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Interval Valued Fuzzy Generalized Semi-Preirresolute Mapping

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Abstract

In this paper, we study some of the properties of interval valued fuzzy generalized semi-preirresolute mappings.

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1 Introduction

The concept of a fuzzy subset was introduced and studied by L. A. Zadeh [8] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C. L. Chang [2] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Tapas Kumar Mondal and S. K. Samantha [5] introduced the topology of interval valued fuzzy sets. T. H. Yalvac introduced the notion of fuzzy irresolute mappings [7]. Prasad, Thakur and Saraf [6] have introduced fuzzy α-irresolute mappings in fuzzy topology. The present author introduced interval valued fuzzy generalized semi-preirresolute mappings and investigate the concept of interval valued fuzzy irresolute mappings and give some characterization and its properties.

2 Preliminaries

Definition 2.1 *[5] Let X be a non empty set and let $D[0,1]$ be the set of all closed subintervals of the closed interval $[0,1]$. Then a mapping $\bar{A} : X \to D[0,1]$ is called an interval valued fuzzy set (briefly $IVFS$) on X. For any $IVFS$ $\bar{A}$, denote $\bar{A}(x) = [A^-(x), A^+(x)]$, where $A^-(x) \leq A^+(x)$, $x \in X$. Then the two fuzzy sets $A^-(x) : X \to [0,1]$ and $A^+(x) : X \to [0,1]$ are called the lower fuzzy set and the upper fuzzy set of $\bar{A}$,respectively.*

Obviously any fuzzy set $\bar{A}$ on X is an $IVFS$ whose lower and upper fuzzy sets coincide with $\bar{A}$, i.e., $\bar{A}(x) = [\bar{A}(x), \bar{A}(x)]$ for all $x \in X$.

Notation 2.2 *The constant fuzzy set on X whose constant 0 is denoted by $\bar{0}$. $IVFSs$ $IVFS$ $\bar{0} = [0,0]$ and $\bar{1} = [1,1]$.*

Notation 2.3 *D^X denotes the set of all interval valued fuzzy subsets of a non empty set X.*

Definition 2.4 *[5] Let $\bar{A}$ and $\bar{B}$ be any two non empty $IVFS$ of X. Let $\bar{A} : X \to D[0,1]$ and $\bar{B} : X \to D[0,1]$ be a fuzzy subsets of X. Let $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$, and $\bar{B} = \{\langle x, [B^-(x), B^+(x)]\rangle : x \in X\} \in D^X$. We define the following relations and operations:*

(i) $\bar{A} \subseteq \bar{B}$ *if and only if* $A^-(x) \leq B^-(x)$ *and* $A^+(x) \leq B^+(x)$, *for all* $x \in X$.

(ii) $\bar{A} = \bar{B}$ *if and only if* $A^-(x) = B^-(x)$, *and* $A^+(x) = B^+(x)$ *for all* $x \in X$.

(iii) $(\bar{A})^c = \bar{1} - \bar{A} = \{\langle x, [1 - A^+(x), 1 - A^-(x)]\rangle : x \in X\}$.

(iv) $\bar{A} \cap \bar{B} = \{\langle x, [\min[A^-(x), B^-(x)], \min[A^+(x), B^+(x)]]\rangle : x \in X\}$.

(v) $\bar{A} \cup \bar{B} = \{\langle x, [\max[A^-(x), B^-(x)], \max[A^+(x), B^+(x)]]\rangle : x \in X\}$.

Definition 2.5 *[5] Let X be a set and $\Im$ be a family of interval valued fuzzy sets ($IVFSs$) of X. The family $\Im$ is called an interval valued fuzzy topology ($IVFT$) on X if and only if $\Im$ satisfies the following axioms:*

(i) $\bar{0}, \bar{1} \in \Im$,

(ii) If $\left\{\bar{A}_i : i \in I\right\} \subseteq \Im$, *then* $\bigcup_{i \in I} \bar{A}_i \in \Im$,

(iii) If $\bar{A}_1, \bar{A}_2, \bar{A}_3, \ldots, \bar{A}_n \in \Im$, *then* $\bigcap_{i=1}^{n} \bar{A}_i \in \Im$.

