holds. Further if x is of first kind then from (14) $m_1 = m_2 = 0$ which implies that $k_1 = k_2 = 0$. Then the first Frenet curvature of the curve x is zero. So x is a part of a straight line. ■

Theorem 3.14 *Let $x : I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve in $\mathbb{E}^3$ and s be its arclenght function. If x is a N-constant curve of second kind, then x is a T-constant curve of first kind with the parametrization*

$$x(s) = \lambda M_1(s) + \mu M_2(s) \tag{16}$$

where λ and μ are real constants or the curve has the parametrization

$$x(s) = (s+b)T(s) + \left(\pm\sqrt{\frac{ck_2^2}{k_1^2+k_2^2}}\right) M_1(s) + \left(\pm\sqrt{\frac{ck_1^2}{k_1^2+k_2^2}}\right) M_2(s) \tag{17}$$

where b and c are real constants.

Proof. Let x is a N-constant curve of second kind then the equation (15) holds. So we get $m_0(k_1 m_1 + k_2 m_2) = 0$. Hence, there are two possible cases; $m_0 = 0$ or $k_1 m_1 + k_2 m_2 = 0$. The first case with the equation (15) implies that $m_1 = \lambda = const$, $m_2 = \mu = const$. So x is a T-constant curve of first kind with the parametrization (16). For the second case by the use of (15), we get

$$\begin{aligned} m_0 &= s + b \\ m_1 &= \pm\sqrt{\frac{ck_2^2}{k_1^2+k_2^2}} \\ m_2 &= \pm\sqrt{\frac{ck_1^2}{k_1^2+k_2^2}} \end{aligned}$$

which completes the proof of the theorem. ■

References

[1] A.T. Ali, Position vectors of general helices in Euclidean 3-space, *Bulletin of Math. Anal. and Appl.*, 3(2) (2011), 198-205.

[2] A.T. Ali, Position vectors of slant helices in Euclidean 3-space, *Journal of the Egyptian Math. Soci.*, 20(2012), 1-6.

[3] L.R. Bishop, There is more than one way to frame a curve, *Amer. Math. Monthly*, 82(3) (1975), 246-251.

[4] Z. Bozkurt, I. Gök, O.Z. Okuyucu and F.N. Ekmekci, Characterization of rectifying, normal and osculating curves in three dimensional compact Lie groups, *Life Sci.*, 10(2013), 819-823.

[5] B. Bukcu and M.K. Karacan, The slant helices according to Bishop frame, *World Academy of Science, Engineering and Technology*, 3(2009), 842-845.

[6] B.Y. Chen, Constant ratio hypersurfaces, *Soochow J. Math.*, 28(2001), 353-362.

[7] B.Y. Chen, Convolution of Riemannian manifolds and its applications, *Bull. Aust. Math. Soc.*, 66(2002), 177-191.

[8] B.Y. Chen, When does the position vector of a space curve always lies in its rectifying plane? *Amer. Math. Monthly*, 110(2003), 147-152.

[9] B.Y. Chen, More on convolution of Riemannian manifolds, *Beitrage Algebra Geom.*, 44(2003), 9-24.

[10] B.Y. Chen and F. Dillen, Rectifying curves as centrodes and extremal curves, *Bull. Inst. Math. Acedemia Sinica*, 33(2005), 77-90.

[11] R. Ezentaş and S. Türkay, Helical versus of rectifying curves in Lorentzian spaces, *Dumlıpınar Univ. Fen Bilim. Ensti. Dergisi*, 6(2004), 239-244.

[12] H. Gluck, Higher curvatures of curves in Euclidean space, *Amer. Math. Monthly*, 73(1966), 699-704.

[13] F. Gökçelik, Z. Bozkurt Z., İ. Gök, F.N. Ekmekçi and Y. Yaylı, Parallel transport frame in 4-dimensional Euclidean space $\mathbb{E}^4$, *Caspian J. of Math. Sci.*, 3(1) (2014), 91-102.

[14] A. Gray, *Modern Differential Geometry of Curves and Surface*, CRS Press Inc., (1993).

[15] S. Gürpınar, K. Arslan and G. Öztürk, A characterization of constant-ratio curves in Euclidean 3-space $\mathbb{E}^3$, arXiv:1410.5577, (2014).

[16] K. Ilarslan and Ö. Boyacıoğlu, Position vectors of a spacelike W-curve in Minkowski space $\mathbb{E}_1^3$, *Bull. Korean Math. Soc.*, 46(2009), 967-978.

[17] K. Ilarslan, E. Nesovic and T.M. Petrovic, Some characterization of rectifying curves in the Minkowski 3-space, *Novi Sad J. Math.*, 32(2003), 23-32.

[18] K. Ilarslan and E. Nesovic, On rectifying curves as centrodes and extremal curves in the Minkowski 3-space $\mathbb{E}_1^3$, *Novi. Sad. J. Math.*, 37(2007), 53-64.

[19] M.K. Karacan and B. Bükcü, On natural curvatures of Bishop frame, *Journal of Vectorial Relativity*, 5(2010), 34-41.

[20] E. Özyılmaz, Classical differential geometry of curves according to type-2 Bishop trihedra, *Mathematical and Computational Applications*, 16(4) (2011), 858-867.

[21] D. Unal, I. Kisi and M. Tosun, Spinor Bishop equations of curves in Euclidean 3-space, *Adv. Appl. Clifford Algebras*, 23(2013), 757-765.

[22] S. Yılmaz, Position vectors of some special spacelike curves according to Bishop frame in Minkowski space $\mathbb{E}_1^3$, *Scientia Magna*, 5(2009), 47-49.

[23] A. Yücesan, N. Ayyıldız and A.C. Çöken, On rectifying dual space curves, *Rev. Math. Comp.*, 20(2007), 497-506.