The pair $(X, \Im)$ is called an interval valued fuzzy topological space ($IVFTS$). The members of $\Im$ are called interval valued fuzzy open sets ($IVFOS$) in X.

An interval valued fuzzy set $\bar{A}$ in X is said to be interval valued fuzzy closed set ($IVFCS$) in X if and only if $(\bar{A})^c$ is an $IVFOS$ in X.

Definition 2.6 *[6] Let $(X, \Im)$ be an IVFTS and $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$ be an $IVFS$ in X. Then the interval valued fuzzy semi-preinterior and interval valued fuzzy semi-preclosure of $\bar{A}$ denoted by $ivfspint(\bar{A})$ and $ivfspcl(\bar{A})$ are defined by*

$$ivfspint(\bar{A}) = \bigcup \{\bar{G} : \bar{G} \text{ in an } IVFSPOS \text{ in } X \text{ and } \bar{G} \subseteq \bar{A}\},$$
$$ivfspcl(\bar{A}) = \bigcap \{\bar{K} : \bar{K} \text{ is an } IVFSPCS \text{ in } X \text{ and } \bar{A} \subseteq \bar{K}\}.$$

For any $IVFS\bar{A}$ in $(X, \Im)$, we have $ivfspcl(\bar{A}^c) = (ivfspint\ (\bar{A}))^c$ and $ivfspint(\bar{A}^c) = (ivfspcl(\bar{A}))^c$.

Definition 2.7 *[1] An $IVFS$ $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$ in an $IVFT$ $(X, \Im)$ is said to be an*

(i) interval valued fuzzy regular closed set ($IVFRCS$ for short) if $\bar{A} = ivfcl(ivfint(\bar{A}))$.

(ii) interval valued fuzzy semi closed set ($IVFSCS$) if $ivfint\ (ivfcl(\bar{A})) \subseteq \bar{A}$.

(iii) interval valued fuzzy preclosed set ($IVFPCS$) if $ivfcl(ivfint\ (\bar{A})) \subseteq \bar{A}$.

Definition 2.8 *An $IVFS$ $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$ in an $IVFT$ $(X, \Im)$ is said to be an*

(i) interval valued fuzzy generalized closed set ($IVFGCS$ for short) if $ivfcl(\bar{A}) = \bar{U}$, whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ in an $IVFOS$.

(ii) interval valued fuzzy regular generalized closed set ($IVFRGCS$ for short) if $ivfcl(\bar{A}) = \bar{U}$, whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ in an $IVFROS$.

Definition 2.9 *An $IVFS$ $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$ in an $IVFT$ $(X, \Im)$ is said to be an*

(i) interval valued fuzzy semi-preclosed set ($IVFSPCS$ for short) if there exist on $IVFPCS$ $\bar{B}$, such that $ivfint\bar{B} \subseteq \bar{A} \subseteq \bar{B}$.

(ii) interval valued fuzzy semi-preopen set ($IVFSPOS$ for short) if there exist on $IVFPOS$ $\bar{B}$, such that $\bar{B} \subseteq \bar{A} \subseteq ivfcl(\bar{B})$,

Definition 2.10 *[4] If every $IVFGSPCS$ in $(X,\Im)$ is an $IVFSPCS$ in $(X,\Im)$, then the space can be called as interval valued fuzzy semi-pre$T_{1/2}$ space ($IVFSPT_{1/2}$ space).*

Definition 2.11 *[3] An $IVFS$ $\bar{A}$ in $IVFTS$ $(X,\Im)$ is said to be an interval valued fuzzy generalized semi-preclosed set ($IVFGSPCS$ for short) if $ivfspcl(\bar{A}) \subseteq \bar{U}$ whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ is an $IVFOS$ in $(X,\Im)$.*

3 Main Results

Definition 3.1 *[3, 4] A Mapping $g : (X,\Im) \to (Y,\sigma)$ is called an interval valued fuzzy generalized semi-preirresolute ($IVFGSP$ irresolute) mapping if $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in $(X,\Im)$ for every $IVFGSPCS$ $\bar{V}$ of (Y,σ).*

Theorem 3.2 *If $g : (X,\Im) \to (Y,\sigma)$ is an $IVFGSP$ irresolute mapping, then g is an $IVFGSP$ continuous mapping.*

Proof. Let g be an $IVFGSP$ irresolute mapping. Let $\bar{V}$ be any $IVFCS$ in Y. Then $\bar{V}$ is an $IVFGSPCS$ and by hypothesis $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFGSP$ continuous mapping. □

Remark 3.3 *The converse of the above theorem 3.2 need not be true from the following example: Let $X = \{a,b\}$, $Y = \{u,v\}$ and*

$$\bar{K_1} = \{\langle a,[0.1,0.2]\rangle, \langle b,[0.3,0.4]\rangle\},$$
$$\bar{L_1} = \{\langle u,[0.8,0.9]\rangle, \langle v,[0.6,0.7]\rangle\}.$$

Then $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ and $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ are $IVFTs$ on X and Y respectively. Define a mapping $g : (X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSP$ continuous mapping but not an $IVFGSP$ irresolute mapping, since the $IVFS$ $\bar{A} = \{\langle a,[0.1,0.2]\rangle, \langle b,[0.3,0.4]\rangle\}$ is an $IVFGSPCS$ in Y, but $g^{-1}(\bar{A}) = \{\langle a,[0.1,0.2]\rangle, \langle b,[0.3,0.4]\rangle\} \subseteq \bar{K_1}$ is not an $IVFGSPCS$ in X, because $ivfspcl(g^{-1}(\bar{A})) = \bar{1} \not\subseteq \bar{K_1}$

Theorem 3.4 *Let $g : (X,\Im) \to (Y,\sigma)$ and $h : (Y,\sigma) \to (Z,\eta)$ be $IVFGSP$ irresolute mappings. Then $h \circ g : (X,\Im) \to (Y,\eta)$ is an $IVFGSP$ irresolute mapping.*

Proof. Let $\bar{V}$ be an $IVFGSPCS$ in Z. Then $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in Y. Since g is an $IVFGSP$ irresolute, $g^{-1}(h^{-1}(\bar{V}))$ is an $IVFGSPCS$ in X. Hence $h \circ g$ is an $IVFGSP$ irresolute mapping. □

Theorem 3.5 *Let $g : (X,\Im) \to (Y,\sigma)$ be an $IVFGSP$ irresolute mapping and $h : (Y,\sigma) \to (Z,\eta)$ be an $IVFGSP$ continuous mapping, then $h \circ g : (X,\Im) \to (Z,\eta)$ is an $IVFGSP$ continuous mapping.*

Proof. Let $\bar{V}$ be an $IVFCS$ in Z. Then $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in Y. Since g is an $IVFGSP$ irresolute mapping, $g^{-1}(h^{-1}(\bar{V}))$ is an $IVFGSPCS$ in X, by hypothesis. Hence $h \circ g$ is an $IVFGSP$ continuous mapping. □

Theorem 3.6 *Let $g : (X, \Im) \rightarrow (Y, \sigma)$ be a mapping from an $IVFT$ X into an $IVFT$ Y. Then the following condition are equivalent if X and Y are $IVFSPT_{1/2}$ spaces:*

(i) g is an $IVFGSP$ irresolute mapping,

(ii) $g^{-1}(\bar{B})$ is an $IVFGSPOS$ in X for each $IVFGSPOS$ in Y,

(iii) $g^{-1}(ivfspint(\bar{B})) \subseteq ivfspint(g^{-1}(\bar{B}))$ for each $IVFS$ $\bar{B}$ of Y,

(iv) $ivfspcl(g^{-1}(\bar{B})) \subseteq g^{-1}(ivfspcl(\bar{B}))$ for each $IVFS$ $\bar{B}$ of Y.

Proof. $(i) \Longleftrightarrow (ii)$ is obvious, since $g^{-1}(\bar{A}^c) = (g^{-1}(\bar{A}))^c$.
$(ii) \Rightarrow (iii)$ Let $\bar{B}$ be any $IVFS$ in Y and $ivfspint(\bar{B}) \subseteq \bar{B}$. Also $g^{-1}(ivf spint(\bar{B})) \subseteq g^{-1}(\bar{B})$. Since $ivfspint(\bar{B})$ is an $IVFSPOS$ in Y, it is an $IVFGSPOS$ in Y. Therefore $g^{-1}(ivfspint(\bar{B}))$ is an $IVFGSPOS$ in X, by hypothesis. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(ivfspint(\bar{B}))$ is an $IVFSPOS$ in X. Hence $g^{-1}(ivfspint(\bar{B})) = ivfspint(g^{-1}(ivfspint(\bar{B}))) \subseteq ivfspint(g^{-1}(\bar{B}))$.
$(iii) \Rightarrow (iv)$ is obvious by taking complement in (iii).
$(iv) \Rightarrow (i)$ Let $\bar{B}$ be an $IVFGSPCS$ in Y. Since Y is an $IVFSPT_{1/2}$ space, $\bar{B}$ is an $IVFSPCS$ in Y and $ivfspcl(\bar{B}) = \bar{B}$. Hence $g^{-1}(\bar{B}) = g^{-1}(ivfspcl(\bar{B})) \supseteq ivfspcl(g^{-1}(\bar{B}))$, by hypothesis. But $g^{-1}(\bar{B}) \subseteq ivf spcl(g^{-1}(\bar{B}))$. Therefore $ivfspcl(g^{-1}(\bar{B})) = g^{-1}(\bar{B})$. This implies $g^{-1}(\bar{B})$ is an $IVFSPCS$ and hence it is an $IVFGSPCS$ in X. Thus g is an $IVFGSP$ irresolute mapping. □

Theorem 3.7 *Let $g : (X, \Im) \rightarrow (Y, \sigma)$ be an $IVFGSP$ irresolute mapping from an $IVFT$ X into an $IVFT$ Y. Then $g^{-1}(\bar{B}) \subseteq ivfspint(g^{-1}(ivfcl(ivf int(ivfcl(\bar{B})))))$ for every $IVFGSPOS$ $\bar{B}$ in Y, if X and Y are $IVFSPT_{1/2}$ spaces.*

Proof. Let $\bar{B}$ be an $IVFGSPOS$ in Y. Then by hypothesis $g^{-1}(\bar{B})$ is an $IVFGSPOS$ in X. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(\bar{B})$ is an $IVFSPOS$ in X. Therefore $ivfspint(g^{-1}(\bar{B})) = g^{-1}(\bar{B})$. Since Y is an $IVFSPT_{1/2}$ space, $\bar{B}$ is an $IVFSPOS$ in Y and $\bar{B} \subseteq ivfcl(ivfint(ivfcl(\bar{B})))$. Now, $g^{-1}(\bar{B}) = ivfspint(g^{-1}(\bar{B}))$ implies, $g^{-1}(\bar{B}) \subseteq ivfspint(g^{-1}(ivfcl(ivfint(ivfcl(\bar{B})))))$. □

Theorem 3.8 *Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFGSP$ irresolute mapping from an $IVFT$ X into an $IVFT$ Y. Then $g^{-1}(\bar{B}) \subseteq ivfspint(ivfcl(ivfint(ivfcl(g^{-1}(\bar{B})))))$ for every $IVFGSPOS$ $\bar{B}$ in Y, if X is an $IVFSPT_{1/2}$ space.*

Proof. Let $\bar{B}$ be an $IVFGSPOS$ in Y. Then by hypothesis $g^{-1}(\bar{B})$ is an $IVFGSPOS$ in X. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(\bar{B})$ is an $IVFSPOS$ in X.Therefore $ivfspint(g^{-1}(\bar{B})) = g^{-1}(\bar{B})$ and $g^{-1}(\bar{B}) \subseteq ivfcl(ivfint(ivfcl(g^{-1}(\bar{B}))))$. Hence $g^{-1}(\bar{B}) \subseteq ivfspint(ivfcl(ivfint(ivfcl(g^{-1}(\bar{B})))))$. □

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